Answers

CHAPTER 1 Graphs and Functions

1.1 Assess Your Understanding *(page 6)*

5. abscissa; ordinate **6.** quadrants **7.** midpoint **8.** F **9.** F **10.** T

11. **(a)** Quadrant II **(d)** Quadrant I
(b) x-axis **(e)** y-axis
(c) Quadrant III **(f)** Quadrant IV

13. The points will be on a vertical line that is 2 units to the right of the y-axis.

15. $\sqrt{5}$

17. $\sqrt{10}$

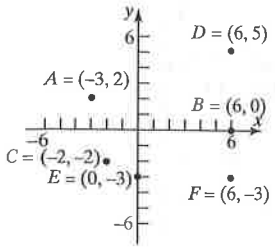
19. $2\sqrt{17}$

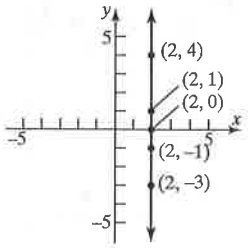
21. $\sqrt{85}$

23. $\sqrt{53}$

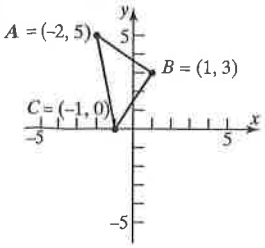
25. $\sqrt{6.89} \approx 2.62$

27. $\sqrt{a^2 + b^2}$

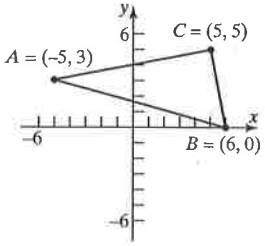




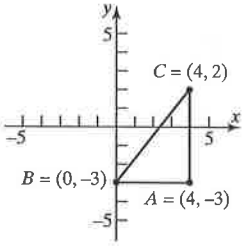
29. $d(A, B) = \sqrt{13}$
$d(B, C) = \sqrt{13}$
$d(A, C) = \sqrt{26}$
$(\sqrt{13})^2 + (\sqrt{13})^2 = (\sqrt{26})^2$
Area $= \frac{13}{2}$ square units



31. $d(A, B) = \sqrt{130}$
$d(B, C) = \sqrt{26}$
$d(A, C) = 2\sqrt{26}$
$(\sqrt{26})^2 + (2\sqrt{26})^2 = (\sqrt{130})^2$
Area = 26 square units



33. $d(A, B) = 4$
$d(B, C) = \sqrt{41}$
$d(A, C) = 5$
$4^2 + 5^2 = (\sqrt{41})^2$
Area = 10 square units



35. $(4, 0)$ **37.** $\left(\frac{3}{2}, 1\right)$ **39.** $(5, -1)$ **41.** $(1.05, 0.7)$ **43.** $\left(\frac{a}{2}, \frac{b}{2}\right)$ **45.** $(2, 2); (2, -4)$ **47.** $(0, 0); (8, 0)$ **49.** $(1, 2)$ **51.** $\sqrt{17}; 2\sqrt{5}; \sqrt{29}$

53. $\left(\frac{s}{2}, \frac{s}{2}\right)$ **55.** $d(P_1, P_2) = 6; d(P_2, P_3) = 4; d(P_1, P_3) = 2\sqrt{13}$; right triangle **57.** $d(P_1, P_2) = 2\sqrt{17}; d(P_2, P_3) = \sqrt{34}; d(P_1, P_3) = \sqrt{34}$; isosceles right triangle **59.** $90\sqrt{2} \approx 127.28$ ft **61.** **(a)** $(90, 0), (90, 90), (0, 90)$ **(b)** $5\sqrt{2161} \approx 232.43$ ft **(c)** $30\sqrt{149} \approx 366.20$ ft

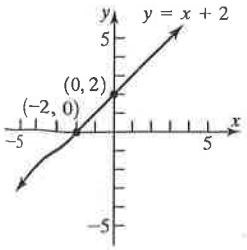
63. $d = 50t$ mi **65.** **(a)** $(2.65, 1.6)$ **(b)** Approximately 1.285 units

1.2 Assess Your Understanding *(page 19)*

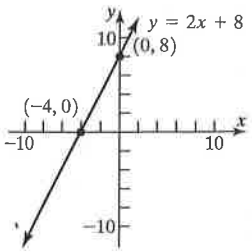
3. intercepts **4.** y-axis **5.** $(-3, 4)$ **6.** T **7.** F **8.** radius **9.** T **10.** F **11.** $(0, 0)$ is on the graph. **13.** $(0, 3)$ is on the graph.

15. $(0, 2)$ and $(\sqrt{2}, \sqrt{2})$ are on the graph.

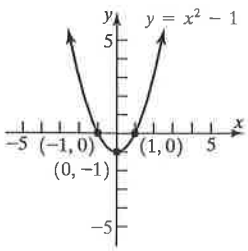
17. $(-2, 0), (0, 2)$



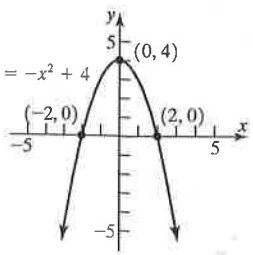
19. $(-4, 0), (0, 8)$



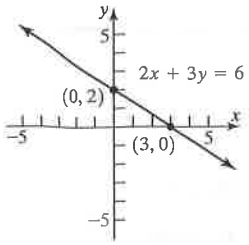
21. $(-1, 0), (1, 0), (0, -1)$



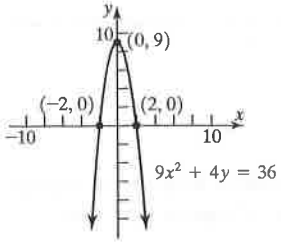
23. $(-2, 0), (2, 0), (0, 4)$



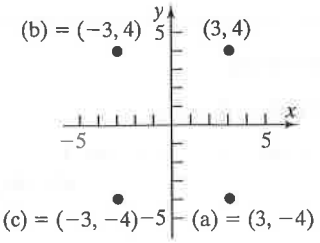
25. (3, 0), (0, 2)



27. (−2, 0), (2, 0), (0, 9)



29.



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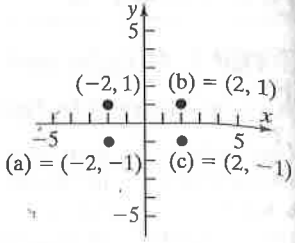
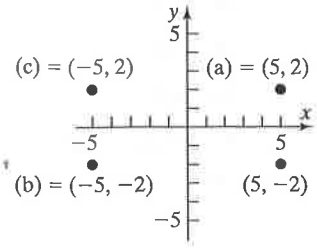


33.



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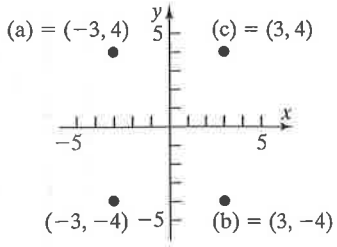
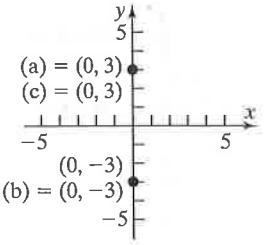


37.



39. **(a)** (−1, 0), (1, 0)

(b) Symmetric with respect to the x-axis, the y-axis, and the origin

41. **(a)** $\left(-\frac{\pi}{2}, 0\right)$, (0, 1), $\left(\frac{\pi}{2}, 0\right)$

(b) Symmetric with respect to the y-axis

43. **(a)** (0, 0)

(b) Symmetric with respect to the x-axis

45. **(a)** (−2, 0), (0, 0), (2, 0)

(b) Symmetric with respect to the origin

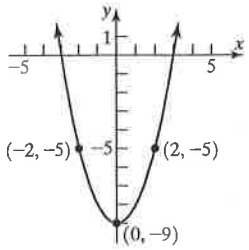
47. **(a)** (−1, 0), (0, −1), (1, 0)

(b) Symmetric with respect to the y-axis

49. **(a)** No intercepts

(b) Symmetric with respect to the origin

51.



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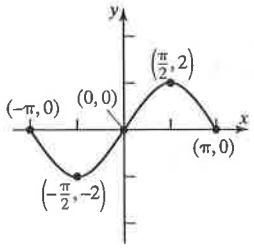
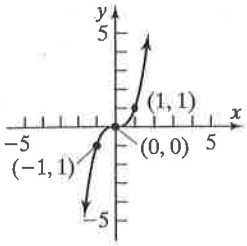
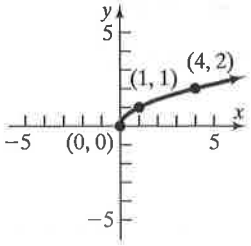


55. (−4, 0), (0, −2), (0, 2); symmetric with respect to the x-axis **57.** (0, 0); symmetric with respect to the origin **59.** (0, 9), (3, 0), (−3, 0); symmetric with respect to the y-axis **61.** (−2, 0), (2, 0), (0, −3), (0, 3); symmetric with respect to the x-axis, y-axis, and origin **63.** (0, −27), (3, 0); no symmetry **65.** (0, −4), (4, 0), (−1, 0); no symmetry **67.** (0, 0); symmetric with respect to the origin **69.** (0, 0); symmetric with respect to the origin

71.



73.



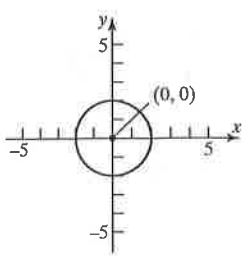
75. $b = 13$

77. $a = -4$ or $a = 1$

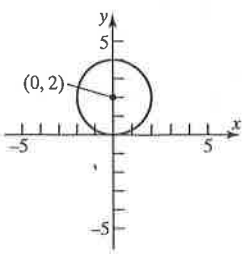
79. Center (2, 1); radius = 2; $(x-2)^2 + (y-1)^2 = 4$

81. Center $\left(\frac{5}{2}, 2\right)$; radius $\frac{3}{2}$; $\left(x-\frac{5}{2}\right)^2 + (y-2)^2 = \frac{9}{4}$

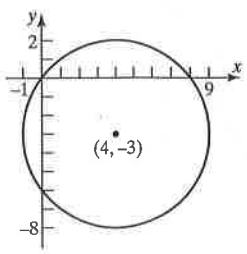
83. $x^2 + y^2 = 4$;
$x^2 + y^2 - 4 = 0$



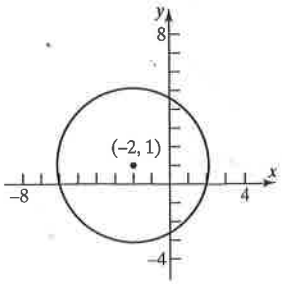
85. $x^2 + (y-2)^2 = 4$;
$x^2 + y^2 - 4y = 0$



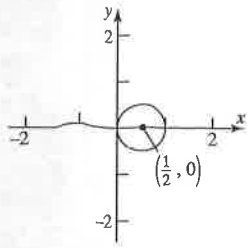
87. $(x-4)^2 + (y+3)^2 = 25$;
$x^2 + y^2 - 8x + 6y = 0$



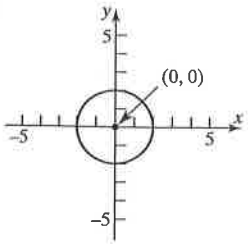
89. $(x+2)^2 + (y-1)^2 = 16$;
$x^2 + y^2 + 4x - 2y - 11 = 0$



91. $\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}$; $x^2 + y^2 - x = 0$

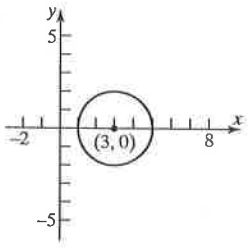


93. **(a)** $(h, k) = (0, 0); r = 2$ **(b)**



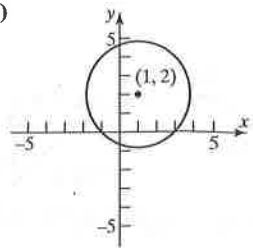
(c) $(\pm 2, 0); (0, \pm 2)$

95. **(a)** $(h, k) = (3, 0); r = 2$ **(b)**



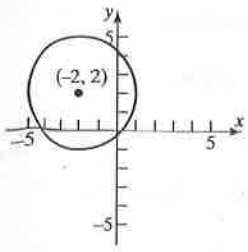
(c) $(1, 0); (5, 0)$

97. **(a)** $(h, k) = (1, 2); r = 3$ **(b)**



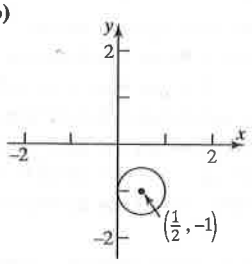
(c) $\left(1 \pm \sqrt{5}, 0\right); \left(0, 2 \pm 2\sqrt{2}\right)$

99. **(a)** $(h, k) = (-2, 2); r = 3$ **(b)**



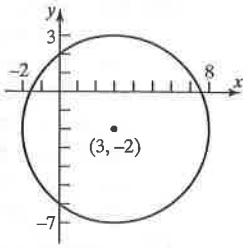
(c) $\left(-2 \pm \sqrt{5}, 0\right); \left(0, 2 \pm \sqrt{5}\right)$

101. **(a)** $(h, k) = \left(\frac{1}{2}, -1\right); r = \frac{1}{2}$ **(b)**



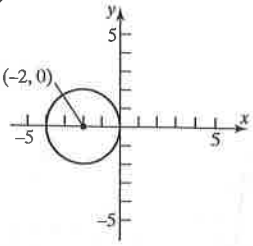
(c) $(0, -1)$

103. **(a)** $(h, k) = (3, -2); r = 5$ **(b)**



(c) $\left(3 \pm \sqrt{21}, 0\right); (0, -6), (0, 2)$

105. **(a)** $(h, k) = (-2, 0); r = 2$ **(b)**



(c) $(0, 0), (-4, 0)$

107. $x^2 + y^2 = 13$ **109.** $(x - 2)^2 + (y - 3)^2 = 9$ **111.** $(x + 1)^2 + (y - 3)^2 = 5$ **113.** $(x + 1)^2 + (y - 3)^2 = 1$ **115.** $(-1, -2)$ **117.** 4

119. 18 units^2 **121.** $x^2 + (y - 139)^2 = 15{,}625$ **123.** $x^2 + y^2 + 2x + 4y - 4168.16 = 0$ **129.** (b), (c), (e), (g)

1.3 Assess Your Understanding *(page 35)*

5. independent; dependent **6.** range **7.** F **8.** T **9.** F **10.** F **11.** vertical **12.** 5; −3 **13.** $a = -2$ **14.** F **15.** F **16.** T

17. Function; Domain: {Elvis, Colleen, Kaleigh, Marissa}, Range: {January 8, March 15, September 17} **19.** Not a function **21.** Not a function

23. Function; Domain: $\{1, 2, 3, 4\}$; Range: $\{3\}$ **25.** Not a function **27.** Function; Domain: $\{-2, -1, 0, 1\}$, Range: $\{0, 1, 4\}$ **29.** Function

31. Function **33.** Not a function **35.** Not a function **37.** Function **39.** Not a function **41.** **(a)** -4 **(b)** 1 **(c)** -3 **(d)** $3x^2 - 2x - 4$ **(e)** $-3x^2 - 2x + 4$ **(f)** $3x^2 + 8x + 1$ **(g)** $12x^2 + 4x - 4$ **(h)** $3x^2 + 6xh + 3h^2 + 2x + 2h - 4$

43. **(a)** 0 **(b)** $\frac{1}{2}$ **(c)** $-\frac{1}{2}$ **(d)** $\frac{-x}{x^2 + 1}$ **(e)** $\frac{-x}{x^2 + 1}$ **(f)** $\frac{x + 1}{x^2 + 2x + 2}$ **(g)** $\frac{2x}{4x^2 + 1}$ **(h)** $\frac{x + h}{x^2 + 2xh + h^2 + 1}$

45. **(a)** 4 **(b)** 5 **(c)** 5 **(d)** $|x| + 4$ **(e)** $-|x| - 4$ **(f)** $|x + 1| + 4$ **(g)** $2|x| + 4$ **(h)** $|x + h| + 4$

47. **(a)** $-\frac{1}{5}$ **(b)** $-\frac{3}{2}$ **(c)** $\frac{1}{8}$ **(d)** $\frac{2x - 1}{3x + 5}$ **(e)** $\frac{-2x - 1}{3x - 5}$ **(f)** $\frac{2x + 3}{3x - 2}$ **(g)** $\frac{4x + 1}{6x - 5}$ **(h)** $\frac{2x + 2h + 1}{3x + 3h - 5}$ **49.** All real numbers

51. All real numbers **53.** $\{x | x \neq -4, x \neq 4\}$ **55.** $\{x | x \neq 0\}$ **57.** $\{x | x \geq 4\}$ **59.** $\{x | x > 9\}$ **61.** 4 **63.** $2x + h - 1$ **65.** $6x + 3h - 2$

67. $3x^2 + 3xh + h^2$ **69.** **(a)** $f(0) = 3; f(-6) = -3$ **(b)** $f(6) = 0; f(11) = 1$ **(c)** Positive **(d)** Negative **(e)** -3, 6, and 10

(f) $-3 < x < 6; 10 < x \leq 11$ **(g)** $\{x | -6 \leq x \leq 11\}$ **(h)** $\{y | -3 \leq y \leq 4\}$ **(i)** $-3, 6, 10$ **(j)** 3 **(k)** 3 times **(l)** Once **(m)** 0, 4 **(n)** $-5, 8$

71. Not a function **73.** Function **(a)** Domain: $\{x | -\pi \leq x \leq \pi\}$; Range: $\{y | -1 \leq y \leq 1\}$ **(b)** $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 0\right), (0, 1)$ **(c)** y-axis

75. Not a function **77.** Function **(a)** Domain: $\{x | x > 0\}$; Range: all real numbers **(b)** $(1, 0)$ **(c)** None

79. Function **(a)** Domain: all real numbers; Range: $\{y | y \leq 2\}$ **(b)** $(-3, 0), (3, 0), (0, 2)$ **(c)** y-axis **81.** Function **(a)** Domain: all real numbers; Range: $\{y | y \geq -3\}$ **(b)** $(1, 0), (3, 0), (0, 9)$ **(c)** None **83.** **(a)** Yes **(b)** $f(-2) = 9; (-2, 9)$ **(c)** $0, \frac{1}{2}; (0, -1), \left(\frac{1}{2}, -1\right)$ **(d)** All real numbers

(e) $-\frac{1}{2}, 1$ **(f)** -1 **85.** **(a)** No **(b)** $f(4) = -3; (4, -3)$ **(c)** $14; (14, 2)$ **(d)** $\{x | x \neq 6\}$ **(e)** -2 **(f)** $-\frac{1}{3}$ **87.** **(a)** Yes **(b)** $f(2) = \frac{8}{17}; \left(2, \frac{8}{17}\right)$

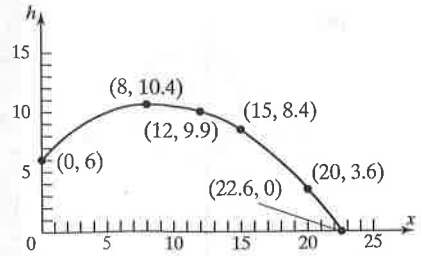
(c) $-1, 1; (-1, 1), (1, 1)$ **(d)** All real numbers **(e)** 0 **(f)** 0 **89.** $A = -\frac{7}{2}$ **91.** $A = -4$ **93.** $A = 8$; undefined at $x = 3$ **95.** $A(x) = \frac{1}{2}x^2$

97. $G(x) = 10x$ **99.** **(a)** P is the dependent variable; a is the independent variable. **(b)** $P(20) = 197.34$ million; In 2005, there are 197.34 million people 20 years of age or older. **(c)** $P(0) = 290.580$ million; In 2005, there are 290.580 million people. **101.** **(a)** 15.1 m, 14.071 m, 12.944 m, 11.719 m

(b) 1.01 sec, 1.43 sec, 1.75 sec **(c)** 2.02 sec **103.** **(a)** \$222 **(b)** \$225 **(c)** \$220 **(d)** \$230

105. **(a)** Approximately 10.4 ft high **(b)** Approximately 9.9 ft high

(c)

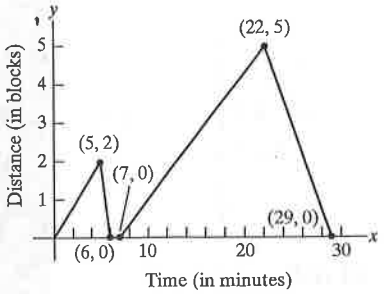


(d) The ball will not go through the hoop; $h(15) \approx 8.4$ ft.
If $v = 30$ ft/sec, $h(15) = 10$ ft.

107. Only $h(x) = 2x$ **109.** The x-intercepts can number anywhere from 0 to infinitely many. There is at most one y-intercept.

111. **(a)** III **(b)** IV **(c)** I **(d)** V **(e)** II

113.



115. **(a)** 2 hr elapsed during which Kevin was between 0 and 3 mi from home.
(b) 0.5 hr elapsed during which Kevin was 3 mi from home.
(c) 0.3 hr elapsed during which Kevin was between 0 and 3 mi from home.
(d) 0.2 hr elapsed during which Kevin was 0 mi from home.
(e) 0.9 hr elapsed during which Kevin was between 0 and 2.8 mi from home.
(f) 0.3 hr elapsed during which Kevin was 2.8 mi from home.
(g) 1.1 hr elapsed during which Kevin was between 0 and 2.8 mi from home.
(h) 3 mi **(i)** 2 times

117. No points whose x-coordinate is 5 or whose y-coordinate is 0 can be on the graph.

1.4 Assess Your Understanding *(page 47)*

4. increasing **5.** even; odd **6.** T **7.** T **8.** F **9.** Yes **11.** No **13.** $(-8, -2)$; $(0, 2)$; $(5, \infty)$ **15.** Yes; 10 **17.** $-2, 2; 6, 10$

19. **(a)** $(-2, 0)$, $(0, 3)$, $(2, 0)$ **(b)** Domain: $\{x|-4 \le x \le 4\}$ or $[-4, 4]$; Range: $\{y|0 \le y \le 3\}$ or $[0, 3]$
(c) Increasing on $(-2, 0)$ and $(2, 4)$; Decreasing on $(-4, -2)$ and $(0, 2)$ **(d)** Even

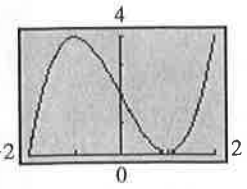
21. **(a)** $(0, 1)$ **(b)** Domain: all real numbers; Range: $\{y|y > 0\}$ or $(0, \infty)$ **(c)** Increasing on $(-\infty, \infty)$ **(d)** Neither

23. **(a)** $(-\pi, 0)$, $(0, 0)$, $(\pi, 0)$ **(b)** Domain: $\{x|-\pi \le x \le \pi\}$ or $[-\pi, \pi]$; Range: $\{y|-1 \le y \le 1\}$ or $[-1, 1]$
(c) Increasing on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; Decreasing on $\left(-\pi, -\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \pi\right)$ **(d)** Odd

25. **(a)** $\left(0, \frac{1}{2}\right), \left(\frac{1}{3}, 0\right), \left(\frac{5}{2}, 0\right)$ **(b)** Domain: $\{x|-3 \le x \le 3\}$ or $[-3, 3]$; Range: $\{y|-1 \le y \le 2\}$ or $[-1, 2]$
(c) Increasing on $(2, 3)$; Decreasing on $(-1, 1)$; Constant on $(-3, -1)$ and $(1, 2)$ **(d)** Neither **27.** **(a)** 0; 3 **(b)** $-2, 2; 0, 0$

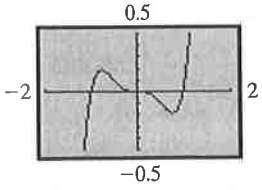
29. **(a)** $\frac{\pi}{2}$; 1 **(b)** $-\frac{\pi}{2}$; -1 **31.** Odd **33.** Even **35.** Odd **37.** Neither **39.** Even **41.** Odd

43.



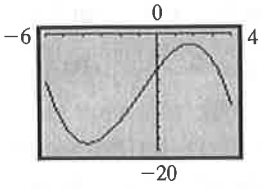
Increasing: $(-2, -1)$, $(1, 2)$
Decreasing: $(-1, 1)$
Local maximum: $(-1, 4)$
Local minimum: $(1, 0)$

45.



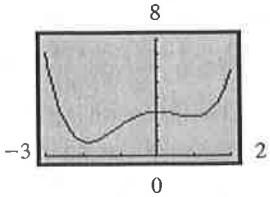
Increasing: $(-2, -0.77)$, $(0.77, 2)$
Decreasing: $(-0.77, 0.77)$
Local maximum: $(-0.77, 0.19)$
Local minimum: $(0.77, -0.19)$

47.



Increasing: $(-3.77, 1.77)$
Decreasing: $(-6, -3.77)$, $(1.77, 4)$
Local maximum: $(1.77, -1.91)$
Local minimum: $(-3.77, -18.89)$

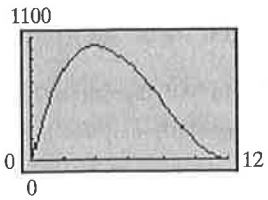
49.



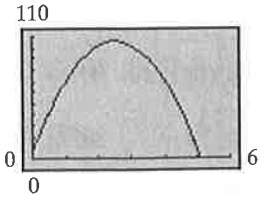
Increasing: $(-1.87, 0)$, $(0.97, 2)$
Decreasing: $(-3, -1.87)$, $(0, 0.97)$
Local maximum: $(0, 3)$
Local minima: $(-1.87, 0.95)$, $(0.97, 2.65)$

51. **(a)** -4 **(b)** -8 **(c)** -10 **53.** **(a)** 17 **(b)** -1 **(c)** 11 **55.** **(a)** 5 **(b)** $y = 5x - 2$ **57.** **(a)** -1; **(b)** $y = -x$ **59.** **(a)** 4 **(b)** $y = 4x - 8$

61. **(a)** $V(x) = x(24 - 2x)^2$
(b) 972 in.^{3} **(c)** 160 in.^{3}
(d) V is largest when $x = 4$.

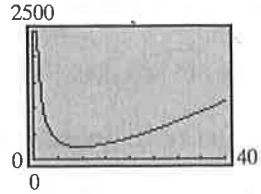


63. **(a)**



(b) 2.5 sec
(c) 106 ft

65. **(a)**



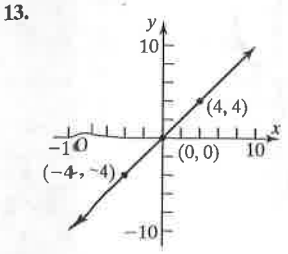
(b) 10 riding lawn mowers/hr
(c) \$239/mower

69. At most one **71.** Yes; the function $f(x) = 0$ is both even and odd.

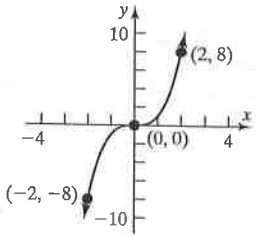
1.5 Assess Your Understanding *(page 55)*

1. piecewise-defined **2.** T **3.** F **4.** F **5.** C **7.** E **9.** B **11.** F

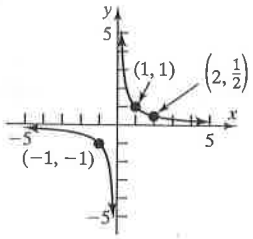
13.



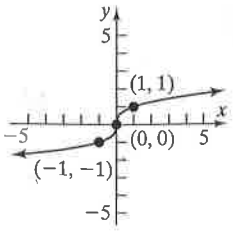
15.



17.

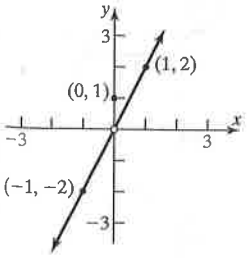


19.



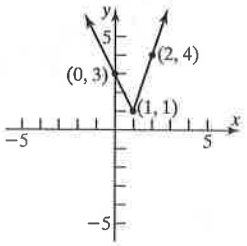
21. (a) 4 (b) 2 (c) 5 **23.** (a) −4 (b) −2 (c) 0 (d) 25

25. (a) All real numbers
(b) (0, 1)
(c)



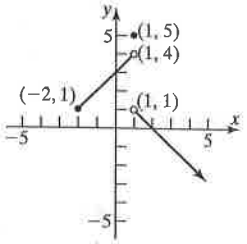
(d) $\{y|y \neq 0\}; (-\infty, 0) \cup (0, \infty)$

27. (a) All real numbers
(b) (0, 3)
(c)



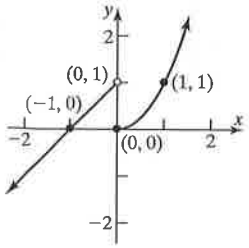
(d) $\{y|y \geq 1\}; [1, \infty)$

29. (a) $\{x|x \geq -2\}; [-2, \infty)$
(b) (0, 3), (2, 0)
(c)



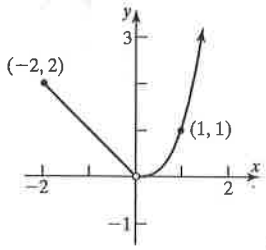
(d) $\{y|y < 4, y = 5\}; (-\infty, 4) \cup \{5\}$

31. (a) All real numbers
(b) (−1, 0), (0, 0)
(c)



(d) All real numbers

33. (a) $\{x|x \geq -2, x \neq 0\}; [-2, 0) \cup (0, \infty)$
(b) No intercepts
(c)



(d) $\{y|y > 0\}; (0, \infty)$

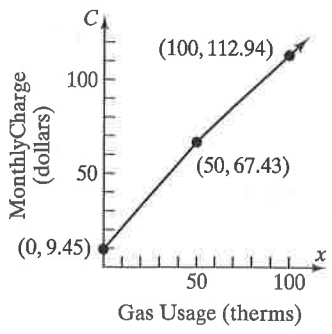
35. $f(x) = \begin{cases} -x & \text{if } -1 \leq x \leq 0 \\ \frac{1}{2}x & \text{if } 0 < x \leq 2 \end{cases}$ (Other answers are possible.)

37. $f(x) = \begin{cases} -x & \text{if } x \leq 0 \\ -x + 2 & \text{if } 0 < x \leq 2 \end{cases}$ (Other answers are possible.)

39. (a) \$35 (b) \$61 (c) \$35.40

41. (a) \$67.43
(b) \$477.04
(c) $C = \begin{cases} 1.15955x + 9.45 & \text{if } 0 \leq x \leq 50 \\ 0.91025x + 21.915 & \text{if } x > 50 \end{cases}$

(d)



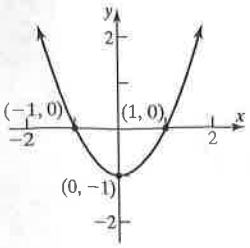
43. Each graph is that of $y = x^2$, but shifted vertically. If $y = x^2 + k, k > 0$, the shift is up k units; if $y = x^2 - k, k > 0$, the shift is down k units. **45.** Each graph is that of $y = |x|$, but either compressed or stretched. If $y = k|x|$ and $k > 1$, the graph is stretched vertically; if $y = k|x|, 0 < k < 1$, the graph is compressed vertically. **47.** The graph of $y = f(-x)$ is the reflection about the y-axis of the graph of $y = f(x)$. **49.** They are all U-shaped and open upward. All three go through the points $(-1, 1), (0, 0)$ and $(1, 1)$. As the exponent increases, the steepness of the curve increases (except near $x = 0$). **51.** y is a function. Domain: all real numbers; $(-\infty, \infty)$; range: $\{y|y = 0$ or $y = 1\}$; y-intercept: $(0, 1)$; x-intercepts: $\{(x, 0)\ x$ is an irrational number$\}$. The function is even.

1.6 Assess Your Understanding *(page 67)*

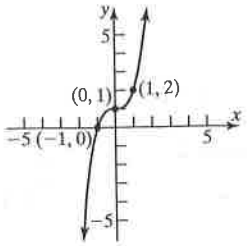
1. horizontal; right **2.** y **3.** $-5; -2; 2$ **4.** T **5.** F **6.** T **7.** B **9.** H **11.** I **13.** L **15.** F **17.** G **19.** $y = (x - 4)^3$ **21.** $y = x^3 + 4$ **23.** $y = -x^3$ **25.** $y = 4x^3$ **27.** (1) $y = \sqrt{x} + 2$; (2) $y = -(\sqrt{x} + 2)$; (3) $y = -(\sqrt{-x} + 2)$

29. (1) $y = -\sqrt{x}$; (2) $y = -\sqrt{x} + 2$; (3) $y = -\sqrt{x+3} + 2$ **31.** (c) **33.** (c)

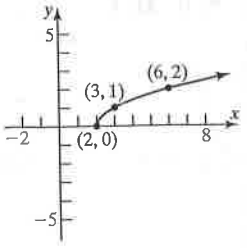
35.



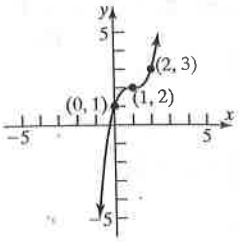
37.



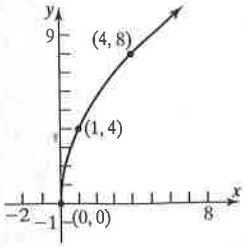
39.



41.



43.



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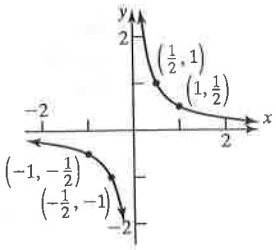
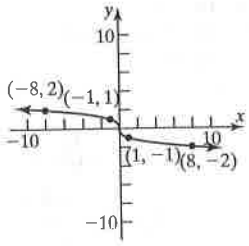
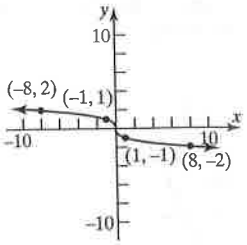


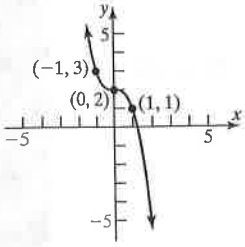
47.



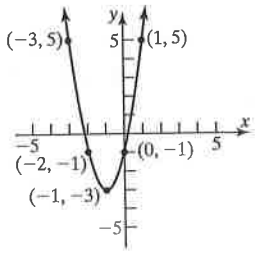
49.



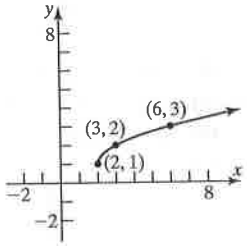
51.



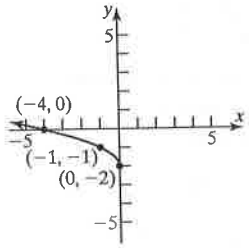
53.



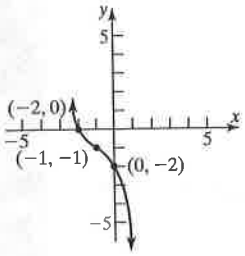
55.



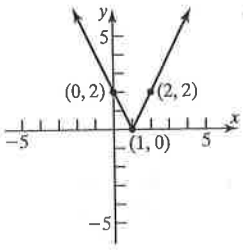
57.



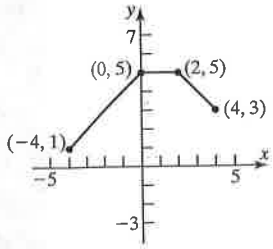
59.



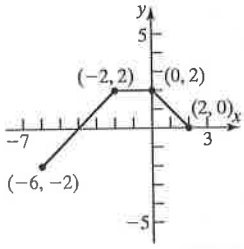
61.



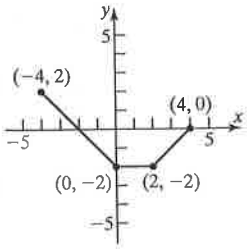
63. (a) $F(x) = f(x) + 3$



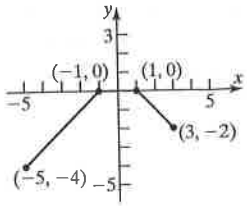
(b) $G(x) = f(x + 2)$



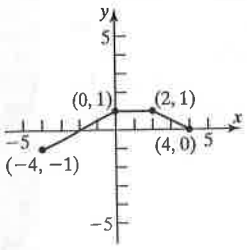
(c) $P(x) = -f(x)$



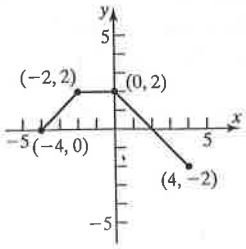
(d) $H(x) = f(x + 1) - 2$



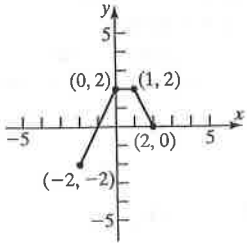
(e) $Q(x) = \frac{1}{2}f(x)$



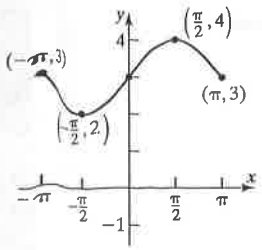
(f) $g(x) = f(-x)$



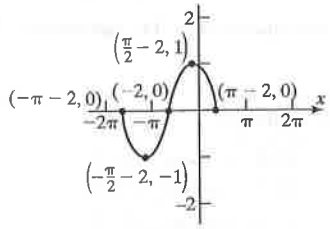
(g) $h(x) = f(2x)$



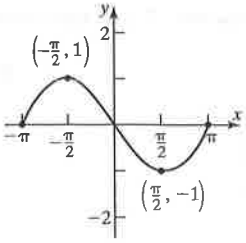
65. **(a)** $F(x) = f(x) + 3$



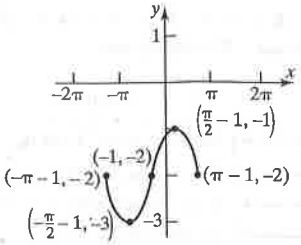
(b) $G(x) = f(x + 2)$



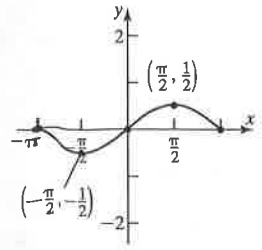
(c) $P(x) = -f(x)$



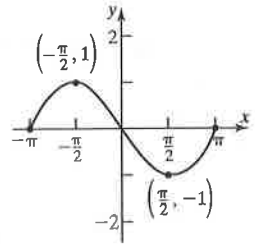
(d) $H(x) = f(x + 1) - 2$



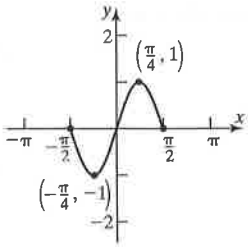
(e) $Q(x) = \frac{1}{2}f(x)$



(f) $g(x) = f(-x)$

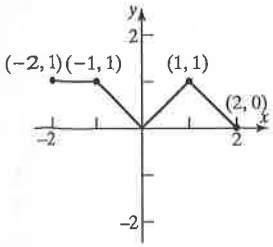


(g) $h(x) = f(2x)$

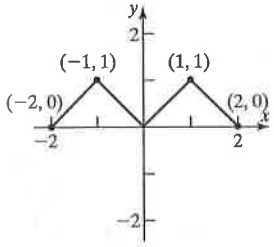


67. **(a)** -7 and 1 **(b)** -3 and 5 **(c)** -5 and 3 **(d)** -3 and 5 **69.** **(a)** $(-3, 3)$ **(b)** $(4, 10)$ **(c)** Decreasing on $(-1, 5)$ **(d)** Decreasing on $(-5, 1)$

71. **(a)**

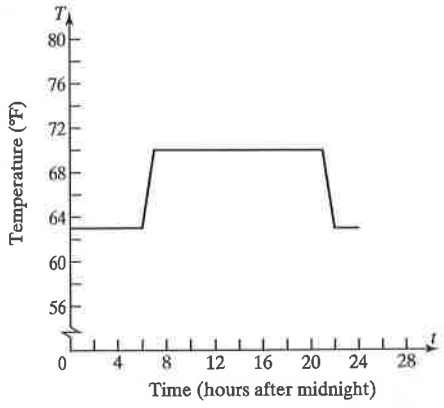


(b)

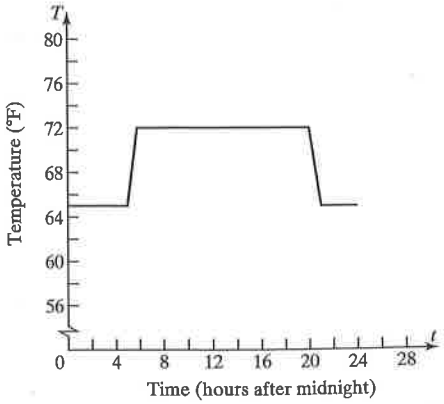


73. **(a)** 72°F; 65°F

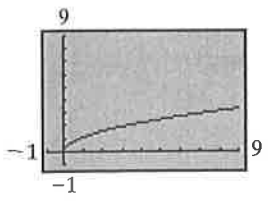
(b) The temperature decreases by 2° to 70°F during the day and 63°F overnight.



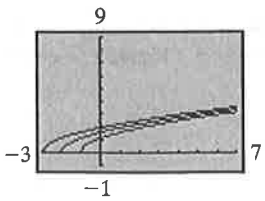
(c) The time at which the temperature adjusts between the daytime and overnight settings is moved to 1 hr sooner. It begins warming up at 5:00 AM instead of 6:00 AM, and it begins cooling down at 8:00 PM instead of 9:00 PM.



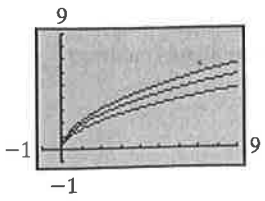
75. **(a)**



(b)



(d)



77. The graph of $y = 4f(x)$ is a vertical stretch by a factor of 4. The graph of $y = f(4x)$ is a horizontal compression by a factor of $\frac{1}{4}$.

1.7 Assess Your Understanding *(page 80)*

1. one-to-one **2.** $y = x$ **3.** $[4, \infty)$ **4.** F **5.** one-to-one **7.** Not one-to-one **9.** Not one-to-one **11.** one-to-one **13.** one-to-one **15.** Not one-to-one **17.** one-to-one

19.

Annual Rainfall	Location
460.00	Mt Waialeale, Hawaii
202.01	Monrovia, Liberia
196.46	Pago Pago, American Samoa
191.02	Moulmein, Burma
182.87	Lae, Papua New Guinea

Domain: {460.00, 202.01, 196.46, 191.02, 182.87}
Range: {Mt Waialeale, Monrovia, Pago Pago, Moulmein, Lae}

21.

Monthly Cost of Life Insurance	Age
\$7.09	30
\$8.40	40
\$11.29	45

Domain: {\$7.09, \$8.40, \$11.29}
Range: {30, 40, 45}

23. $\{(5, -3), (9, -2), (2, -1), (11, 0), (-5, 1)\}$

Domain: $\{5, 9, 2, 11, -5\}$
Range: $\{-3, -2, -1, 0, 1\}$

25. $\{(1, -2), (2, -3), (0, -10), (9, 1), (4, 2)\}$

Domain: $\{1, 2, 0, 9, 4\}$
Range: $\{-2, -3, -10, 1, 2\}$

27. $f(g(x)) = f\left(\frac{1}{3}(x-4)\right) = 3\left[\frac{1}{3}(x-4)\right] + 4$
$= (x-4) + 4 = x$

$g(f(x)) = g(3x+4) = \frac{1}{3}[(3x+4) - 4] = \frac{1}{3}(3x) = x$

29. $f(g(x)) = f\left(\frac{x}{4} + 2\right) = 4\left[\frac{x}{4} + 2\right] - 8 = (x+8) - 8 = x$

$g(f(x)) = g(4x-8) = \frac{4x-8}{4} + 2 = (x-2) + 2 = x$

31. $f(g(x)) = f\left(\sqrt[3]{x+8}\right) = \left(\sqrt[3]{x+8}\right)^3 - 8 = (x+8) - 8 = x$

$g(f(x)) = g(x^3 - 8) = \sqrt[3]{(x^3-8)+8} = \sqrt[3]{x^3} = x$

33. $f(g(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\left(\frac{1}{x}\right)} = x$; $g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{\left(\frac{1}{x}\right)} = x$

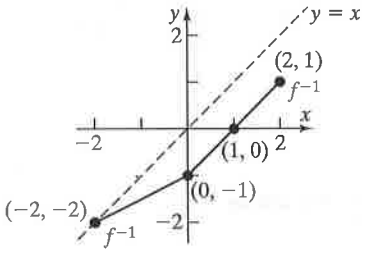
35. $f(g(x)) = f\left(\frac{4x-3}{2-x}\right) = \frac{2\left(\frac{4x-3}{2-x}\right) + 3}{\frac{4x-3}{2-x} + 4}$

$= \frac{2(4x-3) + 3(2-x)}{4x - 3 + 4(2-x)} = \frac{5x}{5} = x$

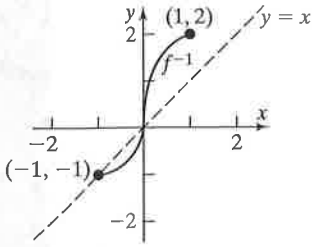
$g(f(x)) = g\left(\frac{2x+3}{x+4}\right) = \frac{4\left(\frac{2x+3}{x+4}\right) - 3}{2 - \frac{2x+3}{x+4}}$

$= \frac{4(2x+3) - 3(x+4)}{2(x+4) - (2x+3)} = \frac{5x}{5} = x$

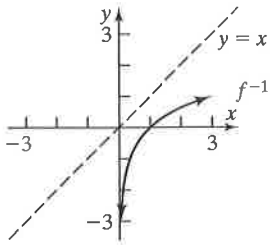
37.



39.



41.

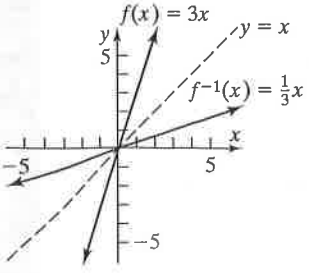


43. $f^{-1}(x) = \frac{1}{3}x$

$f(f^{-1}(x)) = f\left(\frac{1}{3}x\right) = 3\left(\frac{1}{3}x\right) = x$

$f^{-1}(f(x)) = f^{-1}(3x) = \frac{1}{3}(3x) = x$

Domain f = range f^{-1} = all real numbers
Range f = domain f^{-1} = all real numbers

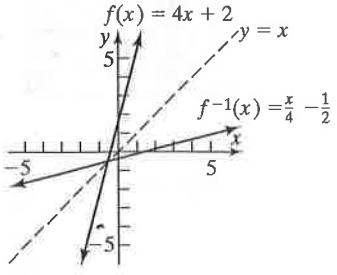


45. $f^{-1}(x) = \frac{x}{4} - \frac{1}{2}$

$f(f^{-1}(x)) = f\left(\frac{x}{4} - \frac{1}{2}\right) = 4\left(\frac{x}{4} - \frac{1}{2}\right) + 2 = (x-2) + 2 = x$

$f^{-1}(f(x)) = f^{-1}(4x+2) = \frac{4x+2}{4} - \frac{1}{2} = \left(x + \frac{1}{2}\right) - \frac{1}{2} = x$

Domain f = range f^{-1} = all real numbers
Range f = domain f^{-1} = all real numbers



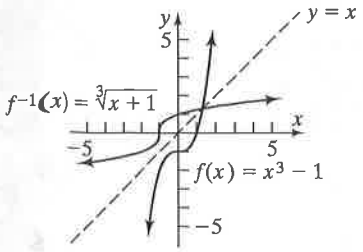
47. $f^{-1}(x) = \sqrt[3]{x+1}$

$f(f^{-1}(x)) = f\left(\sqrt[3]{x+1}\right) = \left(\sqrt[3]{x+1}\right)^3 - 1 = x$

$f^{-1}(f(x)) = f^{-1}(x^3 - 1) = \sqrt[3]{(x^3-1)+1} = x$

Domain f = range f^{-1} = all real numbers

Range f = domain f^{-1} = all real numbers



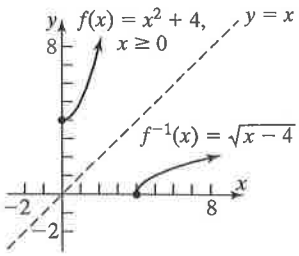
49. $f^{-1}(x) = \sqrt{x-4}, x \geq 4$

$f(f^{-1}(x)) = f\left(\sqrt{x-4}\right) = \left(\sqrt{x-4}\right)^2 + 4 = x$

$f^{-1}(f(x)) = f^{-1}(x^2+4) = \sqrt{(x^2+4)-4} = \sqrt{x^2} = x, x \geq 0$

Domain f = range f^{-1} = $\{x | x \geq 0\}$ or $[0, \infty)$

Range f = domain f^{-1} = $\{x | x \geq 4\}$ or $[4, \infty)$



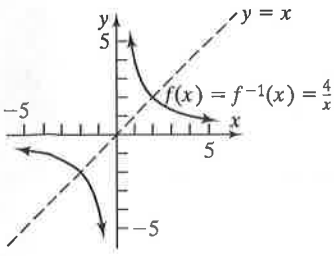
51. $f^{-1}(x) = \dfrac{4}{x}$

$f(f^{-1}(x)) = f\left(\dfrac{4}{x}\right) = \dfrac{4}{\left(\frac{4}{x}\right)} = x$

$f^{-1}(f(x)) = f^{-1}\left(\dfrac{4}{x}\right) = \dfrac{4}{\left(\frac{4}{x}\right)} = x$

Domain f = range f^{-1} = all real numbers except 0

Range f = domain f^{-1} = all real numbers except 0



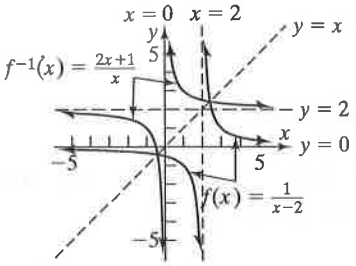
53. $f^{-1}(x) = \dfrac{2x+1}{x}$

$f(f^{-1}(x)) = f\left(\dfrac{2x+1}{x}\right) = \dfrac{1}{\frac{2x+1}{x} - 2} = \dfrac{x}{(2x+1)-2x} = x$

$f^{-1}(f(x)) = f^{-1}\left(\dfrac{1}{x-2}\right) = \dfrac{2\left(\frac{1}{x-2}\right)+1}{\frac{1}{x-2}} = \dfrac{2+(x-2)}{1} = x$

Domain f = range f^{-1} = all real numbers except 2

Range f = domain f^{-1} = all real numbers except 0



55. $f^{-1}(x) = \dfrac{2-3x}{x}$

$f(f^{-1}(x)) = f\left(\dfrac{2-3x}{x}\right) = \dfrac{2}{3 + \frac{2-3x}{x}}$

$= \dfrac{2x}{3x+2-3x} = \dfrac{2x}{2} = x$

$f^{-1}(f(x)) = f^{-1}\left(\dfrac{2}{3+x}\right) = \dfrac{2-3\left(\frac{2}{3+x}\right)}{\frac{2}{3+x}}$

$= \dfrac{2(3+x) - 3 \cdot 2}{2} = \dfrac{2x}{2} = x$

Domain f = all real numbers except -3

Range f = domain f^{-1} = all real numbers except 0

57. $f^{-1}(x) = \dfrac{-2x}{x-3}$

$f(f^{-1}(x)) = f\left(\dfrac{-2x}{x-3}\right) = \dfrac{3\left(\frac{-2x}{x-3}\right)}{\frac{-2x}{x-3} + 2}$

$= \dfrac{3(-2x)}{-2x + 2(x-3)} = \dfrac{-6x}{-6} = x$

$f^{-1}(f(x)) = f^{-1}\left(\dfrac{3x}{x+2}\right) = \dfrac{-2\left(\frac{3x}{x+2}\right)}{\frac{3x}{x+2} - 3}$

$= \dfrac{-2(3x)}{3x - 3(x+2)} = \dfrac{-6x}{-6} = x$

Domain f = all real numbers except -2

Range f = domain f^{-1} = all real numbers except 3

59. $f^{-1}(x) = \dfrac{x}{3x-2}$

$$f(f^{-1}(x)) = f\left(\frac{x}{3x-2}\right) = \frac{2\left(\frac{x}{3x-2}\right)}{3\left(\frac{x}{3x-2}\right)-1}$$

$$= \frac{2x}{3x-(3x-2)} = \frac{2x}{2} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{2x}{3x-1}\right) = \frac{\frac{2x}{3x-1}}{3\left(\frac{2x}{3x-1}\right)-2}$$

$$= \frac{2x}{6x-2(3x-1)} = \frac{2x}{2} = x$$

Domain f = all real numbers except $\frac{1}{3}$

Range f = domain f^{-1} = all real numbers except $\frac{2}{3}$

61. $f^{-1}(x) = \dfrac{3x+4}{2x-3}$

$$f(f^{-1}(x)) = f\left(\frac{3x+4}{2x-3}\right) = \frac{3\left(\frac{3x+4}{2x-3}\right)+4}{2\left(\frac{3x+4}{2x-3}\right)-3}$$

$$= \frac{3(3x+4)+4(2x-3)}{2(3x+4)-3(2x-3)} = \frac{17x}{17} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{3x+4}{2x-3}\right) = \frac{3\left(\frac{3x+4}{2x-3}\right)+4}{2\left(\frac{3x+4}{2x-3}\right)-3}$$

$$= \frac{3(3x+4)+4(2x-3)}{2(3x+4)-3(2x-3)} = \frac{17x}{17} = x$$

Domain f = all real numbers except $\frac{3}{2}$

Range f = domain f^{-1} = all real numbers except $\frac{3}{2}$

63. $f^{-1}(x) = \dfrac{-2x+3}{x-2}$

$$f(f^{-1}(x)) = f\left(\frac{-2x+3}{x-2}\right) = \frac{2\left(\frac{-2x+3}{x-2}\right)+3}{\frac{-2x+3}{x-2}+2}$$

$$= \frac{2(-2x+3)+3(x-2)}{-2x+3+2(x-2)} = \frac{-x}{-1} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{2x+3}{x+2}\right) = \frac{-2\left(\frac{2x+3}{x+2}\right)+3}{\frac{2x+3}{x+2}-2}$$

$$= \frac{-2(2x+3)+3(x+2)}{2x+3-2(x+2)} = \frac{-x}{-1} = x$$

Domain f = all real numbers except -2

Range f = domain f^{-1} = all real numbers except 2

65. $f^{-1}(x) = \dfrac{2}{\sqrt{1-2x}}$

$$f(f^{-1}(x)) = f\left(\frac{2}{\sqrt{1-2x}}\right) = \frac{\frac{4}{1-2x}-4}{2\cdot\frac{4}{1-2x}} = \frac{4-4(1-2x)}{2\cdot 4}$$

$$= \frac{8x}{8} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{x^2-4}{2x^2}\right) = \frac{2}{\sqrt{1-2\left(\frac{x^2-4}{2x^2}\right)}} = \frac{2}{\sqrt{\frac{4}{x^2}}} = \sqrt{x^2}$$

$= x$, since $x > 0$

Domain $f = \{x|x > 0\}$ or $(0, \infty)$

Range f = domain $f^{-1} = \left\{x\middle|x < \frac{1}{2}\right\}$ or $\left(-\infty, \frac{1}{2}\right)$

67. **(a)** 0 **(b)** 2 **(c)** 0 **(d)** 1 **69.** 7 **71.** Domain of f^{-1}: $[-2, \infty)$; range of f^{-1}: $[5, \infty)$ **73.** Domain of g^{-1}: $[0, \infty)$; range of g^{-1}: all real numbers

75. Increasing on the interval $(f(0), f(5))$ **77.** $f^{-1}(x) = \dfrac{1}{m}(x-b), m \neq 0$ **79.** Quadrant I

81. Possible answer: $f(x) = |x|, x \geq 0$, is one-to-one; $f^{-1}(x) = x, x \geq 0$ **83.** **(a)** $r(d) = \dfrac{d+90.39}{6.97}$

(b) $r(d(r)) = \dfrac{6.97r-90.39+90.39}{6.97} = \dfrac{6.97r}{6.97} = r$; $d(r(d)) = 6.97\left(\dfrac{d+90.39}{6.97}\right) - 90.39 = d + 90.39 - 90.39 = d$ **(c)** 56 miles per hour

85. **(a)** 77.6 kg **(b)** $h(W) = \dfrac{W-50}{2.3} + 60 = \dfrac{W+88}{2.3}$ **(c)** $h(W(h)) = \dfrac{50+2.3(h-60)+88}{2.3} = \dfrac{2.3h}{2.3} = h$ **(d)** 73 inches

$$W(h(W)) = 50 + 2.3\left(\frac{W+88}{2.3} - 60\right)$$

$$= 50 + W + 88 - 138 = W$$

87. **(a)** $\{g|30{,}650 \leq g \leq 74{,}200\}$ **(b)** $\{T|4220 \leq T \leq 15{,}107.50\}$ **(c)** $g(T) = \dfrac{T-4220}{0.25} + 30{,}650$

Domain: $\{T|4220 \leq T \leq 15{,}107.50\}$

Range: $\{g|30{,}650 \leq g \leq 74{,}200\}$

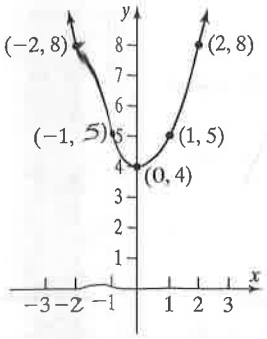
89. $f^{-1}(x) = \dfrac{-dx+b}{cx-a}$; $f = f^{-1}$ if $a = -d$ **93.** No

Review Exercises *(page 86)*

1. **(a)** $2\sqrt{5}$ **(b)** $(2, 1)$ **(c)** $\frac{1}{2}$ **(d)** For each run of 2, there is a rise of 1. **3.** **(a)** 5 **(b)** $\left(-\frac{1}{2}, 1\right)$ **(c)** $-\frac{4}{3}$ **(d)** For each run of 3, there is a rise of -4.

5. **(a)** 12 **(b)** $(4, 2)$ **(c)** undefined **(d)** no change in x

7.



9. $(0, 0)$; symmetric with respect to the x-axis

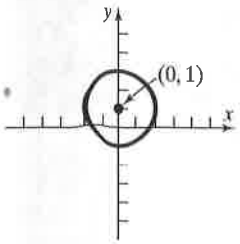
11. $(\pm 4, 0), (0, \pm 2)$; symmetric with respect to the x-axis, y-axis, and origin

13. $(0, 1)$; symmetric with respect to the y-axis

15. $(0, 0), (-1, 0), (0, -2)$; no symmetry

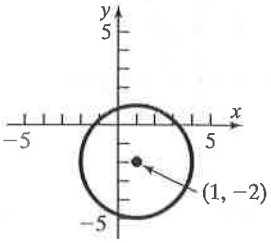
17. $(x + 2)^2 + (y - 3)^2 = 16$ **19.** $(x + 1)^2 + (y + 2)^2 = 1$

21. Center $(0, 1)$; radius $= 2$



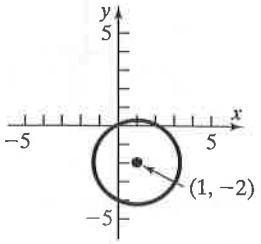
Intercepts: $(-\sqrt{3}, 0), (\sqrt{3}, 0), (0, -1), (0, 3)$

23. Center $(1, -2)$; radius $= 3$



Intercepts: $(1 - \sqrt{5}, 0), (1 + \sqrt{5}, 0), (0, -2 - 2\sqrt{2}), (0, -2 + 2\sqrt{2})$

25. Center $(1, -2)$; radius $= \sqrt{5}$



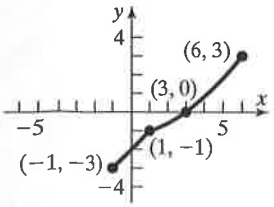
Intercepts: $(0, 0), (2, 0), (0, -4)$

27. Function; domain $\{-1, 2, 4\}$, range $\{0, 3\}$ **29.** **(a)** 2 **(b)** -2 **(c)** $-\dfrac{3x}{x^2 - 1}$ **(d)** $\dfrac{3x}{x^2 - 1}$ **(e)** $\dfrac{3(x - 2)}{x^2 - 4x + 3}$ **(f)** $\dfrac{6x}{4x^2 - 1}$ **31.** **(a)** 0 **(b)** 0 **(c)** $\sqrt{x^2 - 4}$ **(d)** $-\sqrt{x^2 - 4}$ **(e)** $\sqrt{x^2 - 4x}$ **(f)** $2\sqrt{x^2 - 1}$ **33.** **(a)** 0 **(b)** 0 **(c)** $\dfrac{x^2 - 4}{x^2}$ **(d)** $-\dfrac{x^2 - 4}{x^2}$ **(e)** $\dfrac{x(x - 4)}{(x - 2)^2}$ **(f)** $\dfrac{x^2 - 1}{x^2}$

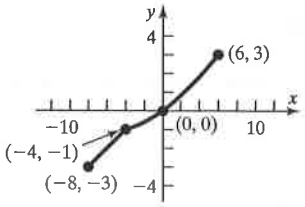
35. $\{x | x \neq -3, x \neq 3\}$ **37.** $\{x | x \leq 2\}$ **39.** $\{x | x > 0\}$ **41.** $\{x | x \neq -3, x \neq 1\}$ **43.** $-4x + 1 - 2h$

45. **(a)** Domain: $\{x | -4 \leq x \leq 3\}$; Range: $\{y | -3 \leq y \leq 3\}$ **(b)** $(0, 0)$ **(c)** -1 **(d)** -4 **(e)** $\{x | 0 < x \leq 3\}$

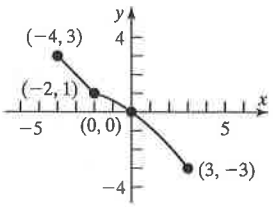
(f)



(g)



(h)



47. **(a)** Domain: $\{x | -4 \leq x \leq 4\}$ or $[-4, 4]$
Range: $\{y | -3 \leq y \leq 1\}$ or $[-3, 1]$

(b) Increasing on $(-4, -1)$ and $(3, 4)$;
Decreasing on $(-1, 3)$

(c) Local maximum is 1 and occurs at $x = -1$.
Local minimum is -3 and occurs at $x = 3$.

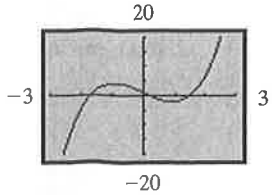
(d) No symmetry

(e) Neither

(f) x-intercepts: $-2, 0, 4$; y-intercept: 0

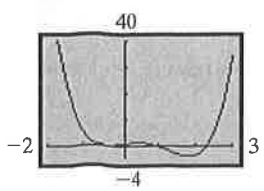
49. Odd **51.** Even **53.** Neither **55.** Odd

57.



Local maximum: $(-0.91, 4.04)$
Local minimum: $(0.91, -2.04)$
Increasing: $(-3, -0.91)$; $(0.91, 3)$
Decreasing: $(-0.91, 0.91)$

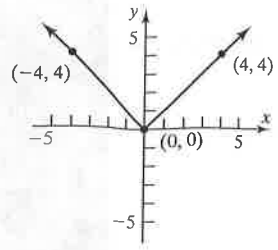
59.



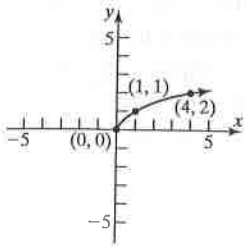
Local maximum: $(0.41, 1.53)$
Local minima: $(-0.34, 0.54)$; $(1.80, -3.56)$
Increasing: $(-0.34, 0.41)$; $(1.80, 3)$
Decreasing: $(-2, -0.34)$; $(0.41, 1.80)$

61. **(a)** 23 **(b)** 7 **(c)** 47 **63.** -5 **65.** -17 **67.** No **69.** Yes

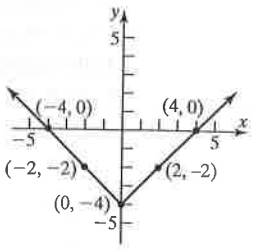
71.



73.

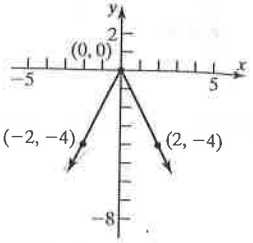


75.



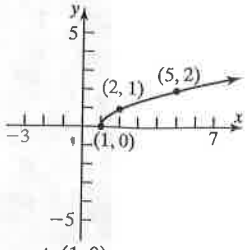
Intercepts: $(-4, 0)$, $(4, 0)$, $(0, -4)$
Domain: all real numbers
Range: $\{y|y \geq -4\}$ or $[-4, \infty)$

77.



Intercept: $(0, 0)$
Domain: all real numbers
Range: $\{y|y \leq 0\}$ or $(-\infty, 0]$

79.

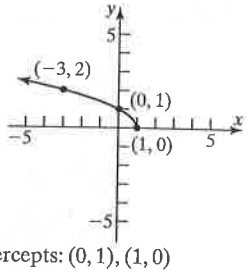


ıtercept: $(1, 0)$

'omain: $\{x|x \geq 1\}$ or $[1, \infty)$

ange: $\{y|y \geq 0\}$ or $[0, \infty)$

81.

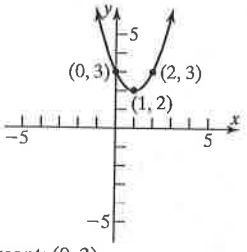


Intercepts: $(0, 1)$, $(1, 0)$

Domain: $\{x|x \leq 1\}$ or $(-\infty, 1]$

Range: $\{y|y \geq 0\}$ or $[0, \infty)$

83.

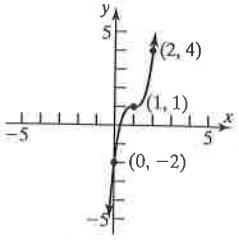


Intercept: $(0, 3)$

Domain: all real numbers

Range: $\{y|y \geq 2\}$ or $[2, \infty)$

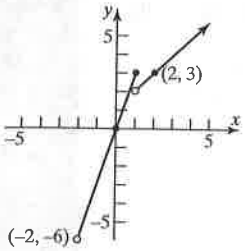
85.



Intercepts: $(0, -2)$,
$\left(1, -\frac{\sqrt[3]{9}}{3}, 0\right)$ or about $(0.3, 0)$
Domain: all real numbers
Range: all real numbers

87. (a) $\{x|x > -2\}$ or $(-2, \infty)$ (b) $(0, 0)$

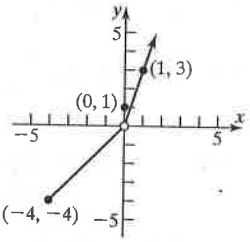
(c)



(d) $\{y|y > -6\}$ or $(-6, \infty)$

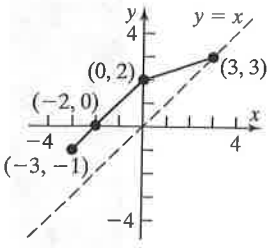
89. (a) $\{x|x \geq -4\}$ or $[-4, \infty)$ (b) $(0, 1)$

(c)



(d) $\{y|-4 \leq y < 0 \text{ or } y > 0\}$ or $[-4, 0) \cup (0, \infty)$

$A = 11$ **93.** (a) One-to-one (b) $\{(2, 1), (5, 3), (8, 5), (10, 6)\}$



97. $f^{-1}(x) = \dfrac{2x + 3}{5x - 2}$

$$f(f^{-1}(x)) = \frac{2\left(\frac{2x + 3}{5x - 2}\right) + 3}{5\left(\frac{2x + 3}{5x - 2}\right) - 2} = x$$

$$f^{-1}(f(x)) = \frac{2\left(\frac{2x + 3}{5x - 2}\right) + 3}{5\left(\frac{2x + 3}{5x - 2}\right) - 2} = x$$

Domain of f = range of f^{-1} = all real numbers except $\frac{2}{5}$

Range of f = domain of f^{-1} = all real numbers except $\frac{2}{5}$

$f^{-1}(x) = \dfrac{x + 1}{x}$

$$f(f^{-1}(x)) = \frac{1}{\frac{x + 1}{x} - 1} = x$$

$$f^{-1}(f(x)) = \frac{\frac{1}{x - 1} + 1}{\frac{1}{x - 1}} = x$$

Domain of f = range of f^{-1} = all real numbers except 1

Range of f = domain of f^{-1} = all real numbers except 0

101. $f^{-1}(x) = \dfrac{27}{x^3}$

$$f(f^{-1}(x)) = \frac{3}{\left(\frac{27}{x^3}\right)^{1/3}} = x$$

$$f^{-1}(f(x)) = \frac{27}{\left(\frac{3}{x^{1/3}}\right)^3} = x$$

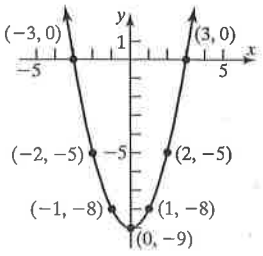
Domain of f = range of f^{-1} = all real numbers except 0

Range of f = domain of f^{-1} = all real numbers except 0

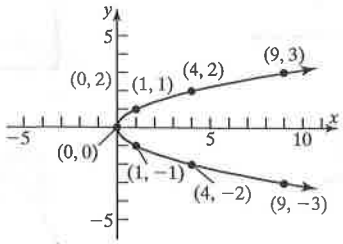
103. $d(A,B) = \sqrt{(1-3)^2 + (1-4)^2} = \sqrt{13}$ and $d(B,C) = \sqrt{(-2-1)^2 + (3-1)^2} = \sqrt{13}$

Chapter Test *(page 89)*

1. $d = 2\sqrt{13}$ **2.** $(2,1)$ **3.**

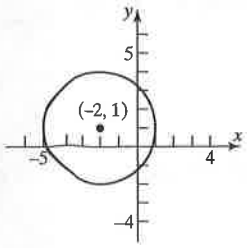


4.



5. Intercepts: $(-3,0), (3,0), (0,9)$; symmetric with respect to the y-axis **6.** $x^2 + y^2 - 8x + 6y = 0$

7. Center: $(-2,1)$; radius: 3



8. (a) Function; domain: $\{2,4,6,8\}$; range: $\{5,6,7,8\}$ **(b)** Not a function **(c)** Not a function

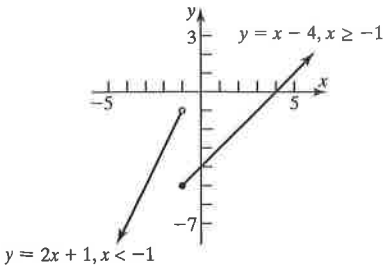
(d) Function; domain; all real numbers; range: $\{y|y \geq 2\}$ **9.** Domain: $\left\{x|x \leq \frac{4}{5}\right\}$; $f(-1) = 3$ **10.** Domain: $\{x|x \neq -2\}$; $g(-1) = 1$

11. Domain: $\{x|x \neq -9, x \neq 4\}$; $h(-1) = \frac{1}{8}$ **12. (a)** Domain: $\{x|-5 \leq x \leq 5\}$; range: $\{y|-3 \leq y \leq 3\}$

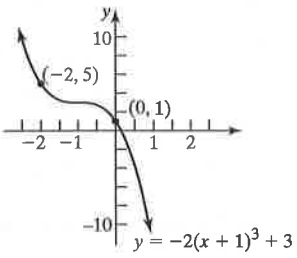
(b) $(0,2), (-2,0)$, and $(2,0)$ **(c)** $f(1) = 3$ **(d)** $x = -5$ and $x = 3$ **(e)** $\{x|-5 \leq x < -2$ or $2 < x \leq 5\}$ or $[-5,-2) \cup (2,5]$

13. Local maxima: $f(-0.85) \approx -0.86$; $f(2.35) \approx 15.55$; Local minima: $f(0) = -2$; the function is increasing on the intervals $(-5,-0.85)$ and $(0,2.35)$ and decreasing on the intervals $(-0.85,0)$ and $(2.35,5)$.

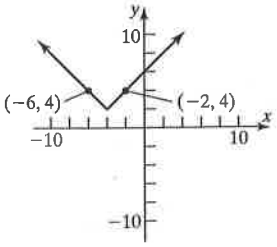
14. (a)



15. 19 **16. (a)**



(b)



(b) $(0,-4), (4,0)$

(c) $g(-5) = -9$

(d) $g(2) = -2$

17. $f^{-1}(x) = \frac{2+5x}{3x}$; domain of $f = \left\{x\middle|x \neq \frac{5}{3}\right\}$, range of $f = \{y|y \neq 0\}$; domain of $f^{-1} = \{x|x \neq 0\}$; range of $f^{-1} = \left\{y\middle|y \neq \frac{5}{3}\right\}$

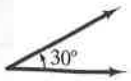
18. The point $(-5,3)$ must be on the graph of f^{-1}.

CHAPTER 2 Trigonometric Functions

2.1 Assess Your Understanding *(page 103)*

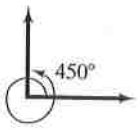
3. standard position **4.** $r\theta$; $\frac{1}{2}r^2\theta$ **5.** $\frac{s}{t}$; $\frac{\theta}{t}$ **6.** F **7.** T **8.** T **9.** T **10.** F

11.

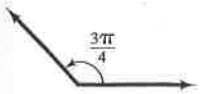


13.

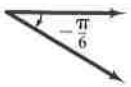
15.



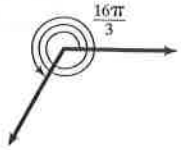
17.



19.



21.



23. 40.17° **25.** 1.03° **27.** 9.15° **29.** 40°19′12″ **31.** 18°15′18″ **33.** 19°59′24″ **35.** $\frac{\pi}{6}$ **37.** $\frac{4\pi}{3}$ **39.** $-\frac{\pi}{3}$ **41.** π **43.** $-\frac{3\pi}{4}$ **45.** $-\frac{\pi}{2}$

47. 60° **49.** −225° **51.** 90° **53.** 15° **55.** −90° **57.** −30° **59.** 0.30 **61.** −0.70 **63.** 2.18 **65.** 179.91° **67.** 114.59° **69.** 362.11° **71.** 5 m

73. 6 ft **75.** 0.6 radian **77.** $\frac{\pi}{3} \approx 1.047$ in. **79.** 25 m^2 **81.** $2\sqrt{3} \approx 3.464$ ft **83.** 0.24 radian **85.** $\frac{\pi}{3} \approx 1.047$ in.^{2} **87.** $s = 2.094$ ft; $A = 2.094$ ft^2

89. $s = 14.661$ yd; $A = 87.965$ yd^2 **91.** $3\pi \approx 9.42$ in; $5\pi \approx 15.71$ in. **93.** $2\pi \approx 6.28$ m^2 **95.** $\frac{675\pi}{2} \approx 1060.29$ ft^2

97. $\omega = \frac{1}{60}$radian/s; $v = \frac{1}{12}$cm/s **99.** Approximately 452.5 rpm **101.** Approximately 359 mi **103.** Approximately 898 mi/h

105. Approximately 2292 mi/h **107.** $\frac{3}{4}$rpm **109.** Approximately 2.86 mi/h **111.** Approximately 31.47 rpm **113.** Approximately 1037 mi/h

115. Radius ≈ 3979 mi; circumference ≈ 25,000 mi **117.** $v_1 = r_1\omega_1$, $v_2 = r_2\omega_2$, and $v_1 = v_2$, so $r_1\omega_1 = r_2\omega_2 \Rightarrow \frac{r_1}{r_2} = \frac{\omega_2}{\omega_1}$.

2.2 Assess Your Understanding *(page 119)*

7. $\frac{3}{2}$ **8.** 0.91 **9.** T **10.** F **11.** $\sin t = \frac{1}{2}$; $\cos t = \frac{\sqrt{3}}{2}$; $\tan t = \frac{\sqrt{3}}{3}$; $\csc t = 2$; $\sec t = \frac{2\sqrt{3}}{3}$; $\cot t = \sqrt{3}$ **13.** $\sin t = \frac{\sqrt{21}}{5}$; $\cos t = -\frac{2}{5}$; $\tan t = -\frac{\sqrt{21}}{2}$; $\csc t = \frac{5\sqrt{21}}{21}$; $\sec t = -\frac{5}{2}$; $\cot t = -\frac{2\sqrt{21}}{21}$ **15.** $\sin t = \frac{\sqrt{2}}{2}$; $\cos t = -\frac{\sqrt{2}}{2}$; $\tan t = -1$; $\csc t = \sqrt{2}$; $\sec t = -\sqrt{2}$; $\cot t = -1$

17. $\sin t = -\frac{1}{3}$; $\cos t = \frac{2\sqrt{2}}{3}$; $\tan t = -\frac{\sqrt{2}}{4}$; $\csc t = -3$; $\sec t = \frac{3\sqrt{2}}{4}$; $\cot t = -2\sqrt{2}$ **19.** −1 **21.** 0 **23.** −1 **25.** 0 **27.** −1 **29.** $\frac{1}{2}\left(\sqrt{2}+1\right)$

31. 2 **33.** $\frac{1}{2}$ **35.** $\sqrt{6}$ **37.** 4 **39.** 0 **41.** 0 **43.** $2\sqrt{2} + \frac{4\sqrt{3}}{3}$ **45.** −1 **47.** 1

49. $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$; $\cos\frac{2\pi}{3} = -\frac{1}{2}$; $\tan\frac{2\pi}{3} = -\sqrt{3}$; $\csc\frac{2\pi}{3} = \frac{2\sqrt{3}}{3}$; $\sec\frac{2\pi}{3} = -2$; $\cot\frac{2\pi}{3} = -\frac{\sqrt{3}}{3}$

51. $\sin 210° = -\frac{1}{2}$; $\cos 210° = -\frac{\sqrt{3}}{2}$; $\tan 210° = \frac{\sqrt{3}}{3}$; $\csc 210° = -2$; $\sec 210° = -\frac{2\sqrt{3}}{3}$; $\cot 210° = \sqrt{3}$

53. $\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$; $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$; $\tan\frac{3\pi}{4} = -1$; $\csc\frac{3\pi}{4} = \sqrt{2}$; $\sec\frac{3\pi}{4} = -\sqrt{2}$; $\cot\frac{3\pi}{4} = -1$

55. $\sin\frac{8\pi}{3} = \frac{\sqrt{3}}{2}$; $\cos\frac{8\pi}{3} = -\frac{1}{2}$; $\tan\frac{8\pi}{3} = -\sqrt{3}$; $\csc\frac{8\pi}{3} = \frac{2\sqrt{3}}{3}$; $\sec\frac{8\pi}{3} = -2$; $\cot\frac{8\pi}{3} = -\frac{\sqrt{3}}{3}$

57. $\sin 405° = \frac{\sqrt{2}}{2}$; $\cos 405° = \frac{\sqrt{2}}{2}$; $\tan 405° = 1$; $\csc 405° = \sqrt{2}$; $\sec 405° = \sqrt{2}$; $\cot 405° = 1$

59. $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$; $\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$; $\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$; $\csc\left(-\frac{\pi}{6}\right) = -2$; $\sec\left(-\frac{\pi}{6}\right) = \frac{2\sqrt{3}}{3}$; $\cot\left(-\frac{\pi}{6}\right) = -\sqrt{3}$

61. $\sin(-45°) = -\frac{\sqrt{2}}{2}$; $\cos(-45°) = \frac{\sqrt{2}}{2}$; $\tan(-45°) = -1$; $\csc(-45°) = -\sqrt{2}$; $\sec(-45°) = \sqrt{2}$; $\cot(-45°) = -1$

63. $\sin\frac{5\pi}{2} = 1$; $\cos\frac{5\pi}{2} = 0$; $\tan\frac{5\pi}{2}$ is undefined; $\csc\frac{5\pi}{2} = 1$; $\sec\frac{5\pi}{2}$ is undefined; $\cot\frac{5\pi}{2} = 0$ **65.** $\sin 720° = 0$; $\cos 720° = 1$; $\tan 720° = 0$; $\csc 720°$ is undefined; $\sec 720° = 1$; $\cot 720°$ is undefined **67.** 0.47 **69.** 0.38 **71.** 1.33 **73.** 0.31 **75.** 3.73 **77.** 1.04 **79.** 0.84 **81.** 0.02

83. $\sin\theta = \frac{4}{5}$; $\cos\theta = -\frac{3}{5}$; $\tan\theta = -\frac{4}{3}$; $\csc\theta = \frac{5}{4}$; $\sec\theta = -\frac{5}{3}$; $\cot\theta = -\frac{3}{4}$

85. $\sin\theta = \frac{3\sqrt{13}}{13}$; $\cos\theta = \frac{2\sqrt{13}}{13}$; $\tan\theta = \frac{3}{2}$; $\csc\theta = \frac{\sqrt{13}}{3}$; $\sec\theta = \frac{\sqrt{13}}{2}$; $\cot\theta = \frac{2}{3}$

87. $\sin\theta = -\frac{\sqrt{2}}{2}$; $\cos\theta = -\frac{\sqrt{2}}{2}$; $\tan\theta = 1$; $\csc\theta = -\sqrt{2}$; $\sec\theta = -\sqrt{2}$; $\cot\theta = 1$

89. $\sin\theta = -\frac{2\sqrt{13}}{13}$; $\cos\theta = -\frac{3\sqrt{13}}{13}$; $\tan\theta = \frac{2}{3}$; $\csc\theta = -\frac{\sqrt{13}}{2}$; $\sec\theta = -\frac{\sqrt{13}}{3}$; $\cot\theta = \frac{3}{2}$

91. $\sin\theta = -\frac{3}{5}; \cos\theta = \frac{4}{5}; \tan\theta = -\frac{3}{4}; \csc\theta = -\frac{5}{3}; \sec\theta = \frac{5}{4}; \cot\theta = -\frac{4}{3}$ **93.** 0 **95.** 0 **97.** -0.1

99. 3 **101.** 5 **103.** $\frac{\sqrt{3}}{2}$ **105.** $\frac{1}{2}$ **107.** $\frac{3}{4}$ **109.** $\frac{\sqrt{3}}{2}$ **111.** $\sqrt{3}$ **113.** $-\frac{\sqrt{3}}{2}$

115.

θ	0.5	0.4	0.2	0.1	0.01	0.001	0.0001	0.00001
$\sin\theta$	0.4794	0.3894	0.1987	0.0998	0.0100	0.0010	0.0001	0.00001
$\frac{\sin\theta}{\theta}$	0.9589	0.9735	0.9933	0.9983	1.0000	1.0000	1.0000	1.0000

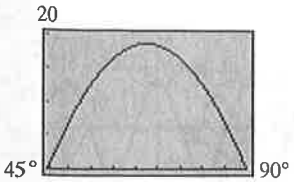
$\frac{\sin\theta}{\theta}$ approaches 1 as θ approaches 0.

117. $R \approx 3110.56$ ft; $H \approx 77.64$ ft **119.** $R \approx 19{,}541.95$ m; $H \approx 2278.14$ m

121. **(a)** 1.20 sec **(b)** 1.12 sec **(c)** 1.20 sec **123.** **(a)** 1.9 hr; 0.57 hr **(b)** 1.69 hr; 0.75 hr **(c)** 1.63 hr; 0.86 hr **(d)** 1.67 hr; $\tan 90°$ is undefined

125. **(a)** 16.56 ft

(b)



(c) 67.5°

127. **(a)** values estimated to the nearest tenth: $\sin 1 \approx 0.8$; $\cos 1 \approx 0.5$; $\tan 1 \approx 1.6$; $\csc 1 \approx 1.3$; $\sec 1 \approx 2.0$; $\cot 1 \approx 0.6$; actual values to the nearest tenth: $\sin 1 \approx 0.8$; $\cos 1 \approx 0.5$; $\tan 1 \approx 1.6$; $\csc 1 \approx 1.2$; $\sec 1 \approx 1.9$; $\cot 1 \approx 0.6$

(b) values estimated to the nearest tenth: $\sin 5.1 \approx -0.9$; $\cos 5.1 \approx 0.4$; $\tan 5.1 \approx -2.3$; $\csc 5.1 \approx -1.1$; $\sec 5.1 \approx 2.5$; $\cot 5.1 \approx -0.4$; actual values to the nearest tenth: $\sin 5.1 \approx -0.9$; $\cos 5.1 \approx 0.4$; $\tan 5.1 \approx -2.4$; $\csc 5.1 \approx -1.1$; $\sec 5.1 \approx 2.6$; $\cot 5.1 \approx -0.4$

2.3 Assess Your Understanding *(page 133)*

5. $2\pi; \pi$ **6.** all real numbers except odd multiples of $\frac{\pi}{2}$ **7.** $[-1, 1]$ **8.** T **9.** F **10.** F **11.** $\frac{\sqrt{2}}{2}$ **13.** 1 **15.** 1 **17.** $\sqrt{3}$

19. $\frac{\sqrt{2}}{2}$ **21.** 0 **23.** $\sqrt{2}$ **25.** $\frac{\sqrt{3}}{3}$ **27.** II **29.** IV **31.** IV **33.** II **35.** $\tan\theta = -\frac{3}{4}; \cot\theta = -\frac{4}{3}; \sec\theta = \frac{5}{4}; \csc\theta = -\frac{5}{3}$

37. $\tan\theta = 2; \cot\theta = \frac{1}{2}; \sec\theta = \sqrt{5}; \csc\theta = \frac{\sqrt{5}}{2}$ **39.** $\tan\theta = \frac{\sqrt{3}}{3}; \cot\theta = \sqrt{3}; \sec\theta = \frac{2\sqrt{3}}{3}; \csc\theta = 2$

41. $\tan\theta = -\frac{\sqrt{2}}{4}; \cot\theta = -2\sqrt{2}; \sec\theta = \frac{3\sqrt{2}}{4}; \csc\theta = -3$ **43.** $\cos\theta = -\frac{5}{13}; \tan\theta = -\frac{12}{5}; \csc\theta = \frac{13}{12}; \sec\theta = -\frac{13}{5}; \cot\theta = -\frac{5}{12}$

45. $\sin\theta = -\frac{3}{5}; \tan\theta = \frac{3}{4}; \csc\theta = -\frac{5}{3}; \sec\theta = -\frac{5}{4}; \cot\theta = \frac{4}{3}$ **47.** $\cos\theta = -\frac{12}{13}; \tan\theta = -\frac{5}{12}; \csc\theta = \frac{13}{5}; \sec\theta = -\frac{13}{12}; \cot\theta = -\frac{12}{5}$

49. $\sin\theta = \frac{2\sqrt{2}}{3}; \tan\theta = -2\sqrt{2}; \csc\theta = \frac{3\sqrt{2}}{4}; \sec\theta = -3; \cot\theta = -\frac{\sqrt{2}}{4}$ **51.** $\cos\theta = -\frac{\sqrt{5}}{3}; \tan\theta = -\frac{2\sqrt{5}}{5}; \csc\theta = \frac{3}{2}; \sec\theta = -\frac{3\sqrt{5}}{5}; \cot\theta = -\frac{\sqrt{5}}{2}$ **53.** $\sin\theta = -\frac{\sqrt{3}}{2}; \cos\theta = \frac{1}{2}; \tan\theta = -\sqrt{3}; \csc\theta = -\frac{2\sqrt{3}}{3}; \cot\theta = -\frac{\sqrt{3}}{3}$ **55.** $\sin\theta = -\frac{3}{5}; \cos\theta = -\frac{4}{5}; \csc\theta = -\frac{5}{3}; \sec\theta = -\frac{5}{4}; \cot\theta = \frac{4}{3}$ **57.** $\sin\theta = \frac{\sqrt{10}}{10}; \cos\theta = -\frac{3\sqrt{10}}{10}; \csc\theta = \sqrt{10}; \sec\theta = -\frac{\sqrt{10}}{3}; \cot\theta = -3$ **59.** $-\frac{\sqrt{3}}{2}$

61. $-\frac{\sqrt{3}}{3}$ **63.** 2 **65.** -1 **67.** -1 **69.** $\frac{\sqrt{2}}{2}$ **71.** 0 **73.** $-\sqrt{2}$ **75.** $\frac{2\sqrt{3}}{3}$ **77.** 1 **79.** 1 **81.** 0 **83.** 1 **85.** -1 **87.** 0 **89.** 0.9 **91.** 9

93. 0 **95.** All real numbers **97.** Odd multiples of $\frac{\pi}{2}$ **99.** Odd multiples of $\frac{\pi}{2}$ **101.** $-1 \le y \le 1$ **103.** All real numbers **105.** $|y| \ge 1$

107. Odd; yes; origin **109.** Odd; yes; origin **111.** Even; yes; y-axis **113.** **(a)** $-\frac{1}{3}$ **(b)** 1 **115.** **(a)** -2 **(b)** 6 **117.** **(a)** -4 **(b)** -12

119. About 15.81 min **121.** Let a be a real number and $P = (x, y)$ be the point on the unit circle that corresponds to t. Consider the equation $\tan t = \frac{y}{x} = a$. Then $y = ax$. But $x^2 + y^2 = 1$, so $x^2 + a^2x^2 = 1$. So, $x = \pm\frac{1}{\sqrt{1+a^2}}$ and $y = \pm\frac{a}{\sqrt{1+a^2}}$; that is, for any real number a, there is a point $P = (x, y)$ on the unit circle for which $\tan t = a$. In other ords, the range of the tangent function is the set of all real numbers.

123. Suppose that there is a number p, $0 < p < 2\pi$, for which $\sin(\theta + p) = \sin\theta$ for all θ. If $\theta = 0$, then $\sin(0 + p) = \sin p = \sin 0 = 0$, so $p = \pi$. If $\theta = \frac{\pi}{2}$, then $\sin\left(\frac{\pi}{2} + p\right) = \sin\left(\frac{\pi}{2}\right)$. But $p = \pi$. Thus, $\sin\left(\frac{3\pi}{2}\right) = -1 = \sin\left(\frac{\pi}{2}\right) = 1$. This is impossible. Therefore, the smallest positive number p for which $\sin(\theta + p) = \sin\theta$ for all θ is 2π. **125.** $\sec\theta = \frac{1}{\cos\theta}$; since $\cos\theta$ has period 2π, so does $\sec\theta$. **127.** If $P = (a, b)$ is the point on the unit circle corresponding to θ, then $Q = (-a, -b)$ is the point on the unit circle corresponding to $\theta + \pi$. Thus, $\tan(\theta + \pi) = \frac{-b}{-a} = \frac{b}{a} = \tan\theta$. Suppose that there exists a number p, $0 < p < \pi$, for which $\tan(\theta + p) = \tan\theta$ for all θ. Then, if $\theta = 0$, then $\tan p = \tan 0 = 0$. But this means that p is a multiple of π. Since no multiple of π exists in the interval $(0, \pi)$, this is a contradiction. Therefore, the period of $f(\theta) = \tan\theta$ is π.

129. Let $P = (a, b)$ be the point on the unit circle corresponding to θ. Then $\csc\theta = \frac{1}{b} = \frac{1}{\sin\theta}$; $\sec\theta = \frac{1}{a} = \frac{1}{\cos\theta}$; $\cot\theta = \frac{a}{b} = \frac{1}{b/a} = \frac{1}{\tan\theta}$.

131. $(\sin\theta\cos\phi)^2 + (\sin\theta\sin\phi)^2 + \cos^2\theta = \sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta$

$= \sin^2\theta(\cos^2\phi + \sin^2\phi) + \cos^2\theta = \sin^2\theta + \cos^2\theta = 1$

2.4 Assess Your Understanding *(page 146)*

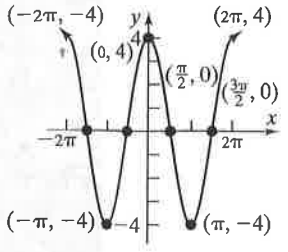
3. $1; \frac{\pi}{2} + 2\pi k, k$ any integer **4.** $3; \pi$ **5.** $3; \frac{\pi}{3}$ **6.** T **7.** F **8.** T **9.** 0 **11.** $-\frac{\pi}{2} < x < \frac{\pi}{2}$ **13.** 1 **15.** $0, \pi, 2\pi$

17. $\sin x = 1$ for $x = -\frac{3\pi}{2}, \frac{\pi}{2}$; $\sin x = -1$ for $x = -\frac{\pi}{2}, \frac{3\pi}{2}$ **19.** Amplitude = 2; period = 2π **21.** Amplitude = 4; period = π

23. Amplitude = 6; period = 2 **25.** Amplitude = $\frac{1}{2}$; period = $\frac{4\pi}{3}$ **27.** Amplitude = $\frac{5}{3}$; period = 3 **29.** F **31.** A **33.** H **35.** C **37.** J

39. A **41.** B

43.



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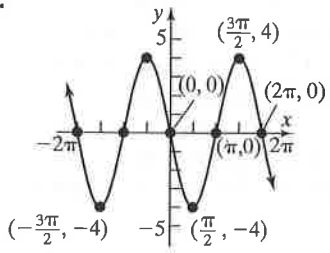

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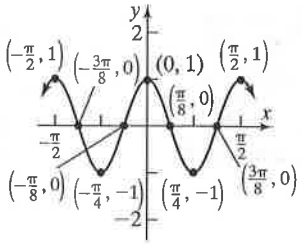

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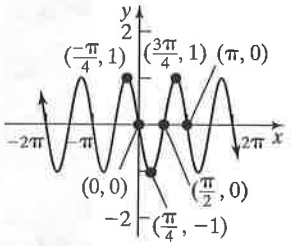

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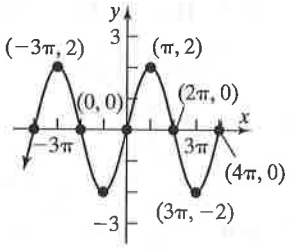

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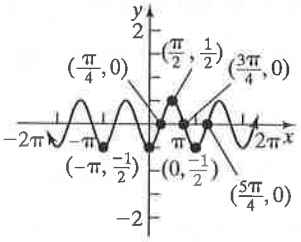

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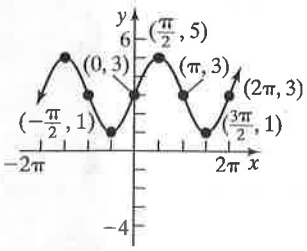

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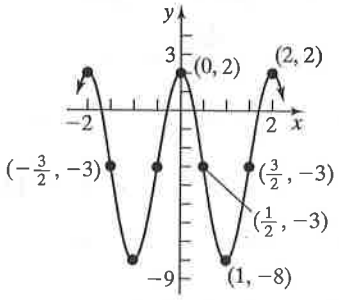

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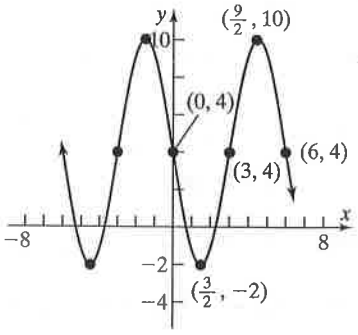

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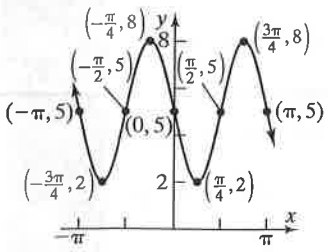

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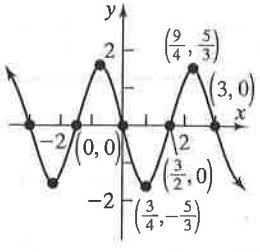

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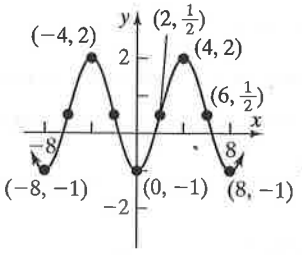
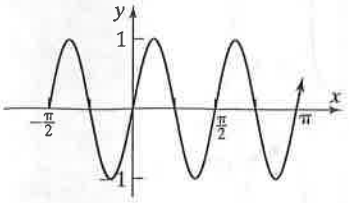


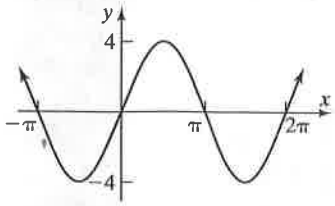
67. $y = \pm 3\sin(2x)$ **69.** $y = \pm 3\sin(\pi x)$ **71.** $y = 5\cos\left(\frac{\pi}{4}x\right)$ **73.** $y = -3\cos\left(\frac{1}{2}x\right)$ **75.** $y = \frac{3}{4}\sin(2\pi x)$ **77.** $y = -\sin\left(\frac{3}{2}x\right)$

79. $y = -\cos\left(\frac{4\pi}{3}x\right) + 1$ **81.** $y = 3\sin\left(\frac{\pi}{2}x\right)$ **83.** $y = -4\cos(3x)$ **85.** $\frac{2}{\pi}$ **87.** $\frac{\sqrt{2}}{\pi}$

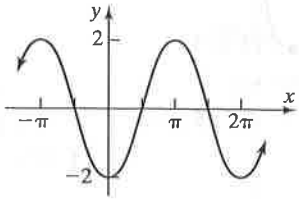
89. $(f \circ g)(x) = \sin(4x)$



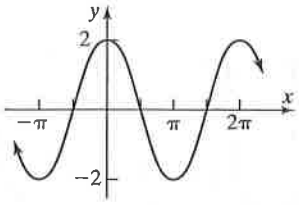
$(g \circ f)(x) = 4 \sin x$



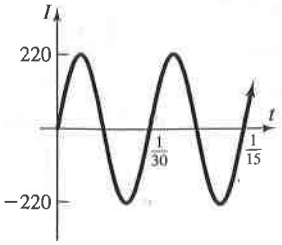
91. $(f \circ g)(x) = -2 \cos x$



$(g \circ f)(x) = \cos(-2x)$

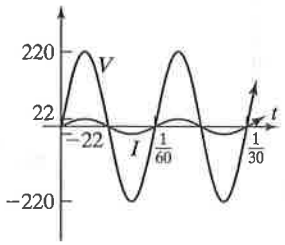


93. Period $= \frac{1}{30}$s; amplitude $= 220$ amp



95. (a) Amplitude $= 220$ V; period $= \frac{1}{60}$s

(b), (e)



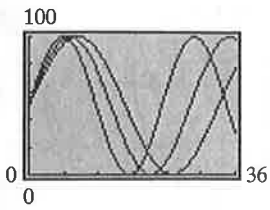
(c) $I(t) = 22 \sin(120\pi t)$

(d) Amplitude $= 22$ amp; period $= \frac{1}{60}$s

97. (a) $P(t) = \dfrac{[V_0 \sin(2\pi ft)]^2}{R} = \dfrac{V_0^2}{R}\sin^2(2\pi ft)$ **(b)** Since the graph of P has amplitude $\dfrac{V_0^2}{2R}$ and period $\dfrac{1}{2f}$ and is of the form $y = A\cos(\omega t) + B$, then $A = -\dfrac{V_0^2}{2R}$ and $B = \dfrac{V_0^2}{2R}$. Since $\dfrac{1}{2f} = \dfrac{2\pi}{\omega}$, then $\omega = 4\pi f$. Therefore, $P(t) = -\dfrac{V_0^2}{2R}\cos(4\pi ft) + \dfrac{V_0^2}{2R} = \dfrac{V_0^2}{2R}[1 - \cos(4\pi ft)]$.

99. (a) Physical potential: $\omega = \dfrac{2\pi}{23}$; emotional potential: $\omega = \dfrac{\pi}{14}$; intellectual potential: $\omega = \dfrac{2\pi}{33}$

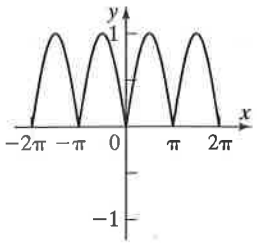
(b)



(c) No

(d) Physical potential peaks at 15 days after 20th birthday. Emotional potential is 50% at 17 days, with a maximum at 10 days and a minum at 24 days. Intellectual potential starts fairly high, drops to a minimum at 13 days, and rises to a maximum at 29 days.

101.

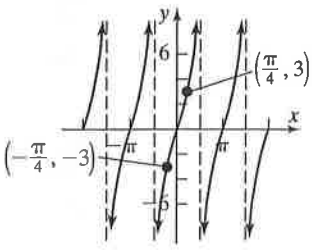


2.5 Assess Your Understanding *(Page 156)*

3. origin; odd multiples of $\frac{\pi}{2}$ **4.** y-axis; odd multiples of $\frac{\pi}{2}$ **5.** $y = \cos x$ **6.** T **7.** 0 **9.** 1

11. $\sec x = 1$ for $x = -2\pi, 0, 2\pi$; $\sec x = -1$ for $x = -\pi, \pi$ **13.** $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ **15.** $-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

17.



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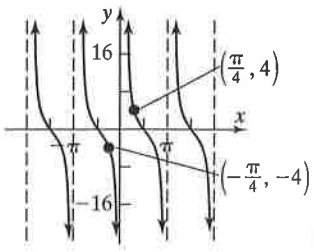

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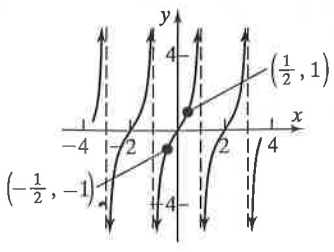

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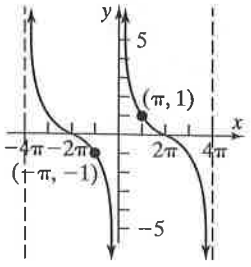

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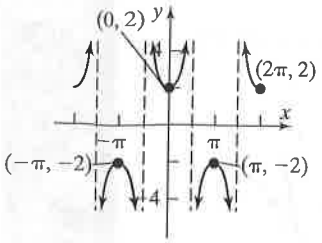

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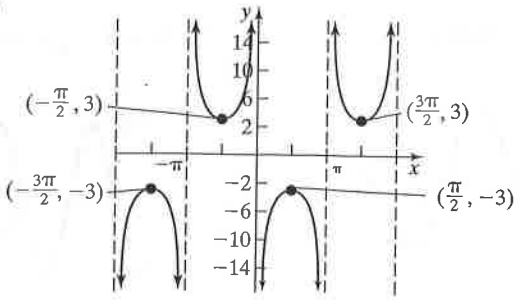

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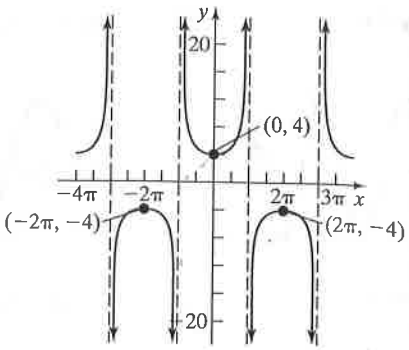

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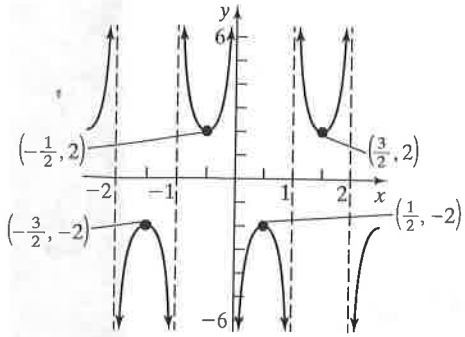

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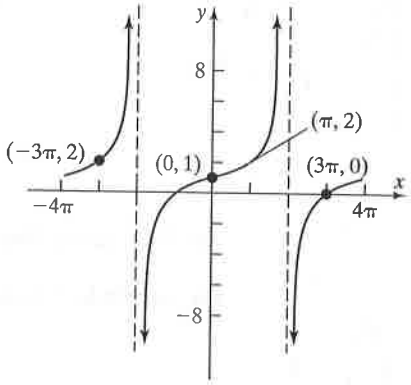
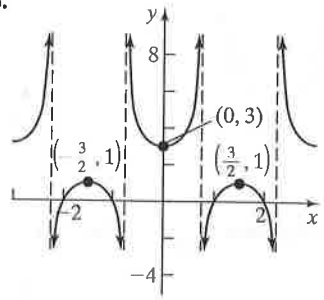
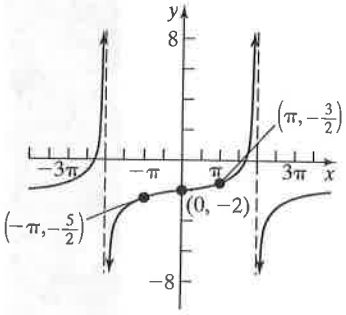


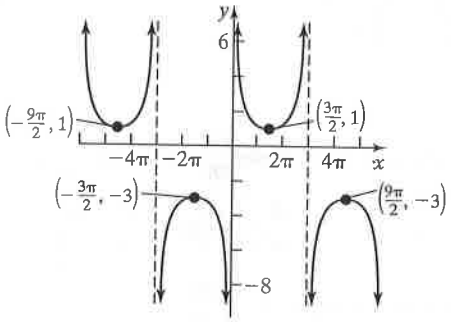
35.



37.

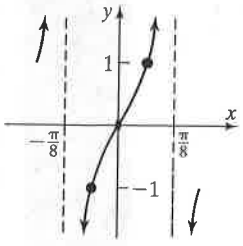


39.

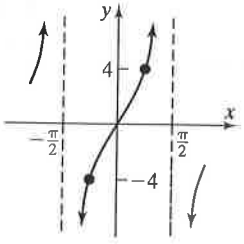


41. $\frac{2\sqrt{3}}{\pi}$ **43.** $\frac{6\sqrt{3}}{\pi}$

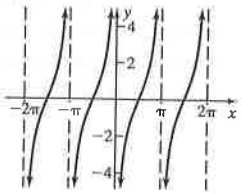
45. $(f \circ g)(x) = \tan(4x)$



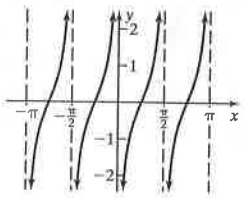
$(g \circ f)(x) = 4\tan x$



47. $(f \circ g)(x) = -2\cot x$

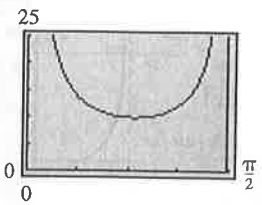


$(g \circ f)(x) = \cot(-2x)$



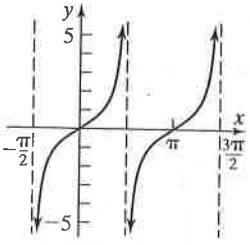
49. (a) $L(\theta) = \dfrac{3}{\cos\theta} + \dfrac{4}{\sin\theta}$
$= 3\sec\theta + 4\csc\theta$

(b)

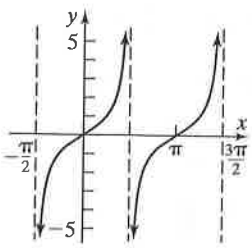


(c) 0.83 (d) 9.86 ft

51.



$y = \tan x$



$y = -\cot\left(x + \frac{\pi}{2}\right)$

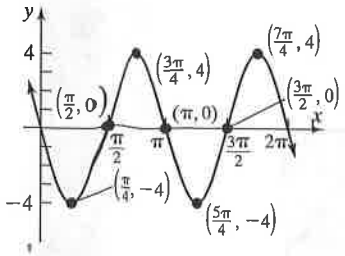
2.6 Assess Your Understanding *(page 166)*

1. phase shift **2.** F

3. Amplitude = 4

Period = π

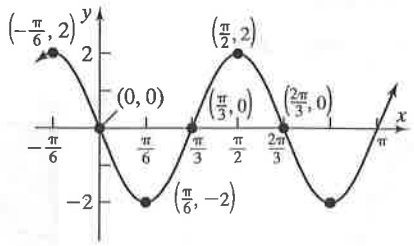
Phase shift = $\frac{\pi}{2}$



5. Amplitude = 2

Period = $\frac{2\pi}{3}$

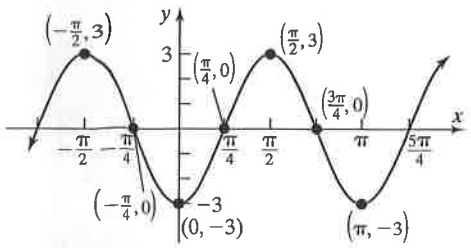
Phase shift = $-\frac{\pi}{6}$



7. Amplitude = 3

Period = π

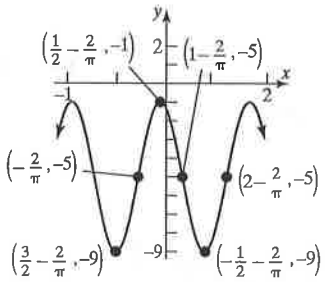
Phase shift = $-\frac{\pi}{4}$



9. Amplitude = 4

Period = 2

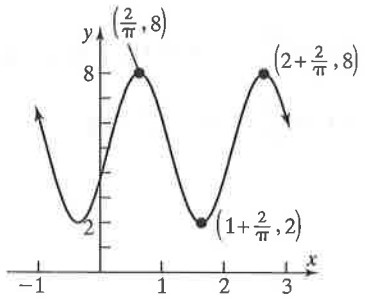
Phase shift = $-\frac{2}{\pi}$



11. Amplitude = 3

Period = 2

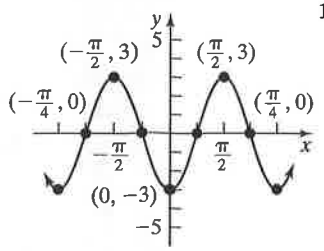
Phase shift = $\frac{2}{\pi}$



13. Amplitude = 3

Period = π

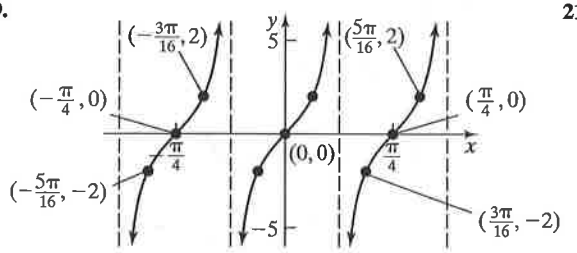
Phase shift = $\frac{\pi}{4}$



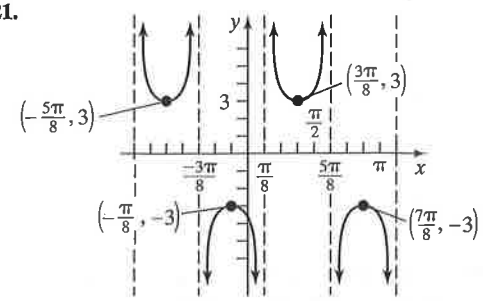
15. $y = 2\sin\left[2\left(x - \frac{1}{2}\right)\right]$ or $y = 2\sin(2x - 1)$

17. $y = 3\sin\left[\frac{2}{3}\left(x + \frac{1}{3}\right)\right]$ or $y = 3\sin\left(\frac{2}{3}x + \frac{2}{9}\right)$

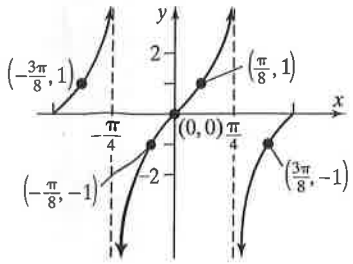
19.



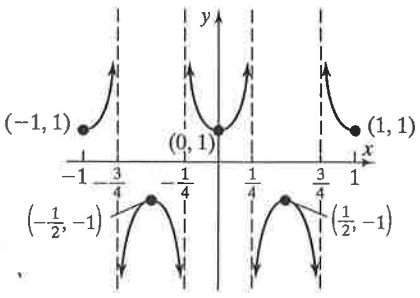
21.



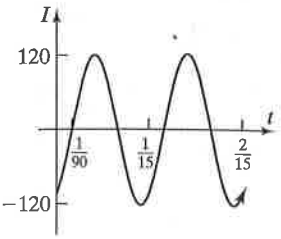
23.



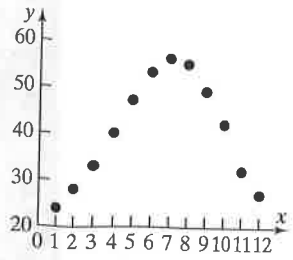
25.



27. Period = $\frac{1}{15}$ s; amplitude = 120 amp; phase shift = $\frac{1}{90}$ s

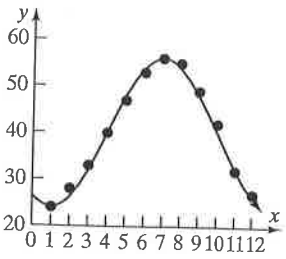


29. (a)



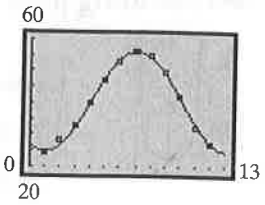
(b) $y = 15.9 \sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 40.1$

(c)

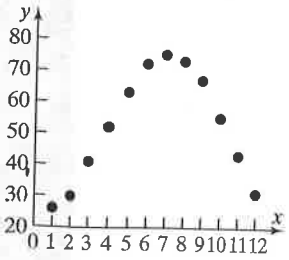


(d) $y = 15.62 \sin(0.517x - 2.096) + 40.377$

(e)

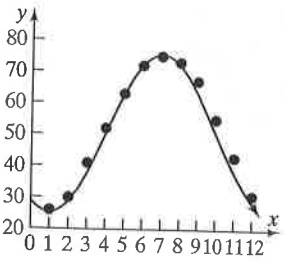


31. (a)



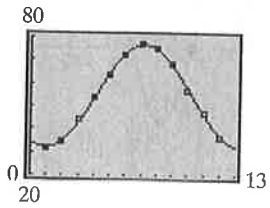
(b) $y = 24.95 \sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 50.45$

(c)

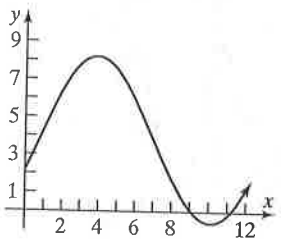


(d) $y = 25.693 \sin(0.476x - 1.814) + 49.854$

(e)



3. (a) 4:08 PM **(b)** $y = 4.4 \sin\left[\frac{4\pi}{25}(x - 0.5083)\right] + 3.8$ or $y = 4.4 \sin\left[\frac{4\pi}{25}x - 0.2555\right] + 3.8$ **(c)**

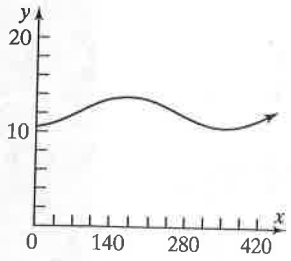


(d) 8.2 ft

. (a) $y = 1.61 \sin\left(\frac{2\pi}{365}x - 1.39\right) + 12.14$

(b) 12.42 h

(c)

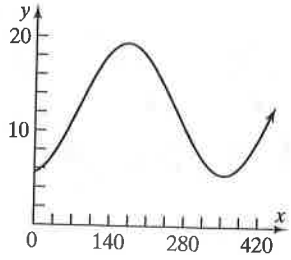


(d) The actual hours of sunlight on April 1, 2005 was 12.43 hours. This is very close to the predicted amount of 12.42 hours.

37. (a) $y = 6.975 \sin\left(\frac{2\pi}{365}x - 1.39\right) + 12.445$

(b) 13.67 h

(c)



(d) The actual hours of sunlight on April 1, 2005 was 13.43 hours. This is close to the predicted amount of 13.67 hours.

eview Exercises *(page 171)*

$\frac{3\pi}{4}$ **3.** $\frac{\pi}{10}$ **5.** 135° **7.** −450° **9.** $\frac{1}{2}$ **11.** $\frac{3\sqrt{2}}{2} - \frac{4\sqrt{3}}{3}$ **13.** $-3\sqrt{2} - 2\sqrt{3}$ **15.** 3 **17.** 0 **19.** 0 **21.** 1 **23.** 1 **25.** 1 **27.** −1 **29.** 1

$\cos\theta = \frac{3}{5}; \tan\theta = \frac{4}{3}; \csc\theta = \frac{5}{4}; \sec\theta = \frac{5}{3}; \cot\theta = \frac{3}{4}$ **33.** $\sin\theta = -\frac{12}{13}; \cos\theta = -\frac{5}{13}; \csc\theta = -\frac{13}{12}; \sec\theta = -\frac{13}{5}; \cot\theta = \frac{5}{12}$

$\sin\theta = \frac{3}{5}; \cos\theta = -\frac{4}{5}; \tan\theta = -\frac{3}{4}; \csc\theta = \frac{5}{3}; \cot\theta = -\frac{4}{3}$ **37.** $\cos\theta = -\frac{5}{13}; \tan\theta = -\frac{12}{5}; \csc\theta = \frac{13}{12}; \sec\theta = -\frac{13}{5}; \cot\theta = -\frac{5}{12}$

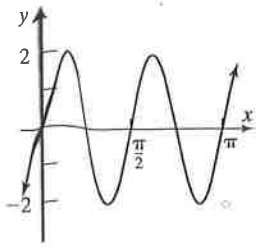
$\cos\theta = \frac{12}{13}; \tan\theta = -\frac{5}{12}; \csc\theta = -\frac{13}{5}; \sec\theta = \frac{13}{12}; \cot\theta = -\frac{12}{5}$

$\sin\theta = -\frac{\sqrt{10}}{10}; \cos\theta = -\frac{3\sqrt{10}}{10}; \csc\theta = -\sqrt{10}; \sec\theta = -\frac{\sqrt{10}}{3}; \cot\theta = 3$

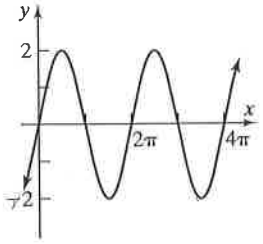
43. $\sin\theta = -\frac{2\sqrt{2}}{3}$; $\cos\theta = \frac{1}{3}$; $\tan\theta = -2\sqrt{2}$; $\csc\theta = -\frac{3\sqrt{2}}{4}$; $\cot\theta = -\frac{\sqrt{2}}{4}$

45. $\sin\theta = \frac{\sqrt{5}}{5}$; $\cos\theta = -\frac{2\sqrt{5}}{5}$; $\tan\theta = -\frac{1}{2}$; $\csc\theta = \sqrt{5}$; $\sec\theta = -\frac{\sqrt{5}}{2}$

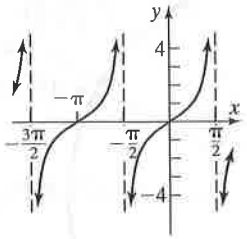
47.



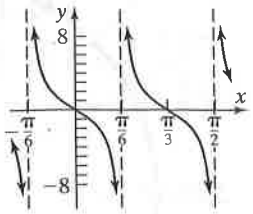
49.



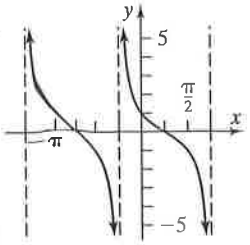
51.



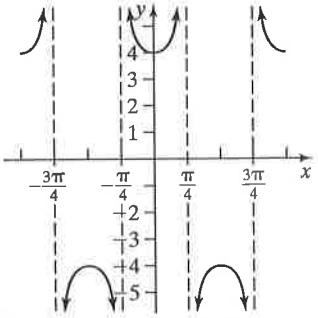
53.



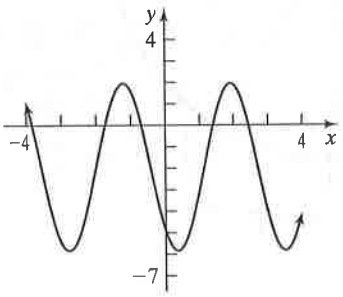
55.



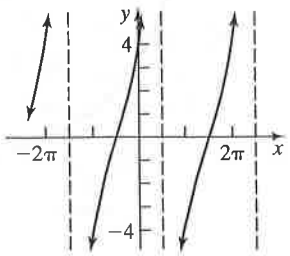
57.



59.



61.

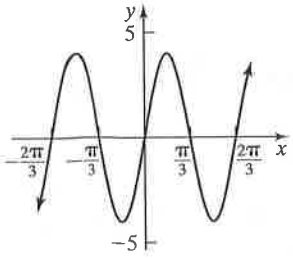


63. Amplitude = 4; period = 2π **65.** Amplitude = 8; period = 4

67. Amplitude = 4

Period = $\frac{2\pi}{3}$

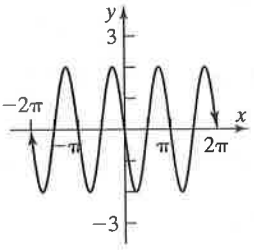
Phase shift = 0



69. Amplitude = 2

Period = π

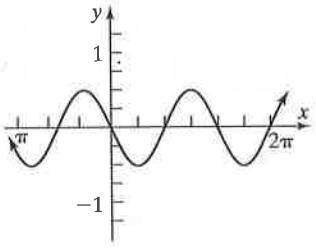
Phase shift = $\frac{\pi}{2}$



71. Amplitude = $\frac{1}{2}$

Period = $\frac{4\pi}{3}$

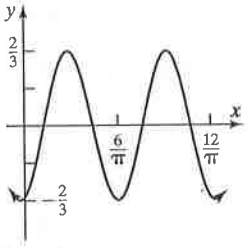
Phase shift = $\frac{2\pi}{3}$



73. Amplitude = $\frac{2}{3}$

Period = 2

Phase shift = $\frac{6}{\pi}$



75. $y = 5\cos\frac{x}{4}$ **77.** $y = -6\cos\left(\frac{\pi}{4}x\right)$ **79.** 0.38 **81.** Sine, cosine, cosecant and secant: negative; tangent and cotangent: positive

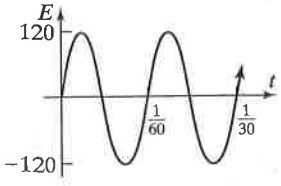
83. $\sin\theta = \frac{2\sqrt{2}}{3}$; $\cos\theta = -\frac{1}{3}$; $\tan\theta = -2\sqrt{2}$; $\csc\theta = \frac{3\sqrt{2}}{4}$; $\sec\theta = -3$; $\cot\theta = -\frac{\sqrt{2}}{4}$

85. Domain: $\left\{x \middle| x \neq \text{odd multiple of } \frac{\pi}{2}\right\}$; range: $\{y||y| \geq 1\}$; period = $2(\pi)$

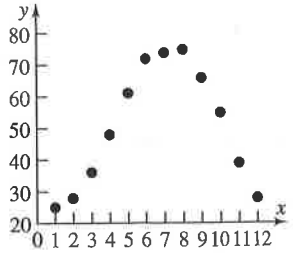
87. $\frac{\pi}{3} \approx 1.05$ ft; $\frac{\pi}{3} \approx 1.05$ ft^2 **89.** Approximately 114.59 revolutions/hr **91.** 0.1 revolution/sec = $\frac{\pi}{5}$ radian/sec

93. **(a)** 120 **(b)** $\frac{1}{60}$

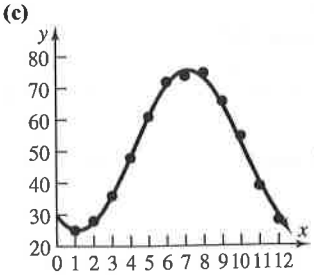
(c)



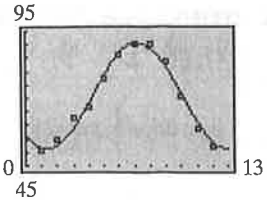
95. **(a)**



(c)

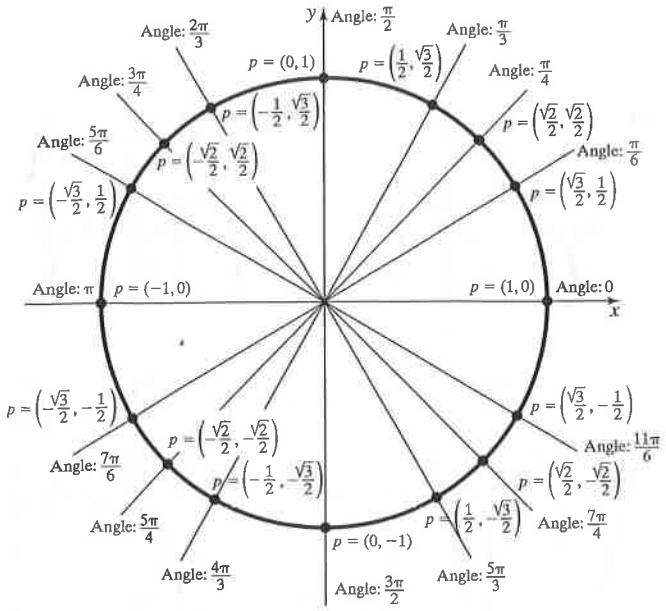


(e)



(b) $y = 19.5\sin\left[\frac{\pi}{6}(x-4)\right] + 70.5$ or $y = 19.5\sin\left(\frac{\pi}{6}x - \frac{2\pi}{3}\right) + 70.5$ **(d)** $y = 19.52\sin(0.54x - 2.28) + 71.01$

97.



Chapter Test *(page 174)*

1. $\frac{13\pi}{9}$ **2.** $-\frac{20\pi}{9}$ **3.** $\frac{13\pi}{180}$ **4.** $-22.5°$ **5.** $810°$ **6.** $135°$ **7.** $\frac{1}{2}$ **8.** 0 **9.** $-\frac{1}{2}$ **10.** $-\frac{\sqrt{3}}{3}$ **11.** 2 **12.** $\frac{3(1-\sqrt{2})}{2}$ **13.** 0.292 **14.** 0.309

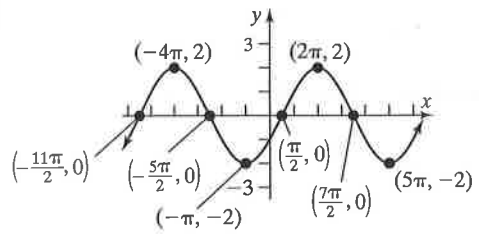
15. -1.524 **16.** 2.747 **17.**

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\sec\theta$	$\csc\theta$	$\cot\theta$
θ in QI	+	+	+	+	+	+
θ in QII	+	−	−	−	+	−
θ in QIII	−	−	+	−	−	+
θ in QIV	−	+	−	+	−	−

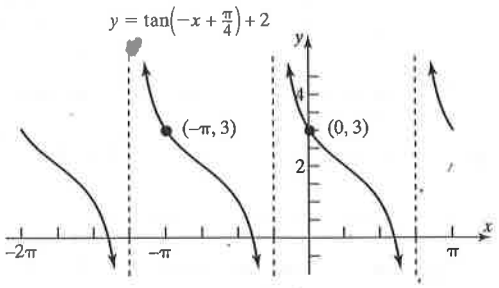
18. $-\frac{3}{5}$

19. $\cos\theta = -\frac{2\sqrt{6}}{7}$; $\tan\theta = -\frac{5\sqrt{6}}{12}$; $\csc\theta = \frac{7}{5}$; $\sec\theta = -\frac{7\sqrt{6}}{12}$; $\cot\theta = -\frac{2\sqrt{6}}{5}$ **20.** $\sin\theta = -\frac{\sqrt{5}}{3}$; $\tan\theta = -\frac{\sqrt{5}}{2}$; $\csc\theta = -\frac{3\sqrt{5}}{5}$; $\sec\theta = \frac{3}{2}$; $\cot\theta = -\frac{2\sqrt{5}}{5}$ **21.** $\sin\theta = \frac{12}{13}$; $\cos\theta = -\frac{5}{13}$; $\csc\theta = \frac{13}{12}$; $\sec\theta = -\frac{13}{5}$; $\cot\theta = -\frac{5}{12}$ **22.** $\frac{7\sqrt{53}}{53}$ **23.** $-\frac{5\sqrt{146}}{146}$ **24.** $-\frac{1}{2}$

25.



26.



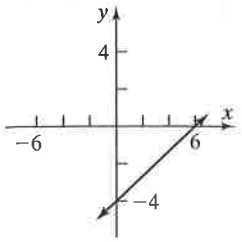
27. $y = -3\sin\left(3x + \frac{3\pi}{4}\right)$

28. 78.93 ft^2 **29.** 143.5 rpm

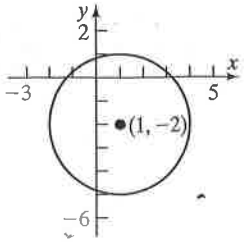
Cumulative Review *(page 175)*

1. $\left\{-1, \frac{1}{2}\right\}$ **2.** $y - 5 = -3(x + 2)$ or $y = -3x - 1$ **3.** $x^2 + (y + 2)^2 = 16$

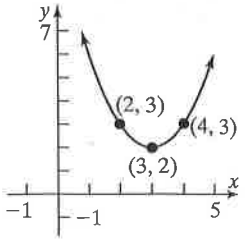
4. A line; slope $\frac{2}{3}$; intercepts $(6, 0)$ and $(0, -4)$



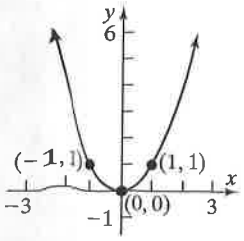
5. A circle; center $(1, -2)$; radius 3



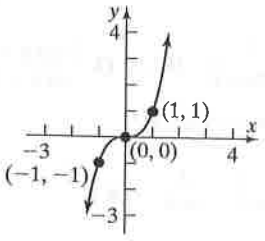
6.



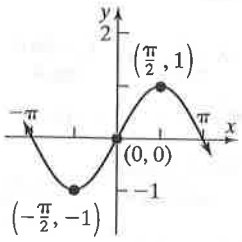
7. (a)



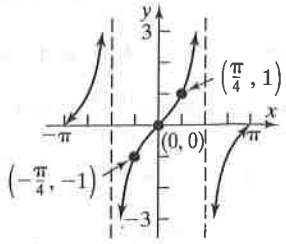
(b)



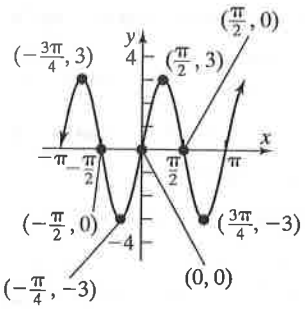
(c)



(d)



8. $f^{-1}(x) = \frac{1}{3}(x + 2)$ **9.** -2 **10.**



11. $3 - \frac{3\sqrt{3}}{2}$

12. $y = 3\cos\left(\frac{\pi}{6}x\right)$

CHAPTER 3 Analytic Trigonometry

3.1 Assess Your Understanding *(page 187)*

7. $x = \sin y$ **8.** $\frac{\pi}{2}$ **9.** $\frac{\pi}{5}$ **10.** F **11.** T **12.** T **13.** 0 **15.** $-\frac{\pi}{2}$ **17.** 0 **19.** $\frac{\pi}{4}$ **21.** $\frac{\pi}{3}$ **23.** $\frac{5\pi}{6}$ **25.** 0.10 **27.** 1.37 **29.** 0.51 **31.** -0.38

33. -0.12 **35.** 1.08 **37.** $\frac{4\pi}{5}$ **39.** $-\frac{3\pi}{8}$ **41.** $-\frac{\pi}{8}$ **43.** $-\frac{\pi}{5}$ **45.** $\frac{1}{4}$ **47.** 4 **49.** Not defined **51.** π

53. $f^{-1}(x) = \sin^{-1}\frac{x-2}{5}$
Domain of f: $(-\infty, \infty)$
Domain of f^{-1}: $[-3, 7]$

55. $f^{-1}(x) = \frac{1}{3}\cos^{-1}\left(-\frac{x}{2}\right)$
Domain of f: $(-\infty, \infty)$
Domain of f^{-1}: $[-2, 2]$

57. $f^{-1}(x) = -\tan^{-1}(x + 3) + 1$
Domain of f: $x \neq \frac{(2k+1)\pi}{2} - 1$; k an integer
Domain of f^{-1}: $(-\infty, \infty)$

59. $f^{-1}(x) = \frac{1}{2}\left[\sin^{-1}\left(\frac{x}{3}\right) - 1\right]$
Domain of f: $(-\infty, \infty)$
Domain of f^{-1}: $[-3, 3]$

61. $\left\{\frac{\sqrt{2}}{2}\right\}$ **63.** $\left\{-\frac{1}{4}\right\}$ **65.** $\{\sqrt{3}\}$ **67.** $\{-1\}$ **69.** (a) 13.92 hr or 13 hr, 55 min (b) 12 hr (c) 13.85 hr or 13 hr, 51 min

71. (a) 13.3 hr or 13 hr, 18 min (b) 12 hr (c) 13.26 hr or 13 hr, 15 min **73.** (a) 12 hr (b) 12 hr (c) 12 hr (d) It is 12 hr. **75.** 3.35 min

77. (a) $\frac{\pi}{3}$ square units (b) $\frac{5\pi}{12}$ square units **79.** 4251 mi

3.2 Assess Your Understanding *(page 194)*

4. $x = \sec y$; ≥ 1; 0; π **5.** $\frac{\sqrt{2}}{2}$ **6.** F **7.** T **8.** T **9.** $\frac{\sqrt{2}}{2}$ **11.** $-\frac{\sqrt{3}}{3}$ **13.** 2 **15.** $\sqrt{2}$ **17.** $-\frac{\sqrt{2}}{2}$ **19.** $\frac{2\sqrt{3}}{3}$ **21.** $\frac{3\pi}{4}$ **23.** $\frac{\pi}{6}$ **25.** $\frac{\sqrt{2}}{4}$

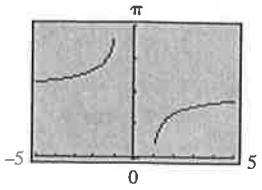
27. $\frac{\sqrt{5}}{2}$ **29.** $-\frac{\sqrt{14}}{2}$ **31.** $-\frac{3\sqrt{10}}{10}$ **33.** $\sqrt{5}$ **35.** $-\frac{\pi}{4}$ **37.** $\frac{\pi}{6}$ **39.** $-\frac{\pi}{2}$ **41.** $\frac{\pi}{6}$ **43.** $\frac{2\pi}{3}$ **45.** 1.32 **47.** 0.46 **49.** -0.34 **51.** 2.72 **53.** -0.73

55. 2.55 **57.** $\frac{1}{\sqrt{1+u^2}}$ **59.** $\frac{u}{\sqrt{1-u^2}}$ **61.** $\frac{\sqrt{u^2-1}}{|u|}$ **63.** $\frac{\sqrt{u^2-1}}{u}$ **65.** $\frac{1}{u}$ **67.** $\frac{5}{13}$ **69.** $\frac{3\pi}{4}$ **71.** $-\frac{3}{4}$ **73.** $\frac{5}{13}$ **75.** $\frac{\pi}{6}$ **77.** $-\sqrt{15}$

79. (a) $\theta = 31.89°$
(b) 54.64 ft in diameter
(c) 37.96 ft high

81. (a) $\theta = 22.3°$
(b) $v_0 = 2940.23$ ft/sec

83.



3.3 Assess Your Understanding *(page 200)*

3. identity; conditional **4.** -1 **5.** 0 **6.** T **7.** F **8.** T **9.** $\frac{1}{\cos\theta}$ **11.** $\frac{1+\sin\theta}{\cos\theta}$ **13.** $\frac{1}{\sin\theta\cos\theta}$ **15.** 2 **17.** $\frac{3\sin\theta+1}{\sin\theta+1}$

19. $\csc\theta\cdot\cos\theta=\frac{1}{\sin\theta}\cdot\cos\theta=\frac{\cos\theta}{\sin\theta}=\cot\theta$ **21.** $1+\tan^2(-\theta)=1+(-\tan\theta)^2=1+\tan^2\theta=\sec^2\theta$

23. $\cos\theta(\tan\theta+\cot\theta)=\cos\theta\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)=\cos\theta\left(\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)=\cos\theta\left(\frac{1}{\cos\theta\sin\theta}\right)=\frac{1}{\sin\theta}=\csc\theta$

25. $\tan u\cot u-\cos^2 u=\frac{\sin u}{\cos u}\cdot\frac{\cos u}{\sin u}-\cos^2 u=1-\cos^2 u=\sin^2 u$

27. $(\sec\theta-1)(\sec\theta+1)=\sec^2\theta-1=\tan^2\theta$ **29.** $(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)=\sec^2\theta-\tan^2\theta=1$

31. $\cos^2\theta(1+\tan^2\theta)=\cos^2\theta+\cos^2\theta\tan^2\theta=\cos^2\theta+\cos^2\theta\cdot\frac{\sin^2\theta}{\cos^2\theta}=\cos^2\theta+\sin^2\theta=1$

33. $(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2=\sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta+\sin^2\theta-2\sin\theta\cos\theta+\cos^2\theta$

$=\sin^2\theta+\cos^2\theta+\sin^2\theta+\cos^2\theta=1+1=2$

35. $\sec^4\theta-\sec^2\theta=\sec^2\theta(\sec^2\theta-1)=(1+\tan^2\theta)\tan^2\theta=\tan^4\theta+\tan^2\theta$

37. $\sec u-\tan u=\frac{1}{\cos u}-\frac{\sin u}{\cos u}=\frac{1-\sin u}{\cos u}\cdot\frac{1+\sin u}{1+\sin u}=\frac{1-\sin^2 u}{\cos u(1+\sin u)}=\frac{\cos^2 u}{\cos u(1+\sin u)}=\frac{\cos u}{1+\sin u}$

39. $3\sin^2\theta+4\cos^2\theta=3\sin^2\theta+3\cos^2\theta+\cos^2\theta=3(\sin^2\theta+\cos^2\theta)+\cos^2\theta=3+\cos^2\theta$

41. $1-\frac{\cos^2\theta}{1+\sin\theta}=1-\frac{1-\sin^2\theta}{1+\sin\theta}=1-\frac{(1+\sin\theta)(1-\sin\theta)}{1+\sin\theta}=1-(1-\sin\theta)=\sin\theta$

43. $\frac{1+\tan v}{1-\tan v}=\frac{1+\frac{1}{\cot v}}{1-\frac{1}{\cot v}}=\frac{\frac{\cot v+1}{\cot v}}{\frac{\cot v-1}{\cot v}}=\frac{\cot v+1}{\cot v-1}$ **45.** $\frac{\sec\theta}{\csc\theta}+\frac{\sin\theta}{\cos\theta}=\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}+\tan\theta=\frac{\sin\theta}{\cos\theta}+\tan\theta=\tan\theta+\tan\theta=2\tan\theta$

47. $\frac{1+\sin\theta}{1-\sin\theta}=\frac{1+\frac{1}{\csc\theta}}{1-\frac{1}{\csc\theta}}=\frac{\frac{\csc\theta+1}{\csc\theta}}{\frac{\csc\theta-1}{\csc\theta}}=\frac{\csc\theta+1}{\csc\theta-1}$

49. $\frac{1-\sin v}{\cos v}+\frac{\cos v}{1-\sin v}=\frac{(1-\sin v)^2+\cos^2 v}{\cos v(1-\sin v)}=\frac{1-2\sin v+\sin^2 v+\cos^2 v}{\cos v(1-\sin v)}=\frac{2-2\sin v}{\cos v(1-\sin v)}=\frac{2(1-\sin v)}{\cos v(1-\sin v)}=\frac{2}{\cos v}=2\sec v$

51. $\frac{\sin\theta}{\sin\theta-\cos\theta}=\frac{1}{\frac{\sin\theta-\cos\theta}{\sin\theta}}=\frac{1}{1-\frac{\cos\theta}{\sin\theta}}=\frac{1}{1-\cot\theta}$

53. $(\sec\theta-\tan\theta)^2=\sec^2\theta-2\sec\theta\tan\theta+\tan^2\theta=\frac{1}{\cos^2\theta}-\frac{2\sin\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}=\frac{1-2\sin\theta+\sin^2\theta}{\cos^2\theta}=\frac{(1-\sin\theta)^2}{1-\sin^2\theta}=\frac{(1-\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$

$=\frac{1-\sin\theta}{1+\sin\theta}$

55. $\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}=\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}+\frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}=\frac{\cos\theta}{\frac{\cos\theta-\sin\theta}{\cos\theta}}+\frac{\sin\theta}{\frac{\sin\theta-\cos\theta}{\sin\theta}}=\frac{\cos^2\theta}{\cos\theta-\sin\theta}+\frac{\sin^2\theta}{\sin\theta-\cos\theta}$

$=\frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}=\frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{\cos\theta-\sin\theta}=\sin\theta+\cos\theta$

57. $\tan\theta+\frac{\cos\theta}{1+\sin\theta}=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=\frac{\sin\theta(1+\sin\theta)+\cos^2\theta}{\cos\theta(1+\sin\theta)}=\frac{\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}=\frac{\sin\theta+1}{\cos\theta(1+\sin\theta)}=\frac{1}{\cos\theta}=\sec\theta$

59. $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{\tan\theta+(\sec\theta-1)}{\tan\theta-(\sec\theta-1)}\cdot\frac{\tan\theta+(\sec\theta-1)}{\tan\theta+(\sec\theta-1)}=\frac{\tan^2\theta+2\tan\theta(\sec\theta-1)+\sec^2\theta-2\sec\theta+1}{\tan^2\theta-(\sec^2\theta-2\sec\theta+1)}$

$=\frac{\sec^2\theta-1+2\tan\theta(\sec\theta-1)+\sec^2\theta-2\sec\theta+1}{\sec^2\theta-1-\sec^2\theta+2\sec\theta-1}=\frac{2\sec^2\theta-2\sec\theta+2\tan\theta(\sec\theta-1)}{-2+2\sec\theta}$

$=\frac{2\sec\theta(\sec\theta-1)+2\tan\theta(\sec\theta-1)}{2(\sec\theta-1)}=\frac{2(\sec\theta-1)(\sec\theta+\tan\theta)}{2(\sec\theta-1)}=\tan\theta+\sec\theta$

61. $\frac{\tan\theta-\cot\theta}{\tan\theta+\cot\theta}=\frac{\frac{\sin\theta}{\cos\theta}-\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}=\frac{\frac{\sin^2\theta-\cos^2\theta}{\cos\theta\sin\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}}=\frac{\sin^2\theta-\cos^2\theta}{1}=\sin^2\theta-\cos^2\theta$

63. $\dfrac{\tan u - \cot u}{\tan u + \cot u} + 1 = \dfrac{\dfrac{\sin u}{\cos u} - \dfrac{\cos u}{\sin u}}{\dfrac{\sin u}{\cos u} + \dfrac{\cos u}{\sin u}} + 1 = \dfrac{\dfrac{\sin^2 u - \cos^2 u}{\cos u \sin u}}{\dfrac{\sin^2 u + \cos^2 u}{\cos u \sin u}} + 1 = \sin^2 u - \cos^2 u + 1 = \sin^2 u + (1 - \cos^2 u) = 2\sin^2 u$

65. $\dfrac{\sec\theta + \tan\theta}{\cot\theta + \cos\theta} = \dfrac{\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}}{\dfrac{\cos\theta}{\sin\theta} + \cos\theta} = \dfrac{\dfrac{1 + \sin\theta}{\cos\theta}}{\dfrac{\cos\theta + \cos\theta\sin\theta}{\sin\theta}} = \dfrac{1 + \sin\theta}{\cos\theta} \cdot \dfrac{\sin\theta}{\cos\theta(1 + \sin\theta)} = \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{1}{\cos\theta} = \tan\theta\sec\theta$

67. $\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta} + 1 = \dfrac{1 - \tan^2\theta}{\sec^2\theta} + 1 = \dfrac{1}{\sec^2\theta} - \dfrac{\tan^2\theta}{\sec^2\theta} + 1 = \cos^2\theta - \dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{1}{\cos^2\theta}} + 1 = \cos^2\theta - \sin^2\theta + 1 = \cos^2\theta + (1 - \sin^2\theta) = 2\cos^2\theta$

69. $\dfrac{\sec\theta - \csc\theta}{\sec\theta\csc\theta} = \dfrac{\sec\theta}{\sec\theta\csc\theta} - \dfrac{\csc\theta}{\sec\theta\csc\theta} = \dfrac{1}{\csc\theta} - \dfrac{1}{\sec\theta} = \sin\theta - \cos\theta$

71. $\sec\theta - \cos\theta = \dfrac{1}{\cos\theta} - \cos\theta = \dfrac{1 - \cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta} = \sin\theta \cdot \dfrac{\sin\theta}{\cos\theta} = \sin\theta\tan\theta$

73. $\dfrac{1}{1 - \sin\theta} + \dfrac{1}{1 + \sin\theta} = \dfrac{1 + \sin\theta + 1 - \sin\theta}{(1 + \sin\theta)(1 - \sin\theta)} = \dfrac{2}{1 - \sin^2\theta} = \dfrac{2}{\cos^2\theta} = 2\sec^2\theta$

75. $\dfrac{\sec\theta}{1 - \sin\theta} = \dfrac{\sec\theta}{1 - \sin\theta} \cdot \dfrac{1 + \sin\theta}{1 + \sin\theta} = \dfrac{\sec\theta(1 + \sin\theta)}{1 - \sin^2\theta} = \dfrac{\sec\theta(1 + \sin\theta)}{\cos^2\theta} = \dfrac{1 + \sin\theta}{\cos^3\theta}$

77. $\dfrac{(\sec v - \tan v)^2 + 1}{\csc v(\sec v - \tan v)} = \dfrac{\sec^2 v - 2\sec v\tan v + \tan^2 v + 1}{\dfrac{1}{\sin v}\left(\dfrac{1}{\cos v} - \dfrac{\sin v}{\cos v}\right)} = \dfrac{2\sec^2 v - 2\sec v\tan v}{\dfrac{1}{\sin v}\left(\dfrac{1 - \sin v}{\cos v}\right)} = \dfrac{\dfrac{2}{\cos^2 v} - \dfrac{2\sin v}{\cos^2 v}}{\dfrac{1 - \sin v}{\sin v\cos v}} = \dfrac{2 - 2\sin v}{\cos^2 v} \cdot \dfrac{\sin v\cos v}{1 - \sin v}$

$= \dfrac{2(1 - \sin v)}{\cos v} \cdot \dfrac{\sin v}{1 - \sin v} = \dfrac{2\sin v}{\cos v} = 2\tan v$

79. $\dfrac{\sin\theta + \cos\theta}{\cos\theta} - \dfrac{\sin\theta - \cos\theta}{\sin\theta} = \dfrac{\sin\theta}{\cos\theta} + 1 - 1 + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta} = \dfrac{1}{\cos\theta\sin\theta} = \sec\theta\csc\theta$

81. $\dfrac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = \dfrac{(\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\cos\theta + \cos^2\theta)}{\sin\theta + \cos\theta} = \sin^2\theta + \cos^2\theta - \sin\theta\cos\theta = 1 - \sin\theta\cos\theta$

83. $\dfrac{\cos^2\theta - \sin^2\theta}{1 - \tan^2\theta} = \dfrac{\cos^2\theta - \sin^2\theta}{1 - \dfrac{\sin^2\theta}{\cos^2\theta}} = \dfrac{\cos^2\theta - \sin^2\theta}{\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}} = \cos^2\theta$

85. $\dfrac{(2\cos^2\theta - 1)^2}{\cos^4\theta - \sin^4\theta} = \dfrac{[2\cos^2\theta - (\sin^2\theta + \cos^2\theta)]^2}{(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)} = \dfrac{(\cos^2\theta - \sin^2\theta)^2}{\cos^2\theta - \sin^2\theta} = \cos^2\theta - \sin^2\theta = (1 - \sin^2\theta) - \sin^2\theta = 1 - 2\sin^2\theta$

87. $\dfrac{1 + \sin\theta + \cos\theta}{1 + \sin\theta - \cos\theta} = \dfrac{(1 + \sin\theta) + \cos\theta}{(1 + \sin\theta) - \cos\theta} \cdot \dfrac{(1 + \sin\theta) + \cos\theta}{(1 + \sin\theta) + \cos\theta} = \dfrac{1 + 2\sin\theta + \sin^2\theta + 2(1 + \sin\theta)\cos\theta + \cos^2\theta}{1 + 2\sin\theta + \sin^2\theta - \cos^2\theta}$

$= \dfrac{1 + 2\sin\theta + \sin^2\theta + 2(1 + \sin\theta)(\cos\theta) + (1 - \sin^2\theta)}{1 + 2\sin\theta + \sin^2\theta - (1 - \sin^2\theta)} = \dfrac{2 + 2\sin\theta + 2(1 + \sin\theta)(\cos\theta)}{2\sin\theta + 2\sin^2\theta}$

$= \dfrac{2(1 + \sin\theta) + 2(1 + \sin\theta)(\cos\theta)}{2\sin\theta(1 + \sin\theta)} = \dfrac{2(1 + \sin\theta)(1 + \cos\theta)}{2\sin\theta(1 + \sin\theta)} = \dfrac{1 + \cos\theta}{\sin\theta}$

89. $(a\sin\theta + b\cos\theta)^2 + (a\cos\theta - b\sin\theta)^2 = a^2\sin^2\theta + 2ab\sin\theta\cos\theta + b^2\cos^2\theta + a^2\cos^2\theta - 2ab\sin\theta\cos\theta + b^2\sin^2\theta$

$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\cos^2\theta + \sin^2\theta) = a^2 + b^2$

91. $\dfrac{\tan\alpha + \tan\beta}{\cot\alpha + \cot\beta} = \dfrac{\tan\alpha + \tan\beta}{\dfrac{1}{\tan\alpha} + \dfrac{1}{\tan\beta}} = \dfrac{\tan\alpha + \tan\beta}{\dfrac{\tan\beta + \tan\alpha}{\tan\alpha\tan\beta}} = (\tan\alpha + \tan\beta) \cdot \dfrac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta} = \tan\alpha\tan\beta$

93. $(\sin\alpha + \cos\beta)^2 + (\cos\beta + \sin\alpha)(\cos\beta - \sin\alpha) = (\sin^2\alpha + 2\sin\alpha\cos\beta + \cos^2\beta) + (\cos^2\beta - \sin^2\alpha)$

$= 2\cos^2\beta + 2\sin\alpha\cos\beta = 2\cos\beta(\cos\beta + \sin\alpha)$

95. $\ln|\sec\theta| = \ln|\cos\theta|^{-1} = -\ln|\cos\theta|$

97. $\ln|1 + \cos\theta| + \ln|1 - \cos\theta| = \ln(|1 + \cos\theta||1 - \cos\theta|) = \ln|1 - \cos^2\theta| = \ln|\sin^2\theta| = 2\ln|\sin\theta|$

99. $g(x) = \sec x - \cos x = \dfrac{1}{\cos x} - \cos x = \dfrac{1}{\cos x} - \dfrac{\cos^2 x}{\cos x} = \dfrac{1 - \cos^2 x}{\cos x} = \dfrac{\sin^2 x}{\cos x} = \sin x \cdot \dfrac{\sin x}{\cos x} = \sin x \cdot \tan x = f(x)$

101. $f(\theta) = \dfrac{1-\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1-\sin\theta}{\cos\theta}\cdot\dfrac{1+\sin\theta}{1+\sin\theta} - \dfrac{\cos\theta}{1+\sin\theta}\cdot\dfrac{\cos\theta}{\cos\theta} = \dfrac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)} - \dfrac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$

$= \dfrac{\cos^2\theta}{\cos\theta(1+\sin\theta)} - \dfrac{\cos^2\theta}{\cos\theta(1+\sin\theta)} = 0 = g(\theta)$

103. $1200\sec\theta\,(2\sec^2\theta - 1) = 1200\dfrac{1}{\cos\theta}\left(\dfrac{2}{\cos^2\theta} - 1\right) = 1200\dfrac{1}{\cos\theta}\left(\dfrac{2}{\cos^2\theta} - \dfrac{\cos^2\theta}{\cos^2\theta}\right) = 1200\dfrac{1}{\cos\theta}\left(\dfrac{2-\cos^2\theta}{\cos^2\theta}\right) = \dfrac{1200(1+1-\cos^2\theta)}{\cos^3\theta}$

$= \dfrac{1200\,(1+\sin^2\theta)}{\cos^3\theta}$

3.4 Assess Your Understanding *(page 210)*

4. – **5.** – **6.** F **7.** F **8.** F **9.** $\frac{1}{4}(\sqrt{6}+\sqrt{2})$ **11.** $\frac{1}{4}(\sqrt{2}-\sqrt{6})$ **13.** $-\frac{1}{4}(\sqrt{2}+\sqrt{6})$ **15.** $2-\sqrt{3}$

17. $-\frac{1}{4}(\sqrt{6}+\sqrt{2})$ **19.** $\sqrt{6}-\sqrt{2}$ **21.** $\frac{1}{2}$ **23.** 0 **25.** 1 **27.** -1 **29.** $\frac{1}{2}$ **31.** (a) $\frac{2\sqrt{5}}{25}$ (b) $\frac{11\sqrt{5}}{25}$ (c) $\frac{2\sqrt{5}}{5}$ (d) 2

33. (a) $\frac{4-3\sqrt{3}}{10}$ (b) $\frac{-3-4\sqrt{3}}{10}$ (c) $\frac{4+3\sqrt{3}}{10}$ (d) $\frac{25\sqrt{3}+48}{39}$ **35.** (a) $-\frac{5+12\sqrt{3}}{26}$ (b) $\frac{12-5\sqrt{3}}{26}$ (c) $\frac{-5+12\sqrt{3}}{26}$ (d) $\frac{-240+169\sqrt{3}}{69}$

37. (a) $-\frac{2\sqrt{2}}{3}$ (b) $\frac{-2\sqrt{2}+\sqrt{3}}{6}$ (c) $\frac{-2\sqrt{2}+\sqrt{3}}{6}$ (d) $\frac{9-4\sqrt{2}}{7}$ **39.** $\frac{1-2\sqrt{6}}{6}$ **41.** $\frac{\sqrt{3}-2\sqrt{2}}{6}$ **43.** $\frac{8\sqrt{2}-9\sqrt{3}}{5}$

45. $\sin\left(\dfrac{\pi}{2}+\theta\right) = \sin\dfrac{\pi}{2}\cos\theta + \cos\dfrac{\pi}{2}\sin\theta = 1\cdot\cos\theta + 0\cdot\sin\theta = \cos\theta$

47. $\sin(\pi-\theta) = \sin\pi\cos\theta - \cos\pi\sin\theta = 0\cdot\cos\theta - (-1)\sin\theta = \sin\theta$

49. $\sin(\pi+\theta) = \sin\pi\cos\theta + \cos\pi\sin\theta = 0\cdot\cos\theta + (-1)\sin\theta = -\sin\theta$

51. $\tan(\pi-\theta) = \dfrac{\tan\pi - \tan\theta}{1+\tan\pi\tan\theta} = \dfrac{0-\tan\theta}{1+0\cdot\tan\theta} = -\tan\theta$ **53.** $\sin\left(\dfrac{3\pi}{2}+\theta\right) = \sin\dfrac{3\pi}{2}\cos\theta + \cos\dfrac{3\pi}{2}\sin\theta = (-1)\cos\theta + 0\cdot\sin\theta = -\cos\theta$

55. $\sin(\alpha+\beta) + \sin(\alpha-\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta + \sin\alpha\cos\beta - \cos\alpha\sin\beta = 2\sin\alpha\cos\beta$

57. $\dfrac{\sin(\alpha+\beta)}{\sin\alpha\cos\beta} = \dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta} = \dfrac{\sin\alpha\cos\beta}{\sin\alpha\cos\beta} + \dfrac{\cos\alpha\sin\beta}{\sin\alpha\cos\beta} = 1 + \cot\alpha\tan\beta$

59. $\dfrac{\cos(\alpha+\beta)}{\cos\alpha\cos\beta} = \dfrac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta} = \dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} - \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta} = 1 - \tan\alpha\tan\beta$

61. $\dfrac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta - \cos\alpha\sin\beta} = \dfrac{\dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\cos\beta}} = \dfrac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} - \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}} = \dfrac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}$

63. $\cot(\alpha+\beta) = \dfrac{\cos(\alpha+\beta)}{\sin(\alpha+\beta)} = \dfrac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\sin\alpha\cos\beta + \cos\alpha\sin\beta} = \dfrac{\dfrac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\sin\alpha\sin\beta}}{\dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\sin\alpha\sin\beta}} = \dfrac{\dfrac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} - \dfrac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}}{\dfrac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta} + \dfrac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}} = \dfrac{\cot\alpha\cot\beta - 1}{\cot\beta + \cot\alpha}$

65. $\sec(\alpha+\beta) = \dfrac{1}{\cos(\alpha+\beta)} = \dfrac{1}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} = \dfrac{\dfrac{1}{\sin\alpha\sin\beta}}{\dfrac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\sin\alpha\sin\beta}} = \dfrac{\dfrac{1}{\sin\alpha}\cdot\dfrac{1}{\sin\beta}}{\dfrac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta} - \dfrac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}} = \dfrac{\csc\alpha\csc\beta}{\cot\alpha\cot\beta - 1}$

67. $\sin(\alpha-\beta)\sin(\alpha+\beta) = (\sin\alpha\cos\beta - \cos\alpha\sin\beta)(\sin\alpha\cos\beta + \cos\alpha\sin\beta) = \sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta$

$= (\sin^2\alpha)(1-\sin^2\beta) - (1-\sin^2\alpha)(\sin^2\beta) = \sin^2\alpha - \sin^2\beta$

69. $\sin(\theta+k\pi) = \sin\theta\cos k\pi + \cos\theta\sin k\pi = (\sin\theta)(-1)^k + (\cos\theta)(0) = (-1)^k\sin\theta$, k any integer

71. $\frac{\sqrt{3}}{2}$ **73.** $-\frac{24}{25}$ **75.** $-\frac{33}{65}$ **77.** $\frac{63}{65}$ **79.** $\frac{48+25\sqrt{3}}{39}$ **81.** $\frac{4}{3}$ **83.** $u\sqrt{1-v^2} - v\sqrt{1-u^2}$: $-1 \le u \le 1$; $-1 \le v \le 1$

85. $\dfrac{u\sqrt{1-v^2}-v}{\sqrt{1+u^2}}$: $-\infty < u < \infty$; $-1 \le v \le 1$ **87.** $\dfrac{uv - \sqrt{1-u^2}\sqrt{1-v^2}}{v\sqrt{1-u^2} + u\sqrt{1-v^2}}$: $-1 \le u \le 1$; $-1 \le v \le 1$

89. Let $\alpha = \sin^{-1} v$ and $\beta = \cos^{-1} v$. Then $\sin\alpha = \cos\beta = v$, and since $\sin\alpha = \cos\left(\frac{\pi}{2} - \alpha\right)$, $\cos\left(\frac{\pi}{2} - \alpha\right) = \cos\beta$.

If $v \geq 0$, then $0 \leq \alpha \leq \frac{\pi}{2}$, so $\left(\frac{\pi}{2} - \alpha\right)$ and β both lie on $\left[0, \frac{\pi}{2}\right]$. If $v < 0$, then $-\frac{\pi}{2} \leq \alpha < 0$, so $\left(\frac{\pi}{2} - \alpha\right)$ and β both lie on $\left(\frac{\pi}{2}, \pi\right]$.

Either way, $\cos\left(\frac{\pi}{2} - \alpha\right) = \cos\beta$ implies $\frac{\pi}{2} - \alpha = \beta$, or $\alpha + \beta = \frac{\pi}{2}$.

91. Let $\alpha = \tan^{-1}\frac{1}{v}$ and $\beta = \tan^{-1} v$. Because $v \neq 0$, $\alpha, \beta \neq 0$. Then $\tan\alpha = \frac{1}{v} = \frac{1}{\tan\beta} = \cot\beta$, and since $\tan\alpha = \cot\left(\frac{\pi}{2} - \alpha\right)$, $\cot\left(\frac{\pi}{2} - \alpha\right) = \cot\beta$. Because $v > 0$, $0 < \alpha < \frac{\pi}{2}$, and so $\left(\frac{\pi}{2} - \alpha\right)$ and β both lie on $\left(0, \frac{\pi}{2}\right)$.

Then $\cot\left(\frac{\pi}{2} - \alpha\right) = \cot\beta$ implies $\frac{\pi}{2} - \alpha = \beta$, or $\alpha = \frac{\pi}{2} - \beta$.

93. $\sin(\sin^{-1} v + \cos^{-1} v) = \sin(\sin^{-1} v)\cos(\cos^{-1} v) + \cos(\sin^{-1} v)\sin(\cos^{-1} v) = (v)(v) + \sqrt{1 - v^2}\sqrt{1 - v^2} = v^2 + 1 - v^2 = 1$

95. $\dfrac{\sin(x + h) - \sin x}{h} = \dfrac{\sin x\cos h + \cos x\sin h - \sin x}{h} = \dfrac{\cos x\sin h - \sin x(1 - \cos h)}{h} = \cos x \cdot \dfrac{\sin h}{h} - \sin x \cdot \dfrac{1 - \cos h}{h}$

97. (a) $\tan(\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3) = \tan((\tan^{-1}1 + \tan^{-1}2) + \tan^{-1}3) = \dfrac{\tan(\tan^{-1}1 + \tan^{-1}2) + \tan(\tan^{-1}3)}{1 - \tan(\tan^{-1}1 + \tan^{-1}2)\tan(\tan^{-1}3)}$

$$= \frac{\dfrac{\tan(\tan^{-1}1) + \tan(\tan^{-1}2)}{1 - \tan(\tan^{-1}1)\tan(\tan^{-1}2)} + 3}{1 - \dfrac{\tan(\tan^{-1}1) + \tan(\tan^{-1}2)}{1 - \tan(\tan^{-1}1)\tan(\tan^{-1}2)} \cdot 3} = \frac{\dfrac{1 + 2}{1 - 1 \cdot 2} + 3}{1 - \dfrac{1 + 2}{1 - 1 \cdot 2} \cdot 3} = \frac{\dfrac{3}{-1} + 3}{1 - \dfrac{3}{-1} \cdot 3} = \frac{-3 + 3}{1 + 9} = \frac{0}{10} = 0$$

(b) From the definition of the inverse tangent function $0 < \tan^{-1}1 < \frac{\pi}{2}$, $0 < \tan^{-1}2 < \frac{\pi}{2}$, and $0 < \tan^{-1}3 < \frac{\pi}{2}$, so $0 < \tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 < \frac{3\pi}{2}$.

On the interval $\left(0, \frac{3\pi}{2}\right)$, $\tan\theta = 0$ if and only if $\theta = \pi$. Therefore, from part (a), $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$.

99. $\tan\theta = \tan(\theta_2 - \theta_1) = \dfrac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_1\tan\theta_2} = \dfrac{m_2 - m_1}{1 + m_1 m_2}$

101. $2\cot(\alpha - \beta) = \dfrac{2}{\tan(\alpha - \beta)} = 2\left(\dfrac{1 + \tan\alpha\tan\beta}{\tan\alpha - \tan\beta}\right) = 2\left(\dfrac{1 + (x + 1)(x - 1)}{(x + 1) - (x - 1)}\right) = 2\left(\dfrac{1 + x^2 - 1}{x + 1 - x + 1}\right) = \dfrac{2x^2}{2} = x^2$

103. $\tan\frac{\pi}{2}$ is not defined; $\tan\left(\frac{\pi}{2} - \theta\right) = \dfrac{\sin\left(\frac{\pi}{2} - \theta\right)}{\cos\left(\frac{\pi}{2} - \theta\right)} = \dfrac{\cos\theta}{\sin\theta} = \cot\theta$.

3.5 Assess Your Understanding *(page 219)*

1. $\sin^2\theta$; $2\cos^2\theta$; $2\sin^2\theta$ **2.** $1 - \cos\theta$ **3.** $\sin\theta$ **4.** T **5.** F **6.** F **7. (a)** $\frac{24}{25}$ **(b)** $\frac{7}{25}$ **(c)** $\frac{\sqrt{10}}{10}$ **(d)** $\frac{3\sqrt{10}}{10}$

9. (a) $\frac{24}{25}$ **(b)** $-\frac{7}{25}$ **(c)** $\frac{2\sqrt{5}}{5}$ **(d)** $-\frac{\sqrt{5}}{5}$ **11. (a)** $-\frac{2\sqrt{2}}{3}$ **(b)** $\frac{1}{3}$ **(c)** $\sqrt{\frac{3 + \sqrt{6}}{6}}$ **(d)** $\sqrt{\frac{3 - \sqrt{6}}{6}}$

13. (a) $\frac{4\sqrt{2}}{9}$ **(b)** $-\frac{7}{9}$ **(c)** $\frac{\sqrt{3}}{3}$ **(d)** $\frac{\sqrt{6}}{3}$ **15. (a)** $-\frac{4}{5}$ **(b)** $\frac{3}{5}$ **(c)** $\sqrt{\frac{5 + 2\sqrt{5}}{10}}$ **(d)** $\sqrt{\frac{5 - 2\sqrt{5}}{10}}$

17. (a) $-\frac{3}{5}$ **(b)** $-\frac{4}{5}$ **(c)** $\frac{1}{2}\sqrt{\frac{10 - \sqrt{10}}{5}}$ **(d)** $-\frac{1}{2}\sqrt{\frac{10 + \sqrt{10}}{5}}$ **19.** $\frac{\sqrt{2 - \sqrt{2}}}{2}$ **21.** $1 - \sqrt{2}$ **23.** $-\frac{\sqrt{2 + \sqrt{3}}}{2}$

25. $\dfrac{2}{\sqrt{2 + \sqrt{2}}} = (2 - \sqrt{2})\sqrt{2 + \sqrt{2}}$ **27.** $-\frac{\sqrt{2 - \sqrt{2}}}{2}$ **29.** $-\frac{4}{5}$ **31.** $\frac{\sqrt{10(5 - \sqrt{5})}}{10}$ **33.** $\frac{4}{3}$ **35.** $-\frac{7}{8}$ **37.** $\frac{\sqrt{10}}{4}$ **39.** $-\frac{\sqrt{15}}{3}$

41. $\sin^4\theta = (\sin^2\theta)^2 = \left(\dfrac{1 - \cos(2\theta)}{2}\right)^2 = \frac{1}{4}[1 - 2\cos(2\theta) + \cos^2(2\theta)] = \frac{1}{4} - \frac{1}{2}\cos(2\theta) + \frac{1}{4}\cos^2(2\theta)$

$= \frac{1}{4} - \frac{1}{2}\cos(2\theta) + \frac{1}{4}\left(\dfrac{1 + \cos(4\theta)}{2}\right) = \frac{1}{4} - \frac{1}{2}\cos(2\theta) + \frac{1}{8} + \frac{1}{8}\cos(4\theta) = \frac{3}{8} - \frac{1}{2}\cos(2\theta) + \frac{1}{8}\cos(4\theta)$

43. $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$ **45.** $\sin(5\theta) = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$ **47.** $\cos^4\theta - \sin^4\theta = (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \cos(2\theta)$

49. $\cot(2\theta) = \dfrac{1}{\tan(2\theta)} = \dfrac{1 - \tan^2\theta}{2\tan\theta} = \dfrac{1 - \dfrac{1}{\cot^2\theta}}{2\left(\dfrac{1}{\cot\theta}\right)} = \dfrac{\dfrac{\cot^2\theta - 1}{\cot^2\theta}}{\dfrac{2}{\cot\theta}} = \dfrac{\cot^2\theta - 1}{\cot^2\theta} \cdot \dfrac{\cot\theta}{2} = \dfrac{\cot^2\theta - 1}{2\cot\theta}$

51. $\sec(2\theta) = \dfrac{1}{\cos(2\theta)} = \dfrac{1}{2\cos^2\theta - 1} = \dfrac{1}{\dfrac{2}{\sec^2\theta} - 1} = \dfrac{1}{\dfrac{2 - \sec^2\theta}{\sec^2\theta}} = \dfrac{\sec^2\theta}{2 - \sec^2\theta}$ **53.** $\cos^2(2u) - \sin^2(2u) = \cos[2(2u)] = \cos(4u)$

55. $\dfrac{\cos(2\theta)}{1 + \sin(2\theta)} = \dfrac{\cos^2\theta - \sin^2\theta}{1 + 2\sin\theta\cos\theta} = \dfrac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta} = \dfrac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\sin\theta + \cos\theta)(\sin\theta + \cos\theta)} = \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

$= \dfrac{\dfrac{\cos\theta - \sin\theta}{\sin\theta}}{\dfrac{\cos\theta + \sin\theta}{\sin\theta}} = \dfrac{\dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\sin\theta}}{\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\sin\theta}} = \dfrac{\cot\theta - 1}{\cot\theta + 1}$

57. $\sec^2\dfrac{\theta}{2} = \dfrac{1}{\cos^2\left(\dfrac{\theta}{2}\right)} = \dfrac{1}{\dfrac{1 + \cos\theta}{2}} = \dfrac{2}{1 + \cos\theta}$

59. $\cot^2\dfrac{v}{2} = \dfrac{1}{\tan^2\left(\dfrac{v}{2}\right)} = \dfrac{1}{\dfrac{1 - \cos v}{1 + \cos v}} = \dfrac{1 + \cos v}{1 - \cos v} = \dfrac{1 + \dfrac{1}{\sec v}}{1 - \dfrac{1}{\sec v}} = \dfrac{\dfrac{\sec v + 1}{\sec v}}{\dfrac{\sec v - 1}{\sec v}} = \dfrac{\sec v + 1}{\sec v} \cdot \dfrac{\sec v}{\sec v - 1} = \dfrac{\sec v + 1}{\sec v - 1}$

61. $\dfrac{1 - \tan^2\left(\dfrac{\theta}{2}\right)}{1 + \tan^2\left(\dfrac{\theta}{2}\right)} = \dfrac{1 - \dfrac{1 - \cos\theta}{1 + \cos\theta}}{1 + \dfrac{1 - \cos\theta}{1 + \cos\theta}} = \dfrac{\dfrac{1 + \cos\theta - (1 - \cos\theta)}{1 + \cos\theta}}{\dfrac{1 + \cos\theta + 1 - \cos\theta}{1 + \cos\theta}} = \dfrac{2\cos\theta}{1 + \cos\theta} \cdot \dfrac{1 + \cos\theta}{2} = \cos\theta$

63. $\dfrac{\sin(3\theta)}{\sin\theta} - \dfrac{\cos(3\theta)}{\cos\theta} = \dfrac{\sin(3\theta)\cos\theta - \cos(3\theta)\sin\theta}{\sin\theta\cos\theta} = \dfrac{\sin(3\theta - \theta)}{\frac{1}{2}(2\sin\theta\cos\theta)} = \dfrac{2\sin(2\theta)}{\sin(2\theta)} = 2$

65. $\tan(3\theta) = \tan(\theta + 2\theta) = \dfrac{\tan\theta + \tan(2\theta)}{1 - \tan\theta\tan(2\theta)} = \dfrac{\tan\theta + \dfrac{2\tan\theta}{1 - \tan^2\theta}}{1 - \dfrac{\tan\theta(2\tan\theta)}{1 - \tan^2\theta}} = \dfrac{\tan\theta - \tan^3\theta + 2\tan\theta}{1 - \tan^2\theta - 2\tan^2\theta} = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

67. $\dfrac{1}{2}(\ln|1 - \cos(2\theta)| - \ln 2) = \ln\left(\dfrac{|1 - \cos(2\theta)|}{2}\right)^{1/2} = \ln|\sin^2\theta|^{1/2} = \ln|\sin\theta|$ **69.** $\dfrac{\sqrt{3}}{2}$ **71.** $\dfrac{7}{25}$ **73.** $\dfrac{24}{7}$ **75.** $\dfrac{24}{25}$ **77.** $\dfrac{1}{5}$ **79.** $\dfrac{25}{7}$

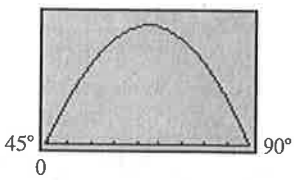
81. (a) $W = 2D(\csc\theta - \cot\theta) = 2D\left(\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\right) = 2D\dfrac{1 - \cos\theta}{\sin\theta} = 2D\tan\dfrac{\theta}{2}$ **(b)** $\theta = 24.45°$

83. (a) $R = \dfrac{v_0^2\sqrt{2}}{16}\cos\theta(\sin\theta - \cos\theta)$

$= \dfrac{v_0^2\sqrt{2}}{32}(2\cos\theta\sin\theta - 2\cos^2\theta)$

$= \dfrac{v_0^2\sqrt{2}}{32}[\sin(2\theta) - \cos(2\theta) - 1]$

(b)

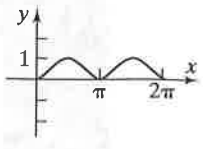


(c) $\theta = 67.5°$ makes R largest.

85. $A = \dfrac{1}{2}h(\text{base}) = h\left(\dfrac{1}{2}\text{base}\right) = s\cos\dfrac{\theta}{2} \cdot s\sin\dfrac{\theta}{2} = \dfrac{1}{2}s^2\sin\theta$ **87.** $\sin(2\theta) = \dfrac{4x}{4 + x^2}$ **89.** $-\dfrac{1}{4}$

91. $\dfrac{2z}{1 + z^2} = \dfrac{2\tan\left(\dfrac{\alpha}{2}\right)}{1 + \tan^2\left(\dfrac{\alpha}{2}\right)} = \dfrac{2\tan\left(\dfrac{\alpha}{2}\right)}{\sec^2\left(\dfrac{\alpha}{2}\right)} = \dfrac{2\sin\left(\dfrac{\alpha}{2}\right)}{\dfrac{\cos\left(\dfrac{\alpha}{2}\right)}{\cos^2\left(\dfrac{\alpha}{2}\right)}} = 2\sin\left(\dfrac{\alpha}{2}\right)\cos\left(\dfrac{\alpha}{2}\right) = \sin\left(2 \cdot \dfrac{\alpha}{2}\right) = \sin\alpha$

93.



95. $\sin\dfrac{\pi}{24} = \dfrac{\sqrt{2}}{4}\sqrt{4 - \sqrt{6} - \sqrt{2}}$;

$\cos\dfrac{\pi}{24} = \dfrac{\sqrt{2}}{4}\sqrt{4 + \sqrt{6} + \sqrt{2}}$

97. $\sin^3\theta + \sin^3(\theta + 120°) + \sin^3(\theta + 240°) = \sin^3\theta + (\sin\theta\cos 120° + \cos\theta\sin 120°)^3 + (\sin\theta\cos 240° + \cos\theta\sin 240°)^3$

$= \sin^3\theta + \left(-\dfrac{1}{2}\sin\theta + \dfrac{\sqrt{3}}{2}\cos\theta\right)^3 + \left(-\dfrac{1}{2}\sin\theta - \dfrac{\sqrt{3}}{2}\cos\theta\right)^3$

$= \sin^3\theta + \dfrac{1}{8}\left(3\sqrt{3}\cos^3\theta - 9\cos^2\theta\sin\theta + 3\sqrt{3}\cos\theta\sin^2\theta - \sin^3\theta\right) - \dfrac{1}{8}\left(\sin^3\theta + 3\sqrt{3}\sin^2\theta\cos\theta + 9\sin\theta\cos^2\theta + 3\sqrt{3}\cos^3\theta\right)$

$= \dfrac{3}{4}\sin^3\theta - \dfrac{9}{4}\cos^2\theta\sin\theta = \dfrac{3}{4}[\sin^3\theta - 3\sin\theta(1 - \sin^2\theta)] = \dfrac{3}{4}(4\sin^3\theta - 3\sin\theta) = -\dfrac{3}{4}\sin(3\theta)$ (from Example 2)

3.6 Assess Your Understanding *(page 224)*

1. $\frac{1}{2}[\cos(2\theta) - \cos(6\theta)]$ **3.** $\frac{1}{2}[\sin(6\theta) + \sin(2\theta)]$ **5.** $\frac{1}{2}[\cos(2\theta) + \cos(8\theta)]$ **7.** $\frac{1}{2}[\cos\theta - \cos(3\theta)]$ **9.** $\frac{1}{2}[\sin(2\theta) + \sin\theta]$ **11.** $2\sin\theta\cos(3\theta)$

13. $2\cos(3\theta)\cos\theta$ **15.** $2\sin(2\theta)\cos\theta$ **17.** $2\sin\theta\sin\frac{\theta}{2}$ **19.** $\frac{\sin\theta + \sin(3\theta)}{2\sin(2\theta)} = \frac{2\sin(2\theta)\cos\theta}{2\sin(2\theta)} = \cos\theta$

21. $\frac{\sin(4\theta) + \sin(2\theta)}{\cos(4\theta) + \cos(2\theta)} = \frac{2\sin(3\theta)\cos\theta}{2\cos(3\theta)\cos\theta} = \frac{\sin(3\theta)}{\cos(3\theta)} = \tan(3\theta)$ **23.** $\frac{\cos\theta - \cos(3\theta)}{\sin\theta + \sin(3\theta)} = \frac{2\sin(2\theta)\sin\theta}{2\sin(2\theta)\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

25. $\sin\theta[\sin\theta + \sin(3\theta)] = \sin\theta[2\sin(2\theta)\cos\theta] = \cos\theta[2\sin(2\theta)\sin\theta] = \cos\theta\left[2\cdot\frac{1}{2}[\cos\theta - \cos(3\theta)]\right] = \cos\theta[\cos\theta - \cos(3\theta)]$

27. $\frac{\sin(4\theta) + \sin(8\theta)}{\cos(4\theta) + \cos(8\theta)} = \frac{2\sin(6\theta)\cos(2\theta)}{2\cos(6\theta)\cos(2\theta)} = \frac{\sin(6\theta)}{\cos(6\theta)} = \tan(6\theta)$

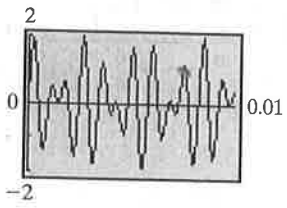
29. $\frac{\sin(4\theta) + \sin(8\theta)}{\sin(4\theta) - \sin(8\theta)} = \frac{2\sin(6\theta)\cos(-2\theta)}{2\sin(-2\theta)\cos(6\theta)} = \frac{\sin(6\theta)}{\cos(6\theta)}\cdot\frac{\cos(2\theta)}{-\sin(2\theta)} = \tan(6\theta)[-\cot(2\theta)] = -\frac{\tan(6\theta)}{\tan(2\theta)}$

31. $\frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \frac{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}{2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}} = \frac{\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha+\beta}{2}}\cdot\frac{\cos\frac{\alpha-\beta}{2}}{\sin\frac{\alpha-\beta}{2}} = \tan\frac{\alpha+\beta}{2}\cot\frac{\alpha-\beta}{2}$

33. $\frac{\sin\alpha + \sin\beta}{\cos\alpha + \cos\beta} = \frac{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}}{2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}} = \frac{\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha+\beta}{2}} = \tan\frac{\alpha+\beta}{2}$

35. $1 + \cos(2\theta) + \cos(4\theta) + \cos(6\theta)$
$= [1 + \cos(6\theta)] + [\cos(2\theta) + \cos(4\theta)]$
$= 2\cos^2(3\theta) + 2\cos(3\theta)\cos(-\theta)$
$= 2\cos(3\theta)[\cos(3\theta) + \cos\theta]$
$= 2\cos(3\theta)[2\cos(2\theta)\cos\theta]$
$= 4\cos\theta\cos(2\theta)\cos(3\theta)$

37. **(a)** $y = 2\sin(2061\pi t)\cos(357\pi t)$ **(b)** $y_{\max} = 2$ **(c)**



39. $I_u = I_x\cos^2\theta + I_y\sin^2\theta - 2I_{xy}\sin\theta\cos\theta = I_x\cos^2\theta + I_y\sin^2\theta - I_{xy}\sin 2\theta$
$= I_x\left(\frac{\cos 2\theta + 1}{2}\right) + I_y\left(\frac{1 - \cos 2\theta}{2}\right) - I_{xy}\sin 2\theta$
$= \frac{I_x}{2}\cos 2\theta + \frac{I_x}{2} + \frac{I_y}{2} - \frac{I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$
$= \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$

$I_v = I_x\sin^2\theta + I_y\cos^2\theta + 2I_{xy}\sin\theta\cos\theta = I_x\left(\frac{1 - \cos 2\theta}{2}\right) + I_y\left(\frac{\cos 2\theta + 1}{2}\right) + I_{xy}\sin 2\theta$
$= \frac{I_x}{2} - \frac{I_x}{2}\cos 2\theta + \frac{I_y}{2}\cos 2\theta + \frac{I_y}{2} + I_{xy}\sin 2\theta$
$= \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$

41. $\sin(2\alpha) + \sin(2\beta) + \sin(2\gamma) = 2\sin(\alpha+\beta)\cos(\alpha-\beta) + \sin(2\gamma) = 2\sin(\alpha+\beta)\cos(\alpha-\beta) + 2\sin\gamma\cos\gamma$
$= 2\sin(\pi - \gamma)\cos(\alpha-\beta) + 2\sin\gamma\cos\gamma = 2\sin\gamma\cos(\alpha-\beta) + 2\sin\gamma\cos\gamma = 2\sin\gamma[\cos(\alpha-\beta) + \cos\gamma]$
$= 2\sin\gamma\left(2\cos\frac{\alpha-\beta+\gamma}{2}\cos\frac{\alpha-\beta-\gamma}{2}\right) = 4\sin\gamma\cos\frac{\pi - 2\beta}{2}\cos\frac{2\alpha - \pi}{2} = 4\sin\gamma\cos\left(\frac{\pi}{2} - \beta\right)\cos\left(\alpha - \frac{\pi}{2}\right)$
$= 4\sin\gamma\sin\beta\sin\alpha$

43.
$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$\sin(\alpha - \beta) + \sin(\alpha + \beta) = 2\sin\alpha\cos\beta$
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

45. $2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = 2\cdot\frac{1}{2}\left[\cos\left(\frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2}\right) + \cos\left(\frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2}\right)\right] = \cos\frac{2\alpha}{2} + \cos\frac{2\beta}{2} = \cos\alpha + \cos\beta$

3.7 Assess Your Understanding *(page 229)*

3. $\frac{\pi}{6}, \frac{5\pi}{6}$ **4.** $\left\{\theta \middle| \theta = \frac{\pi}{6} + 2\pi k, \theta = \frac{5\pi}{6} + 2\pi k, k \text{ any integer}\right\}$ **5.** F **6.** F **7.** $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ **9.** $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ **11.** $\left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

13. $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ **15.** $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ **17.** $\left\{\frac{4\pi}{9}, \frac{8\pi}{9}, \frac{16\pi}{9}\right\}$ **19.** $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ **21.** $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ **23.** $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

25. $\left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}$ **27.** $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ **29.** $\left\{\frac{11\pi}{6}\right\}$ **31.** $\left\{\theta \middle| \theta = \frac{\pi}{6} + 2k\pi, \theta = \frac{5\pi}{6} + 2k\pi\right\}; \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$

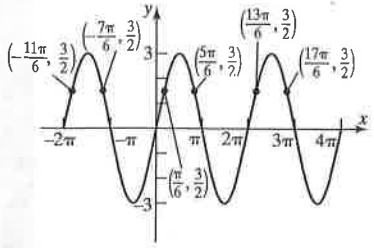
33. $\left\{\theta \middle| \theta = \frac{5\pi}{6} + k\pi\right\}; \frac{5\pi}{6}, \frac{11\pi}{6}, \frac{17\pi}{6}, \frac{23\pi}{6}, \frac{29\pi}{6}, \frac{35\pi}{6}$ **35.** $\left\{\theta \middle| \theta = \frac{\pi}{2} + 2k\pi, \theta = \frac{3\pi}{2} + 2k\pi\right\}; \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}$

37. $\left\{\theta \middle| \theta = \frac{\pi}{3} + k\pi, \theta = \frac{2\pi}{3} + k\pi\right\}; \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$ **39.** $\left\{\theta \middle| \theta = \frac{8\pi}{3} + 4k\pi, \theta = \frac{10\pi}{3} + 4k\pi\right\}; \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{20\pi}{3}, \frac{22\pi}{3}, \frac{32\pi}{3}, \frac{34\pi}{3}$

41. {0.41, 2.73} **43.** {1.37, 4.51} **45.** {2.69, 3.59} **47.** {1.82, 4.46} **49.** {2.08, 5.22} **51.** {0.73, 2.41} **53.** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

55. **(a)** $-2\pi, -\pi, 0, \pi, 2\pi, 3\pi, 4\pi$

(b)



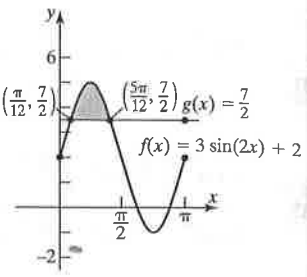
(c) $\left\{-\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}\right\}$

(d) $\left\{x \middle| -\frac{11\pi}{6} < x < -\frac{7\pi}{6} \text{ or } \frac{\pi}{6} < x < \frac{5\pi}{6} \text{ or } \frac{13\pi}{6} < x < \frac{17\pi}{6}\right\}$

57. **(a)** $\left\{x \middle| x = -\frac{\pi}{4} + k\pi\right\}$

(b) $-\frac{\pi}{2} < x < -\frac{\pi}{4}$ or $\left(-\frac{\pi}{2}, -\frac{\pi}{4}\right)$

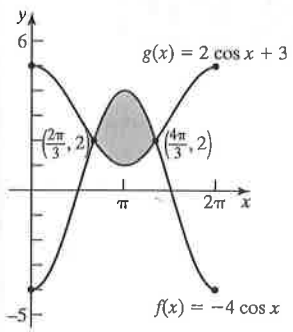
59. **(a), (d)**



(b) $\left\{\frac{\pi}{12}, \frac{5\pi}{12}\right\}$

(c) $\left\{x \middle| \frac{\pi}{12} < x < \frac{5\pi}{12}\right\}$ or $\left(\frac{\pi}{12}, \frac{5\pi}{12}\right)$

61. **(a), (d)**



(b) $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

(c) $\left\{x \middle| \frac{2\pi}{3} < x < \frac{4\pi}{3}\right\}$ or $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

63. **(a)** 10 sec; 30 sec **(b)** 20 sec; 60 sec **(c)** $10 < x < 30$ or (10, 30)

65. **(a)** 150 mi **(b)** 6.06, 8.44, 15.72, 18.11 min **(c)** Before 6.06 min, between 8.44 and 15.72 min, and after 18.11 min **(d)** No

67. 28.90° **69.** Yes; it varies from 1.25 to 1.34.

71. 1.47

73. If θ is the original angle of incidence and ϕ is the angle of refraction, then $\frac{\sin\theta}{\sin\phi} = n_2$.

The angle of incidence of the emerging beam is also ϕ, and the index of refraction is $\frac{1}{n_2}$.

Thus, θ is the angle of refraction of the emerging beam.

3.8 Assess Your Understanding *(page 237)*

5. $\left\{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\right\}$ **7.** $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ **9.** $\left\{0, \frac{\pi}{4}, \frac{5\pi}{4}\right\}$ **11.** $\left\{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\right\}$ **13.** $\{\pi\}$ **15.** $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ **17.** $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$ **19.** $\left\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$

21. $\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$ **23.** $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ **25.** $\left\{0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}\right\}$ **27.** $\left\{0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi, \frac{6\pi}{5}, \frac{7\pi}{5}, \frac{8\pi}{5}, \frac{9\pi}{5}\right\}$ **29.** $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$ **31.** $\left\{\frac{\pi}{2}\right\}$

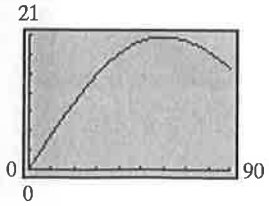
33. {0} **35.** $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ **37.** No real solution **39.** No real solution **41.** $\left\{\frac{\pi}{2}, \frac{7\pi}{6}\right\}$ **43.** $\left\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$ **45.** $\left\{\frac{\pi}{4}\right\}$ **47.** $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

49. $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ **51.** π **53.** −1.31, 1.98, 3.84 **55.** 0.52 **57.** 1.26 **59.** −1.02, 1.02 **61.** 0, 2.15 **63.** 0.76, 1.35

65. **(a)** 60° **(b)** 60°

(c) $A(60°) = 12\sqrt{3}$ in.^{2}

(d)



$\theta_{max} = 60°$

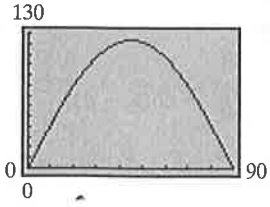
Maximum area = 20.78 in.^{2}

67. 2.03, 4.91

69. **(a)** 30°, 60°

(b) 123.6 m

(c)



Review Exercises *(page 240)*

1. $\frac{\pi}{2}$ **3.** $\frac{\pi}{4}$ **5.** $\frac{5\pi}{6}$ **7.** $\frac{\pi}{4}$ **9.** $\frac{3\pi}{8}$ **11.** $-\frac{\pi}{3}$ **13.** $\frac{\pi}{7}$ **15.** 0.9 **17.** −0.3 **19.** Not defined **21.** $-\frac{\pi}{6}$ **23.** $-\frac{\pi}{4}$ **25.** $-\sqrt{3}$ **27.** $\frac{2\sqrt{3}}{3}$

29. $\frac{4}{5}$ **31.** $-\frac{4}{3}$ **33.** $f^{-1}(x) = \frac{1}{3}\sin^{-1}\left(\frac{x}{2}\right)$; Domain of f: $(-\infty, \infty)$; Domain of f^{-1}: $[-2, 2]$

35. $f^{-1}(x) = \cos^{-1}(3 - x)$; Domain of f: $(-\infty, \infty)$; Domain of f^{-1}: $[2, 4]$ **37.** $\sqrt{1 - u^2}$ **39.** $\frac{1}{u}$ **41.** $\tan\theta\cot\theta - \sin^2\theta = 1 - \sin^2\theta = \cos^2\theta$

43. $\sin^2\theta(1 + \cot^2\theta) = \sin^2\theta\csc^2\theta = 1$ **45.** $5\cos^2\theta + 3\sin^2\theta = 2\cos^2\theta + 3(\cos^2\theta + \sin^2\theta) = 3 + 2\cos^2\theta$

47. $\dfrac{1 - \cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1 - \cos\theta} = \dfrac{(1 - \cos\theta)^2 + \sin^2\theta}{\sin\theta(1 - \cos\theta)} = \dfrac{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1 - \cos\theta)} = \dfrac{2(1 - \cos\theta)}{\sin\theta(1 - \cos\theta)} = 2\csc\theta$

49. $\dfrac{\cos\theta}{\cos\theta - \sin\theta} = \dfrac{\dfrac{\cos\theta}{\cos\theta}}{\dfrac{\cos\theta - \sin\theta}{\cos\theta}} = \dfrac{1}{1 - \dfrac{\sin\theta}{\cos\theta}} = \dfrac{1}{1 - \tan\theta}$

51. $\dfrac{\csc\theta}{1 + \csc\theta} = \dfrac{\dfrac{1}{\sin\theta}}{1 + \dfrac{1}{\sin\theta}} = \dfrac{1}{1 + \sin\theta} = \dfrac{1}{1 + \sin\theta}\cdot\dfrac{1 - \sin\theta}{1 - \sin\theta} = \dfrac{1 - \sin\theta}{1 - \sin^2\theta} = \dfrac{1 - \sin\theta}{\cos^2\theta}$

53. $\csc\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1 - \sin^2\theta}{\sin\theta} = \dfrac{\cos^2\theta}{\sin\theta} = \cos\theta\cdot\dfrac{\cos\theta}{\sin\theta} = \cos\theta\cot\theta$

55. $\dfrac{1 - \sin\theta}{\sec\theta} = \cos\theta(1 - \sin\theta)\cdot\dfrac{1 + \sin\theta}{1 + \sin\theta} = \dfrac{\cos\theta(1 - \sin^2\theta)}{1 + \sin\theta} = \dfrac{\cos^3\theta}{1 + \sin\theta}$

57. $\cot\theta - \tan\theta = \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} = \dfrac{1 - 2\sin^2\theta}{\sin\theta\cos\theta}$

59. $\dfrac{\cos(\alpha + \beta)}{\cos\alpha\sin\beta} = \dfrac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\sin\beta} = \dfrac{\cos\alpha\cos\beta}{\cos\alpha\sin\beta} - \dfrac{\sin\alpha\sin\beta}{\cos\alpha\sin\beta} = \cot\beta - \tan\alpha$

61. $\dfrac{\cos(\alpha - \beta)}{\cos\alpha\cos\beta} = \dfrac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha\cos\beta} = \dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta} = 1 + \tan\alpha\tan\beta$

63. $(1 + \cos\theta)\left(\tan\dfrac{\theta}{2}\right) = (1 + \cos\theta)\cdot\dfrac{\sin\theta}{1 + \cos\theta} = \sin\theta$

65. $2\cot\theta\cot 2\theta = 2\left(\dfrac{\cos\theta}{\sin\theta}\right)\left(\dfrac{\cos 2\theta}{\sin 2\theta}\right) = \dfrac{2\cos\theta(\cos^2\theta - \sin^2\theta)}{2\sin^2\theta\cos\theta} = \dfrac{\cos^2\theta - \sin^2\theta}{\sin^2\theta} = \cot^2\theta - 1$

67. $1 - 8\sin^2\theta\cos^2\theta = 1 - 2(2\sin\theta\cos\theta)^2 = 1 - 2\sin^2(2\theta) = \cos(4\theta)$ **69.** $\dfrac{\sin(2\theta) + \sin(4\theta)}{\cos(2\theta) + \cos(4\theta)} = \dfrac{2\sin(3\theta)\cos(-\theta)}{2\cos(3\theta)\cos(-\theta)} = \tan(3\theta)$

71. $\dfrac{\cos(2\theta) - \cos(4\theta)}{\cos(2\theta) + \cos(4\theta)} - \tan\theta\tan(3\theta) = \dfrac{-2\sin(3\theta)\sin(-\theta)}{2\cos(3\theta)\cos(-\theta)} - \tan\theta\tan(3\theta) = \tan(3\theta)\tan\theta - \tan\theta\tan(3\theta) = 0$

73. $\frac{1}{4}(\sqrt{6} - \sqrt{2})$ **75.** $\frac{1}{4}(\sqrt{6} - \sqrt{2})$ **77.** $\frac{1}{2}$ **79.** $\sqrt{2} - 1$ **81.** **(a)** $-\frac{33}{65}$ **(b)** $-\frac{56}{65}$ **(c)** $-\frac{63}{65}$ **(d)** $\frac{33}{56}$ **(e)** $\frac{24}{25}$ **(f)** $\frac{119}{169}$ **(g)** $\frac{5\sqrt{26}}{26}$ **(h)** $\frac{2\sqrt{5}}{5}$ **83.** **(a)** $-\frac{16}{65}$ **(b)** $-\frac{63}{65}$ **(c)** $-\frac{56}{65}$ **(d)** $\frac{16}{63}$ **(e)** $\frac{24}{25}$ **(f)** $\frac{119}{169}$ **(g)** $\frac{\sqrt{26}}{26}$ **(h)** $-\frac{\sqrt{10}}{10}$ **85.** **(a)** $-\frac{63}{65}$ **(b)** $\frac{16}{65}$ **(c)** $\frac{33}{65}$ **(d)** $-\frac{63}{16}$ **(e)** $\frac{24}{25}$ **(f)** $-\frac{119}{169}$ **(g)** $\frac{2\sqrt{13}}{13}$ **(h)** $-\frac{\sqrt{10}}{10}$ **87.** **(a)** $\frac{-\sqrt{3} - 2\sqrt{2}}{6}$ **(b)** $\frac{1 - 2\sqrt{6}}{6}$ **(c)** $\frac{-\sqrt{3} + 2\sqrt{2}}{6}$ **(d)** $\frac{8\sqrt{2} + 9\sqrt{3}}{23}$ **(e)** $-\frac{\sqrt{3}}{2}$ **(f)** $-\frac{7}{9}$ **(g)** $\frac{\sqrt{3}}{3}$ **(h)** $\frac{\sqrt{3}}{2}$ **89.** **(a)** 1 **(b)** 0 **(c)** $-\frac{1}{9}$ **(d)** Not defined **(e)** $\frac{4\sqrt{5}}{9}$ **(f)** $-\frac{1}{9}$ **(g)** $\frac{\sqrt{30}}{6}$ **(h)** $\frac{\sqrt{6}\sqrt{3 - \sqrt{5}}}{6}$ **91.** $\frac{4 + 3\sqrt{3}}{10}$

93. $-\frac{48 + 25\sqrt{3}}{39}$ **95.** $-\frac{24}{25}$ **97.** $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ **99.** $\left\{\frac{3\pi}{4}, \frac{5\pi}{4}\right\}$ **101.** $\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$ **103.** $\left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$ **105.** $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ **107.** $\{0, \pi\}$

109. $\left\{0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$ **111.** $\left\{0, \frac{\pi}{6}, \frac{5\pi}{6}\right\}$ **113.** $\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}\right\}$ **115.** $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ **117.** $\left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}\right\}$ **119.** $\left\{\frac{\pi}{2}, \pi\right\}$ **121.** 0.78 **123.** −1.11

125. 1.23 **127.** {1.11} **129.** {0.87} **131.** {2.22} **133.** $\left\{-\frac{\sqrt{3}}{2}\right\}$

Chapter Test *(page 242)*

1. $\frac{\pi}{6}$ **2.** $-\frac{\pi}{4}$ **3.** $\frac{\pi}{5}$ **4.** $\frac{7}{3}$ **5.** 3 **6.** $-\frac{4}{3}$ **7.** ≈0.392 **8.** ≈0.775 **9.** ≈1.249 **10.** ≈0.197

11. $\dfrac{(\csc\theta + \cot\theta)}{(\sec\theta + \tan\theta)} = \dfrac{(\csc\theta + \cot\theta)}{(\sec\theta + \tan\theta)}\cdot\dfrac{(\csc\theta - \cot\theta)}{(\csc\theta - \cot\theta)} = \dfrac{(\csc^2\theta - \cot^2\theta)}{(\sec\theta + \tan\theta)(\csc\theta - \cot\theta)} = \dfrac{1}{(\sec\theta + \tan\theta)(\csc\theta - \cot\theta)}$

$= \dfrac{1}{(\sec\theta + \tan\theta)(\csc\theta - \cot\theta)}\cdot\dfrac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta} = \dfrac{\sec\theta - \tan\theta}{(\sec^2\theta - \tan^2\theta)(\csc\theta - \cot\theta)} = \dfrac{\sec\theta - \tan\theta}{\csc\theta - \cot\theta}$

12. $\sin\theta\tan\theta+\cos\theta=\sin\theta\cdot\dfrac{\sin\theta}{\cos\theta}+\cos\theta=\dfrac{\sin^2\theta}{\cos\theta}+\dfrac{\cos^2\theta}{\cos\theta}=\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta}=\dfrac{1}{\cos\theta}=\sec\theta$

13. $\tan\theta+\cot\theta=\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}=\dfrac{\sin^2\theta}{\sin\theta\cos\theta}+\dfrac{\cos^2\theta}{\sin\theta\cos\theta}=\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}=\dfrac{1}{\sin\theta\cos\theta}=\dfrac{2}{2\sin\theta\cos\theta}=\dfrac{2}{\sin(2\theta)}=2\csc(2\theta)$

14. $\dfrac{\sin(\alpha+\beta)}{\tan\alpha+\tan\beta}=\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\dfrac{\sin\alpha}{\cos\alpha}+\dfrac{\sin\beta}{\cos\beta}}=\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}=\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}$

$=\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{1}\cdot\dfrac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta+\cos\alpha\sin\beta}=\cos\alpha\cos\beta$

15. $\sin(3\theta)=\sin(\theta+2\theta)=\sin\theta\cos(2\theta)+\cos\theta\sin(2\theta)=\sin\theta\cdot(\cos^2\theta-\sin^2\theta)+\cos\theta\cdot2\sin\theta\cos\theta=\sin\theta\cos^2\theta-\sin^3\theta+2\sin\theta\cos^2\theta$

$=3\sin\theta\cos^2\theta-\sin^3\theta=3\sin\theta(1-\sin^2\theta)-\sin^3\theta=3\sin\theta-3\sin^3\theta-\sin^3\theta=3\sin\theta-4\sin^3\theta$

16. $\dfrac{(\tan\theta-\cot\theta)}{(\tan\theta+\cot\theta)}=\dfrac{\dfrac{\sin\theta}{\cos\theta}-\dfrac{\cos\theta}{\sin\theta}}{\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}}=\dfrac{\dfrac{\sin^2\theta-\cos^2\theta}{\sin\theta\cos\theta}}{\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}=\dfrac{\sin^2\theta-\cos^2\theta}{\sin^2\theta+\cos^2\theta}=\dfrac{-\cos(2\theta)}{1}=-(2\cos^2\theta-1)=1-2\cos^2\theta$

17. $\dfrac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)$ **18.** $2+\sqrt{3}$ **19.** $\dfrac{\sqrt{5}}{5}$ **20.** $\dfrac{12\sqrt{85}}{49}$ **21.** $\dfrac{2\sqrt{13}\left(\sqrt{5}-3\right)}{39}$ **22.** $\dfrac{2+\sqrt{3}}{4}$ **23.** $\dfrac{\sqrt{6}}{2}$ **24.** $\dfrac{\sqrt{2}}{2}$

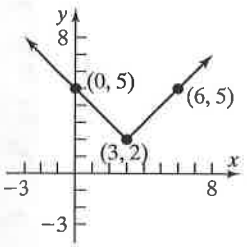
25. $\left\{\dfrac{\pi}{3},\dfrac{2\pi}{3},\dfrac{4\pi}{3},\dfrac{5\pi}{3}\right\}$ **26.** $\{0, 1.911, \pi, 4.373\}$ **27.** $\left\{\dfrac{3\pi}{8},\dfrac{7\pi}{8},\dfrac{11\pi}{8},\dfrac{15\pi}{8}\right\}$ **28.** $\{0.285, 3.427\}$ **29.** $\{0.253, 2.889\}$

30. The change in elevation during the time trial was about 1.18 km.

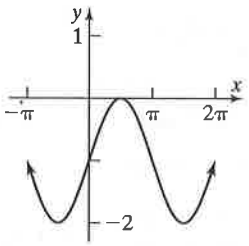
Cumulative Review *(page 243)*

1. $\left\{\dfrac{-1-\sqrt{13}}{6},\dfrac{-1+\sqrt{13}}{6}\right\}$ **2.** $y+1=-1(x-4)$ or $x+y=3$; $6\sqrt{2}$; $(1,2)$ **3.** x-axis symmetry; $(0,-3)$, $(0,3)$, $(3,0)$

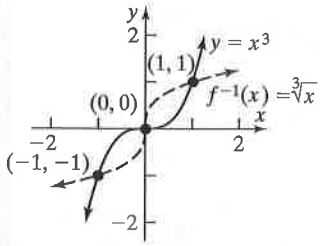
4.



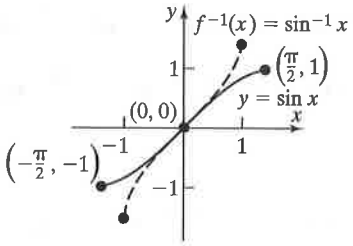
5.



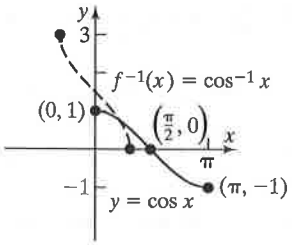
6. (a)



(b)



(c)



7. (a) $-\dfrac{2\sqrt{2}}{3}$ (b) $\dfrac{\sqrt{2}}{4}$ (c) $\dfrac{4\sqrt{2}}{9}$ (d) $\dfrac{7}{9}$ (e) $\sqrt{\dfrac{3+2\sqrt{2}}{6}}$ (f) $-\sqrt{\dfrac{3-2\sqrt{2}}{6}}$ **8.** $\dfrac{\sqrt{5}}{5}$

9. (a) $-\dfrac{2\sqrt{2}}{3}$ (b) $-\dfrac{2\sqrt{2}}{3}$ (c) $\dfrac{7}{9}$ (d) $\dfrac{4\sqrt{2}}{9}$ (e) $\dfrac{\sqrt{6}}{3}$

CHAPTER 4 Applications of Trigonometric Functions

4.1 Assess Your Understanding *(page 254)*

4. F **5.** T **6.** angle of elevation **7.** T **8.** F **9.** $\sin\theta=\dfrac{5}{13}$; $\cos\theta=\dfrac{12}{13}$; $\tan\theta=\dfrac{5}{12}$; $\cot\theta=\dfrac{12}{5}$; $\sec\theta=\dfrac{13}{12}$; $\csc\theta=\dfrac{13}{5}$

11. $\sin\theta=\dfrac{2\sqrt{13}}{13}$; $\cos\theta=\dfrac{3\sqrt{13}}{13}$; $\tan\theta=\dfrac{2}{3}$; $\cot\theta=\dfrac{3}{2}$; $\sec\theta=\dfrac{\sqrt{13}}{3}$; $\csc\theta=\dfrac{\sqrt{13}}{2}$

13. $\sin\theta=\dfrac{\sqrt{3}}{2}$; $\cos\theta=\dfrac{1}{2}$; $\tan\theta=\sqrt{3}$; $\cot\theta=\dfrac{\sqrt{3}}{3}$; $\sec\theta=2$; $\csc\theta=\dfrac{2\sqrt{3}}{3}$

15. $\sin\theta = \frac{\sqrt{6}}{3}$; $\cos\theta = \frac{\sqrt{3}}{3}$; $\tan\theta = \sqrt{2}$; $\cot\theta = \frac{\sqrt{2}}{2}$; $\sec\theta = \sqrt{3}$; $\csc\theta = \frac{\sqrt{6}}{2}$

17. $\sin\theta = \frac{\sqrt{5}}{5}$; $\cos\theta = \frac{2\sqrt{5}}{5}$; $\tan\theta = \frac{1}{2}$; $\cot\theta = 2$; $\sec\theta = \frac{\sqrt{5}}{2}$; $\csc\theta = \sqrt{5}$

19. 0 **21.** 1 **23.** 0 **25.** 0 **27.** 1 **29.** $a \approx 13.74, c \approx 14.62, A = 70°$ **31.** $b \approx 5.03, c \approx 7.83, A = 50°$ **33.** $a \approx 0.71, c \approx 4.06, B = 80°$
35. $b \approx 10.72, c \approx 11.83, B = 65°$ **37.** $b \approx 3.08, a \approx 8.46, A = 70°$ **39.** $c \approx 5.83, A \approx 59.0°, B \approx 31.0°$ **41.** $b \approx 4.58, A \approx 23.6°, B \approx 66.4°$
43. 23.6° and 66.4° **45.** 4.59 in.; 6.55 in. **47. (a)** 5.52 in. or 11.83 in. **49.** 70.02 ft **51.** 985.91 ft **53.** 137.37 m **55.** 80.5°
57. (a) 111.96 ft/sec or 76.3 mi/hr **(b)** 82.42 ft/sec or 56.2 mi/hr **(c)** Under 18.8° **59.** 203.52 ft **61.** 554.52 ft **63.** S76.6°E
65. The embankment is 30.5 m high. **67.** 3.83 mi **69.** 1978.09 ft **71.** 60.27 ft **73.** The buildings are 7984 ft apart. **75.** 69.0° **77.** 38.9°
79. The white ball should hit the top cushion 4.125 ft from the upper left corner.

4.2 Assess Your Understanding *(page 265)*

4. oblique **5.** $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ **6.** F **7.** T **8.** F **9.** $a \approx 3.23, b \approx 3.55, A = 40°$ **11.** $a \approx 3.25, c \approx 4.23, B = 45°$
13. $C = 95°, c \approx 9.86, a \approx 6.36$ **15.** $A = 40°, a = 2, c \approx 3.06$ **17.** $C = 120°, b \approx 1.06, c \approx 2.69$ **19.** $A = 100°, a \approx 5.24, c \approx 0.92$
21. $B = 40°, a \approx 5.64, b \approx 3.86$ **23.** $C = 100°, a \approx 1.31, b \approx 1.31$ **25.** One triangle; $B \approx 30.7°, C \approx 99.3°, c \approx 3.86$ **27.** One triangle;
$C \approx 36.2°, A \approx 43.8°, a \approx 3.51$ **29.** No triangle **31.** Two triangles; $C_1 \approx 30.9°, A_1 \approx 129.1°, a_1 \approx 9.07$ or $C_2 \approx 149.1°, A_2 \approx 10.9°, a_2 \approx 2.20$
33. No triangle **35.** Two triangles; $A_1 \approx 57.7°, B_1 \approx 97.3°, b_1 \approx 2.35$ or $A_2 \approx 122.3°, B_2 \approx 32.7°, b_2 \approx 1.28$ **37. (a)** Station Able is about 143.33 mi from the ship; Station Baker is about 135.58 mi from the ship. **(b)** Approximately 41 min **39.** 1490.48 ft **41.** 381.69 ft **43.** The tree is 39.4 ft high.
45. Adam receives 88.3 more frequent flyer miles. **47.** 84.7°; 183.72 ft **49.** 2.64 mi **51.** 38.5 in. **53.** 449.36 ft **55.** 187,600,000 km or 101,440,000 km
57. The diameter is 252 ft.

59. $$\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c} = \frac{\sin A}{\sin C} - \frac{\sin B}{\sin C} = \frac{\sin A - \sin B}{\sin C} = \frac{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{2\sin\frac{C}{2}\cos\frac{C}{2}} = \frac{\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)}{\sin\frac{C}{2}\cos\frac{C}{2}} = \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\frac{C}{2}}$$

61. $$\frac{a-b}{a+b} = \frac{\frac{a-b}{c}}{\frac{a+b}{c}} = \frac{\frac{\sin\left[\frac{1}{2}(A-B)\right]}{\cos\frac{C}{2}}}{\frac{\cos\left[\frac{1}{2}(A-B)\right]}{\sin\frac{C}{2}}} = \frac{\tan\left[\frac{1}{2}(A-B)\right]}{\cot\frac{C}{2}} = \frac{\tan\left[\frac{1}{2}(A-B)\right]}{\tan\left(\frac{\pi}{2}-\frac{C}{2}\right)} = \frac{\tan\left[\frac{1}{2}(A-B)\right]}{\tan\left[\frac{1}{2}(A+B)\right]}$$

4.3 Assess Your Understanding *(page 272)*

3. Cosines **4.** Sines **5.** Cosines **6.** F **7.** F **8.** T **9.** $b \approx 2.95, A \approx 28.7°, C \approx 106.3°$ **11.** $c \approx 3.75, A \approx 32.1°, B \approx 52.9°$
13. $A \approx 48.5°, B \approx 38.6°, C \approx 92.9°$ **15.** $A \approx 127.2°, B \approx 32.1°, C \approx 20.7°$ **17.** $c \approx 2.57, A \approx 48.6°, B \approx 91.4°$
19. $a \approx 2.99, B \approx 19.2°, C \approx 80.8°$ **21.** $b \approx 4.14, A \approx 43.0°, C \approx 27.0°$ **23.** $c \approx 1.69, A = 65.0°, B = 65.0°$ **25.** $A \approx 67.4°, B = 90°, C \approx 22.6°$
27. $A = 60°, B = 60°, C = 60°$ **29.** $A \approx 33.6°, B \approx 62.2°, C \approx 84.3°$ **31.** $A \approx 97.9°, B \approx 52.4°, C \approx 29.7°$ **33.** 165 yd **35. (a)** 26.4° **(b)** 30.8 hr
37. (a) 63.7 ft **(b)** 66.8 ft **(c)** 92.8° **39. (a)** 492.6 ft **(b)** 269.3 ft **41.** 342.3 ft **43.** The footings should be 7.65 ft apart.

45. Suppose $0 < \theta < \pi$. Then by the Law of Cosines: $d^2 = r^2 + r^2 - 2r^2\cos\theta = 4r^2\left(\frac{1-\cos\theta}{2}\right) \Rightarrow d = 2r\sqrt{\frac{1-\cos\theta}{2}} = 2r\sin\frac{\theta}{2}$. Since for any angle in $(0, \pi)$, d is strictly less than the length of the arc subtended by θ, that is, $d < r\theta$, then $2r\sin\frac{\theta}{2} < r\theta$, or $2\sin\frac{\theta}{2} < \theta$. Since $\cos\frac{\theta}{2} < 1$, then, for $0 < \theta < \pi$, $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} < 2\sin\frac{\theta}{2} < \theta$. If $\theta \geq \pi$, then since $\sin\theta \leq 1$, $\sin\theta < \theta$. Thus $\sin\theta < \theta$ for all $\theta > 0$.

47. $$\sin\frac{C}{2} = \sqrt{\frac{1-\cos C}{2}} = \sqrt{\frac{1-\frac{a^2+b^2-c^2}{2ab}}{2}} = \sqrt{\frac{2ab-a^2-b^2+c^2}{4ab}} = \sqrt{\frac{c^2-(a-b)^2}{4ab}} = \sqrt{\frac{(c+a-b)(c+b-a)}{4ab}}$$
$$= \sqrt{\frac{(2s-2b)(2s-2a)}{4ab}} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

4.4 Assess Your Understanding *(page 278)*

2. Heron's **3.** F **4.** T **5.** 2.83 **7.** 2.99 **9.** 14.98 **11.** 9.56 **13.** 3.86 **15.** 1.48 **17.** 2.82 **19.** 30 **21.** 1.73 **23.** 19.90
25. $K = \frac{1}{2}ab\sin C = \frac{1}{2}a\sin C\left(\frac{a\sin B}{\sin A}\right) = \frac{a^2\sin B\sin C}{2\sin A}$ **27.** 0.92 **29.** 2.27 **31.** 5.44 **33.** 9.03 sq ft **35.** $5446.38
37. The area of home plate is about 216.5 in^2. **39.** $K = \frac{1}{2}r^2(\theta + \sin\theta)$ **41.** The ground area is 7517.4 ft^2.

43. (a) Area $\Delta OAC = \frac{1}{2}|OC||AC| = \frac{1}{2}\cdot\frac{|OC|}{1}\cdot\frac{|AC|}{1} = \frac{1}{2}\sin\alpha\cos\alpha$

(b) Area $\Delta OCB = \frac{1}{2}|BC||OC| = \frac{1}{2}|OB|^2\frac{|BC|}{|OB|}\cdot\frac{|OC|}{|OB|} = \frac{1}{2}|OB|^2\sin\beta\cos\beta$

(c) Area $\Delta OAB = \frac{1}{2}|BD||OA| = \frac{1}{2}|OB|\frac{|BD|}{|OB|} = \frac{1}{2}|OB|\sin(\alpha+\beta)$

(d) $\dfrac{\cos\alpha}{\cos\beta} = \dfrac{\frac{|OC|}{1}}{\frac{|OC|}{|OB|}} = |OB|$

(e) Area ΔOAB = Area ΔOAC + Area ΔOCB

$$\frac{1}{2}|OB|\sin(\alpha+\beta) = \frac{1}{2}\sin\alpha\cos\alpha + \frac{1}{2}|OB|^2\sin\beta\cos\beta$$

$$\sin(\alpha+\beta) = \frac{1}{|OB|}\sin\alpha\cos\alpha + |OB|\sin\beta\cos\beta$$

$$\sin(\alpha+\beta) = \frac{\cos\beta}{\cos\alpha}\sin\alpha\cos\alpha + \frac{\cos\alpha}{\cos\beta}\sin\beta\cos\beta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

45. 31,145 ft^2 **47.** (a) The perimeter and area are both 36. (b) The perimeter and area are both 60.

49. $K = \frac{1}{2}ah = \frac{1}{2}ab\sin C \Rightarrow h = b\sin C = \dfrac{a\sin B\sin C}{\sin A}$

51. $\cot\dfrac{C}{2} = \dfrac{\cos\frac{C}{2}}{\sin\frac{C}{2}} = \dfrac{\frac{c\sin\frac{A}{2}\sin\frac{B}{2}}{r}}{\sqrt{\frac{(s-a)(s-b)}{ab}}} = \dfrac{c\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{(s-a)(s-c)}{ac}}}{r\sqrt{\frac{(s-a)(s-b)}{ab}}} = \frac{c}{r}\sqrt{\frac{(s-c)^2}{c^2}} = \frac{s-c}{r}$

53. K = area of triangle QOR + area of ROP + area of $POQ = \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr = r\frac{1}{2}(a+b+c) = rs$, so

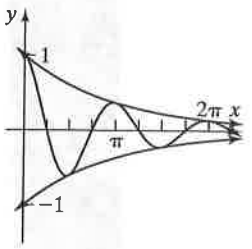
$$r = \frac{K}{s} = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s} = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

4.5 Assess Your Understanding *(page 288)*

2. Simple harmonic; amplitude **3.** Simple harmonic; damped **4.** T **5.** $d = -5\cos(\pi t)$ **7.** $d = -6\cos(2t)$ **9.** $d = -5\sin(\pi t)$

11. $d = -6\sin(2t)$ **13.** (a) Simple harmonic (b) 5 m (c) $\frac{2\pi}{3}$ sec (d) $\frac{3}{2\pi}$ oscillation/sec **15.** (a) Simple harmonic (b) 6 m (c) 2 sec (d) $\frac{1}{2}$ oscillation/sec **17.** (a) Simple harmonic (b) 3 m (c) 4π sec (d) $\frac{1}{4\pi}$ oscillation/sec **19.** (a) Simple harmonic (b) 2 m (c) 1 sec (d) 1 oscillation/sec

21.



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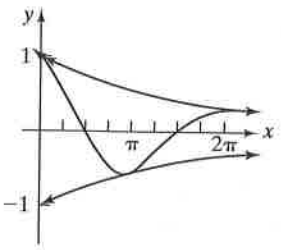

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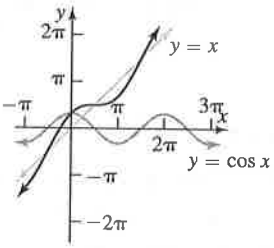

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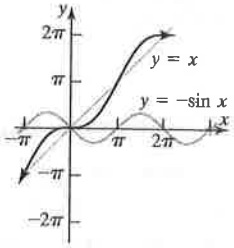

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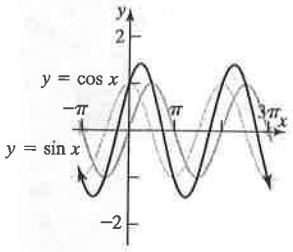

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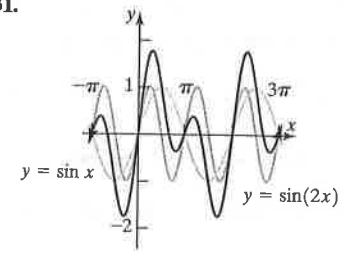
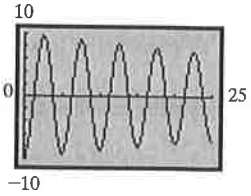


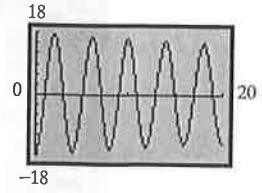
33. (a) $d = -10e^{-0.7t/50}\cos\left(\sqrt{\dfrac{4\pi^2}{25} - \dfrac{0.49}{2500}}\,t\right)$

(b)



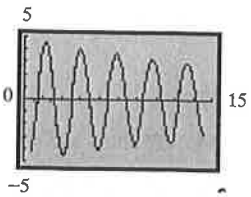
35. (a) $d = -18e^{-0.6t/60}\cos\left(\sqrt{\dfrac{\pi^2}{4} - \dfrac{0.36}{3600}}\,t\right)$

(b)



37. (a) $d = -5e^{-0.8t/20}\cos\left(\sqrt{\dfrac{4\pi^2}{9} - \dfrac{0.64}{400}}\,t\right)$

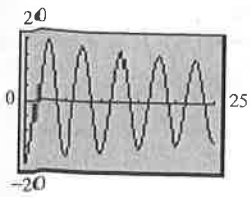
(b)



39. **(a)** The motion is damped. The bob has mass $m = 20$ kg with a damping factor of 0.7 kg/sec.

(b) 20 m leftward

(c)

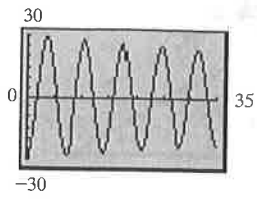


(d) 18.33 m leftward **(e)** $d \to 0$

41. **(a)** The motion is damped. The bob has mass $m = 40$ kg with a damping factor of 0.6 kg/sec.

(b) 30 m leftward

(c)

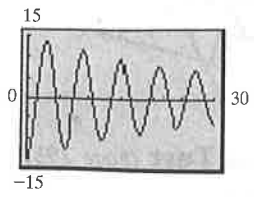


(d) 28.47 m leftward **(e)** $d \to 0$

43. **(a)** The motion is damped. The bob has mass $m = 15$ kg with a damping factor of 0.9 kg/sec.

(b) 15 m leftward

(c)

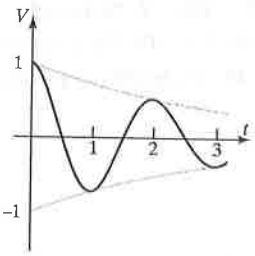


(d) 12.53 m leftward **(e)** $d \to 0$

45. $\omega = 1040\pi$; $d = 0.80 \cos(1040\pi t)$

47. $\omega = 880\pi$; $d = 0.01 \sin(880\pi t)$

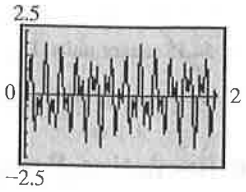
49. **(a)**



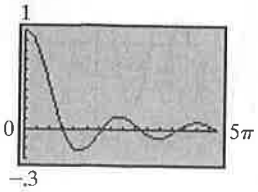
(b) At $t = 0, 2$; at $t = 1, t = 3$

(c) During the approximate intervals $0.35 < t < 0.67$, $1.29 < t < 1.75$, and $2.19 < t \leq 3$

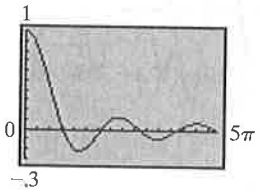
51.



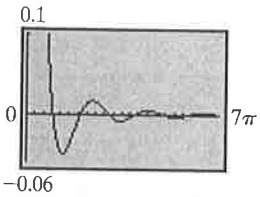
55.



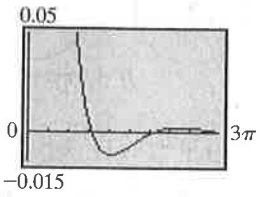
57. $y = \frac{1}{x}\sin x$



$y = \frac{1}{x^2}\sin x$



$y = \frac{1}{x^3}\sin x$



Review Exercises *(page 291)*

1. $\sin\theta = \frac{4}{5}$; $\cos\theta = \frac{3}{5}$; $\tan\theta = \frac{4}{3}$; $\cot\theta = \frac{3}{4}$; $\sec\theta = \frac{5}{3}$; $\csc\theta = \frac{5}{4}$

3. $\sin\theta = \frac{\sqrt{3}}{2}$; $\cos\theta = \frac{1}{2}$; $\tan\theta = \sqrt{3}$; $\cot\theta = \frac{\sqrt{3}}{3}$; $\sec\theta = 2$; $\csc\theta = \frac{2\sqrt{3}}{3}$ **5.** 0 **7.** 1 **9.** 1 **11.** $A = 70°, b \approx 3.42, a \approx 9.40$

13. $a \approx 4.58, A \approx 66.4°, B \approx 23.6°$ **15.** $C = 100°, b \approx 0.65, c \approx 1.29$ **17.** $B \approx 56.8°, C \approx 23.2°, b \approx 4.25$ **19.** No triangle **21.** $b \approx 3.32, A \approx 62.8°, C \approx 17.2°$ **23.** No triangle **25.** $c \approx 2.32, A \approx 16.1°, B \approx 123.9°$ **27.** $B \approx 36.2°, C \approx 63.8°, c \approx 4.55$ **29.** $A \approx 39.6°, B \approx 18.6°, C \approx 121.9°$

31. Two triangles: $B_1 \approx 13.4°, C_1 \approx 156.6°, c_1 \approx 6.86$ or $B_2 \approx 166.6°, C_2 \approx 3.4°, c_2 \approx 1.02$ **33.** $a \approx 5.23, B \approx 46.0°, C \approx 64.0°$

35. 1.93 **37.** 18.79 **39.** 6 **41.** 3.80 **43.** 0.32 **45.** 1.92 in^2 **47.** 23.32 ft **49.** 2.15 mi **51.** 132.55 ft/min **53.** 29.97 ft **55.** 6.22 mi

57. 71.12 ft **59.** \$222,983.51 **61.** 76.94 in. **63.** $d = -3\cos\left(\frac{\pi}{2}t\right)$ **65.** **(a)** Simple harmonic **(b)** 6 ft **(c)** π sec **(d)** $\frac{1}{\pi}$ oscillation/sec

67. **(a)** Simple harmonic

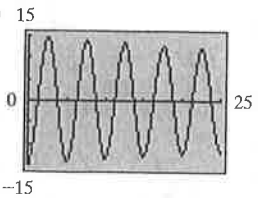
(b) 2 ft

(c) 2 sec

(d) $\frac{1}{2}$ oscillation/sec

69. **(a)** $d = -15e^{-0.75t/80}\cos\left(\sqrt{\frac{4\pi^2}{25} - \frac{0.5625}{6400}}\,t\right)$

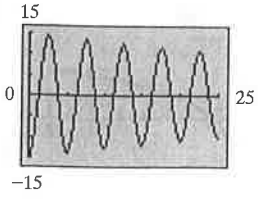
(b)



71. **(a)** The motion is damped. The bob has mass $m = 20$ kg with a damping factor of 0.6 kg/sec.

(b) 15 m leftward

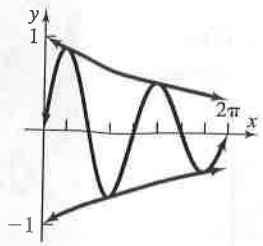
(c)



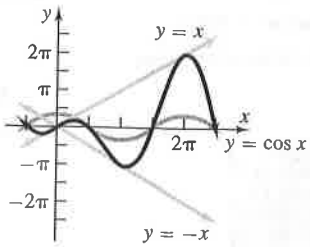
(d) 13.92 m leftward

(e) $d \to 0$

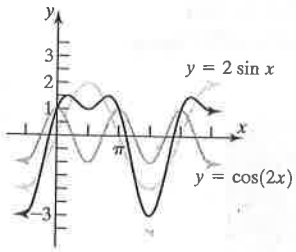
73.



75.



77.



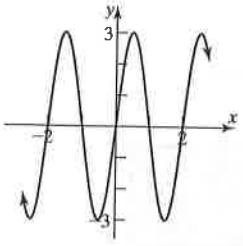
Chapter Test *(page 295)*

1. $\sin\theta = \frac{\sqrt{5}}{5}; \cos\theta = \frac{2\sqrt{5}}{5}; \tan\theta = \frac{1}{2}; \csc\theta = \sqrt{5}; \sec\theta = \frac{\sqrt{5}}{2}; \cot\theta = 2$ **2.** 0 **3.** $a = 15.88, B \approx 57.5°, C \approx 70.5°$
4. $b \approx 6.85, C = 117°, c \approx 16.30$ **5.** $A \approx 52.4°, B \approx 29.7°, C \approx 97.9°$ **6.** $b \approx 4.72, c \approx 1.67, B = 105°$ **7.** No triangle
8. $c \approx 7.62,\ A \approx 80.5°, B \approx 29.5°$ **9.** 15.04 square units **10.** 19.81 square units **11.** 61.0° **12.** 1.3° **13.** The area of the shaded region is 9.26 cm^2.
14. 54.15 square units **15.** Madison will have to swim about 2.23 miles. **16.** 12.63 square units **17.** The lengths of the sides are 15, 18, and 21.
18. $d = 5(\sin 42°)\sin\left(\frac{\pi t}{3}\right)$ or $d \approx 3.346\sin\left(\frac{\pi t}{3}\right)$

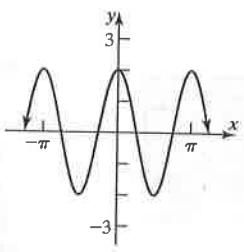
Cumulative Review *(page 296)*

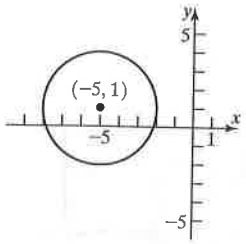
1. $\left\{\frac{1}{3}, 1\right\}$ **2.** $(x+5)^2 + (y-1)^2 = 9$ **3.** $\{x | x \le -1 \text{ or } x \ge 4\}$

4.



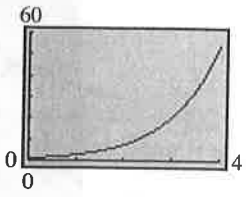
5.



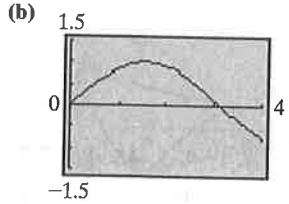


6. (a) $-\frac{2\sqrt{5}}{5}$ (b) $\frac{\sqrt{5}}{5}$ (c) $-\frac{4}{5}$ (d) $-\frac{3}{5}$ (e) $\sqrt{\frac{5-\sqrt{5}}{10}}$ (f) $-\sqrt{\frac{5+\sqrt{5}}{10}}$

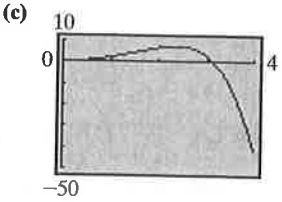
7. (a)



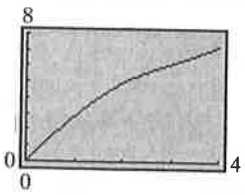
(b)



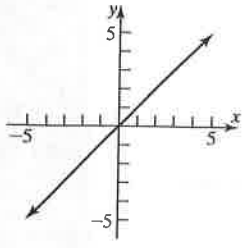
(c)



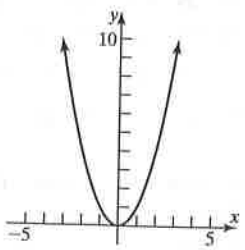
(d)



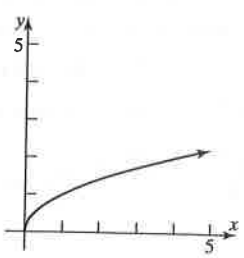
8. (a)



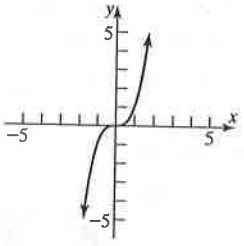
(b)



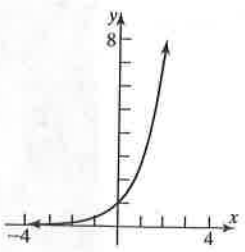
(c)



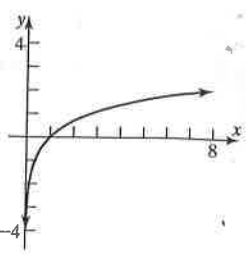
(d)



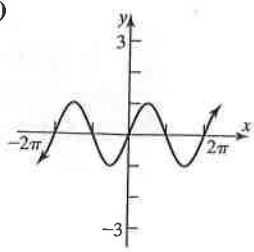
(e)



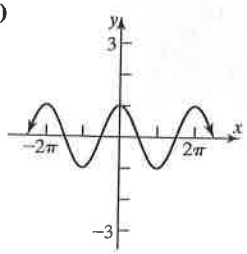
(f)



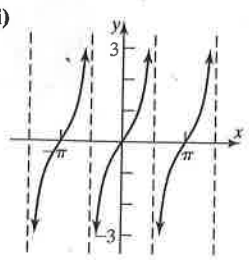
(g)



(h)



(i)



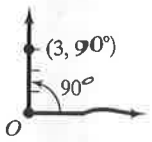
Two triangles: $A_1 \approx 59.0°, B_1 \approx 81.0°, b_1 \approx 23.05$ or $A_2 \approx 121.0°, B_2 \approx 19.0°, b_2 \approx 7.59$ **10.** $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ **11.** π ft ≈ 3.14 ft

CHAPTER 5 Polar Coordinates; Vectors

5.1 Assess Your Understanding *(page 306)*

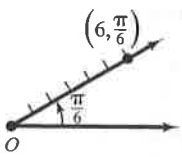
5. pole; polar axis **6.** -2 **7.** $\left(-\sqrt{3}, -1\right)$ **8.** F **9.** T **10.** T **11.** *A* **13.** *C* **15.** *B* **17.** *A*

19.

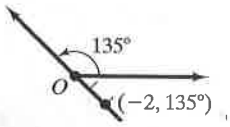


21.

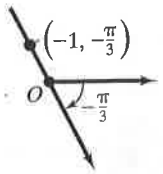
23.



25.

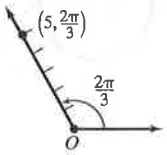


27.

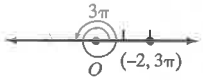


29.

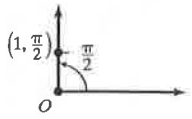
31.



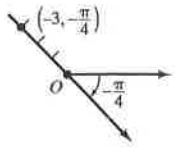
33.



35.



37.



(a) $\left(5, -\frac{4\pi}{3}\right)$ **(b)** $\left(-5, \frac{5\pi}{3}\right)$ **(c)** $\left(5, \frac{8\pi}{3}\right)$

(a) $(2, -2\pi)$ **(b)** $(-2, \pi)$ **(c)** $(2, 2\pi)$

(a) $\left(1, -\frac{3\pi}{2}\right)$ **(b)** $\left(-1, \frac{3\pi}{2}\right)$ **(c)** $\left(1, \frac{5\pi}{2}\right)$

(a) $\left(3, -\frac{5\pi}{4}\right)$ **(b)** $\left(-3, \frac{7\pi}{4}\right)$ **(c)** $\left(3, \frac{11\pi}{4}\right)$

39. $(0, 3)$ **41.** $(-2, 0)$ **43.** $\left(-3\sqrt{3}, 3\right)$ **45.** $\left(\sqrt{2}, -\sqrt{2}\right)$ **47.** $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **49.** $(2, 0)$ **51.** $(-2.57, 7.05)$ **53.** $(-4.98, -3.85)$ **55.** $(3, 0)$

57. $(1, \pi)$ **59.** $\left(\sqrt{2}, -\frac{\pi}{4}\right)$ **61.** $\left(2, \frac{\pi}{6}\right)$ **63.** $(2.47, -1.02)$ **65.** $(9.30, 0.47)$ **67.** $r^2 = \frac{3}{2}$ or $r = \frac{\sqrt{6}}{2}$ **69.** $r^2 \cos^2\theta - 4r \sin\theta = 0$

71. $r^2 \sin 2\theta = 1$ **73.** $r \cos\theta = 4$ **75.** $x^2 + y^2 - x = 0$ or $\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}$ **77.** $(x^2 + y^2)^{3/2} - x = 0$ **79.** $x^2 + y^2 = 4$ **81.** $y^2 = 8(x + 2)$

83. (a) $(-10, 36)$ **(b)** $\left(2\sqrt{349}, 180° + \tan^{-1}\left(-\frac{18}{5}\right)\right) \approx (37.36, 105.5°)$ **(c)** $(-3, -35)$ **(d)** $\left(\sqrt{1234}, 180° + \tan^{-1}\left(\frac{35}{3}\right)\right) \approx (35.13, 265.1°)$

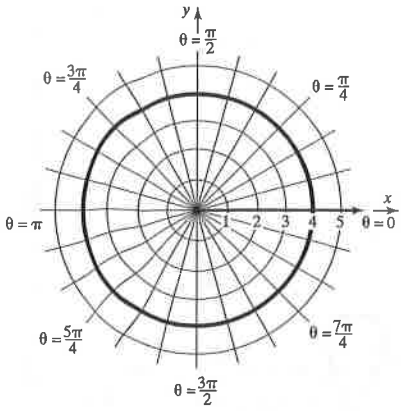
5.2 Assess Your Understanding *(page 321)*

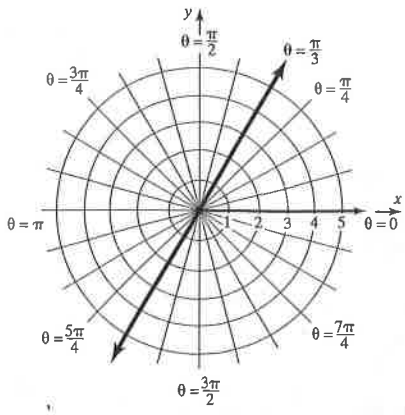
7. polar equation **8.** $r = 2\cos\theta$ **9.** $-r$ **10.** F **11.** F **12.** F

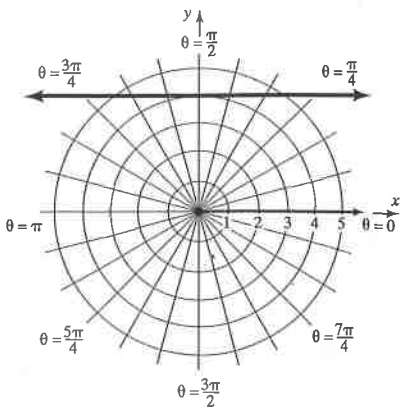
13. $x^2 + y^2 = 16$; circle, radius 4, center at pole

15. $y = \sqrt{3}x$; line through pole, making an angle of $\frac{\pi}{3}$ with polar axis

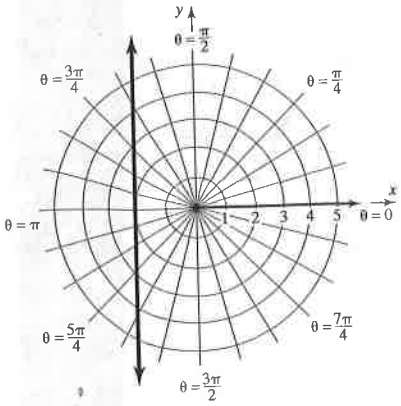
17. $y = 4$; horizontal line 4 units above the pole



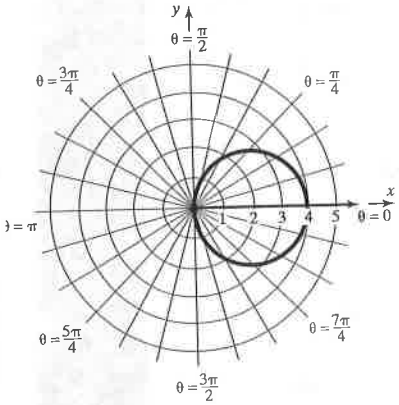




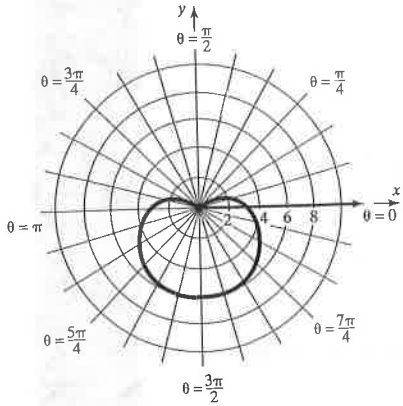
$x = -2$; vertical line 2 units to the left of the pole



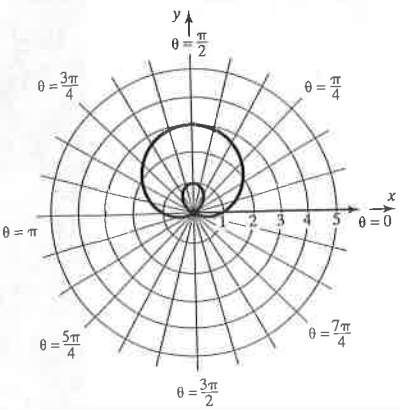
5. $(x - 2)^2 + y^2 = 4$; circle, radius 2, center at $(2, 0)$ in rectangular coordinates



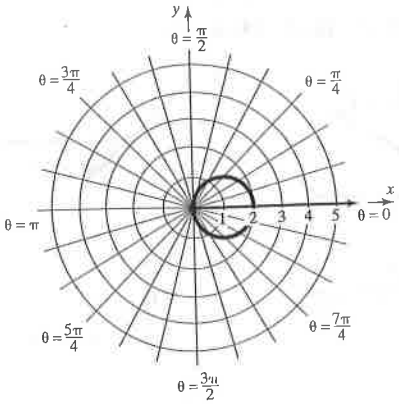
39. Cardioid



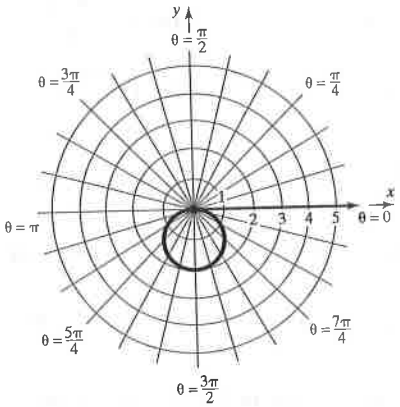
45. Limaçon with inner loop



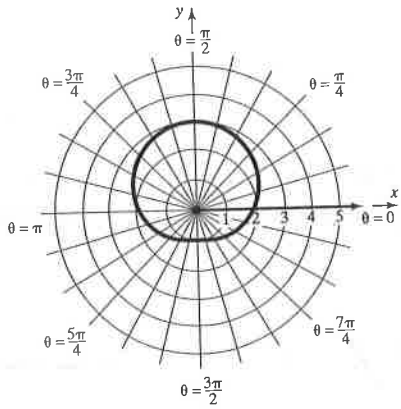
21. $(x - 1)^2 + y^2 = 1$; circle, radius 1, center $(1, 0)$ in rectangular coordinates



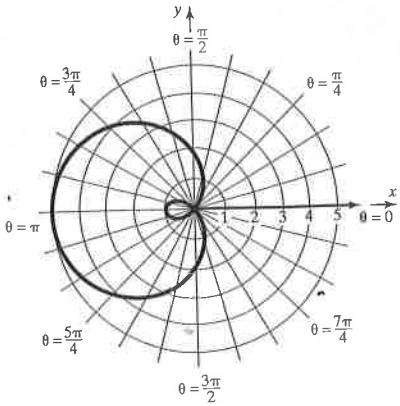
27. $x^2 + (y + 1)^2 = 1$; circle, radius 1, center at $(0, -1)$ in rectangular coordinates



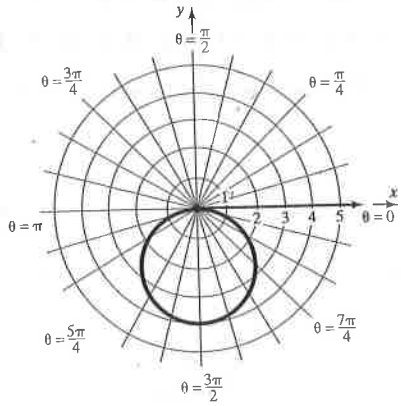
41. Limaçon without inner loop



47. Limaçon with inner loop

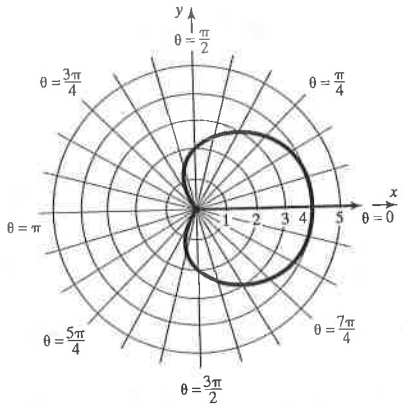


23. $x^2 + (y + 2)^2 = 4$; circle, radius 2, center at $(0, -2)$ in rectangular coordinates

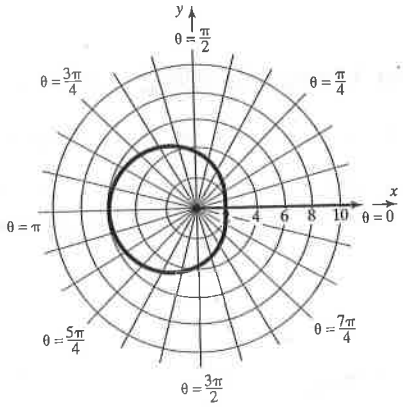


29. E **31.** F **33.** II **35.** D

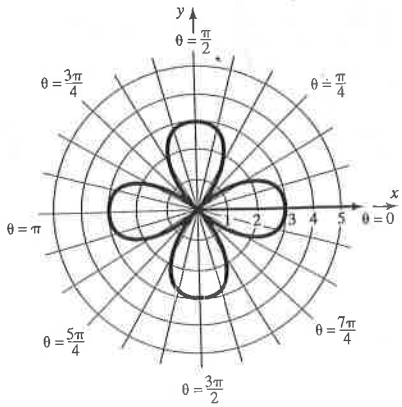
37. Cardioid



43. Limaçon without inner loop



49. Rose



51. F

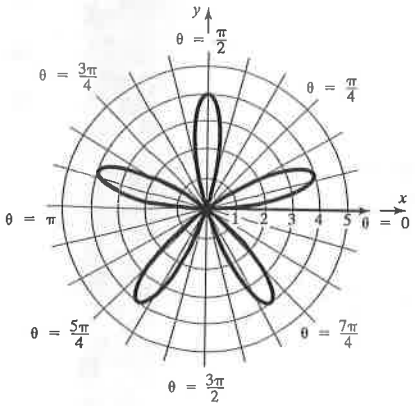
57.

67.

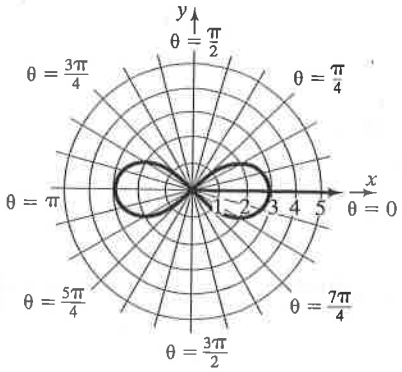
73.

81.

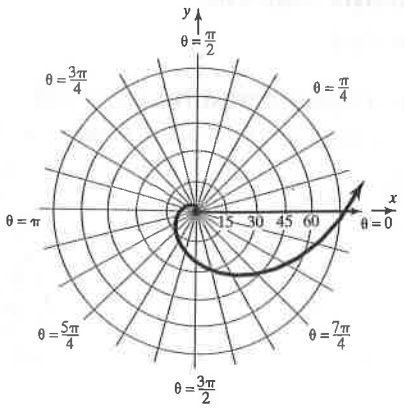
51. Rose



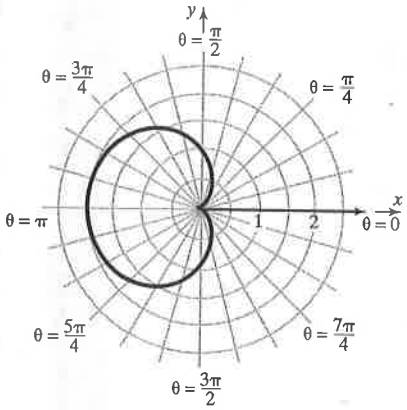
53. Lemniscate



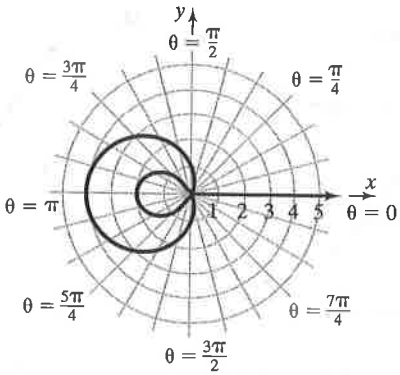
55. Spiral



57. Cardioid



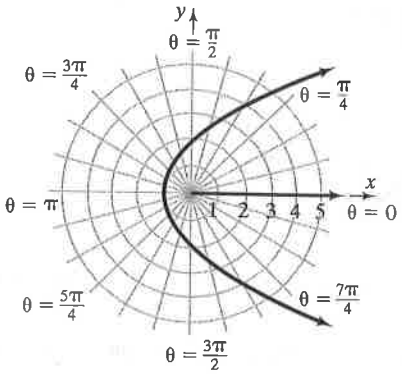
59. Limaçon with inner loop



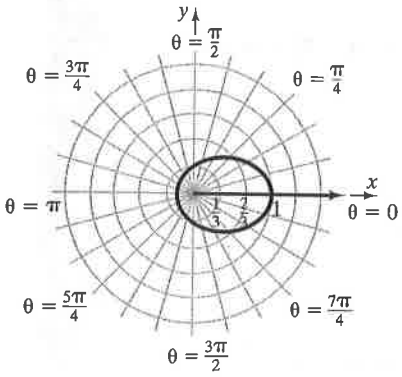
61. $r = 3 + 3\cos\theta$

63. $r = 4 + \sin\theta$

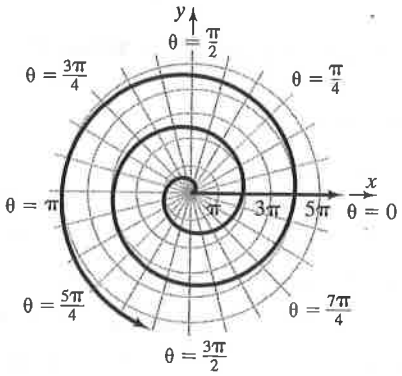
65.



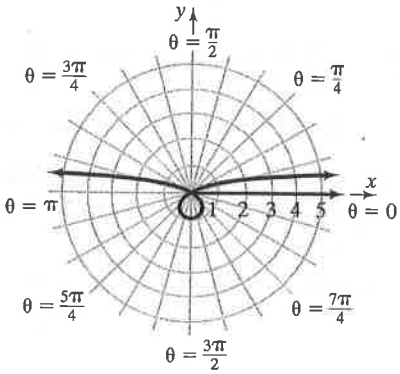
67.



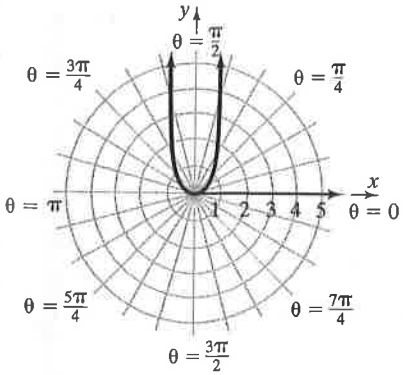
69.



71.



73.



75. $r \sin\theta = a$

$y = a$

77.

$r = 2a \sin\theta$

$r^2 = 2ar \sin\theta$

$x^2 + y^2 = 2ay$

$x^2 + y^2 - 2ay = 0$

$x^2 + (y - a)^2 = a^2$

Circle, radius a, center at $(0, a)$ in rectangular coordinates

79.

$r = 2a \cos\theta$

$r^2 = 2ar \cos\theta$

$x^2 + y^2 = 2ax$

$x^2 - 2ax + y^2 = 0$

$(x - a)^2 + y^2 = a^2$

Circle, radius a, center at $(a, 0)$ in rectangular coordinates

81. (a) $r^2 = \cos\theta$: $r^2 = \cos(\pi - \theta)$

$r^2 = -\cos\theta$

Not equivalent; test fails.

$(-r)^2 = \cos(-\theta)$

$r^2 = \cos\theta$

New test works.

(b) $r^2 = \sin\theta$: $r^2 = \sin(\pi - \theta)$

$r^2 = \sin\theta$

Test works.

$(-r)^2 = \sin(-\theta)$

$r^2 = -\sin\theta$

Not equivalent; new test fails.

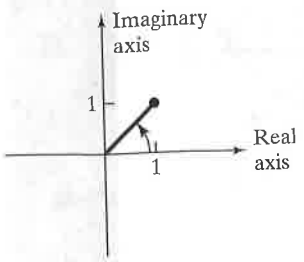
Historical Problems *(page 330)*

1. (a) $1 + 4i, 1 + i$ (b) $-1, 2 + i$

5.3 Assess Your Understanding *(page 330)*

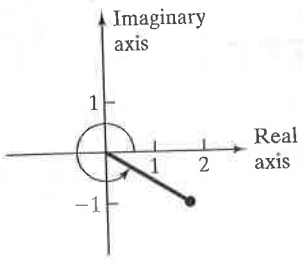
5. magnitude; modulus; argument 6. De Moivre's 7. three 8. T 9. F 10. T

11.



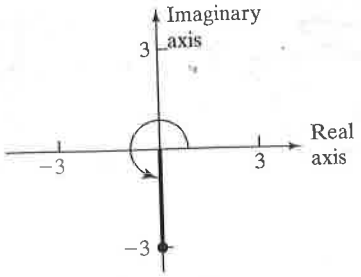
$\sqrt{2}(\cos 45° + i \sin 45°)$

13.



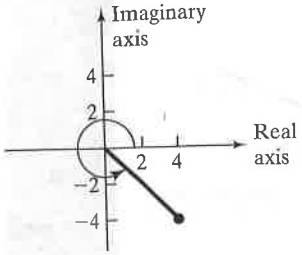
$2(\cos 330° + i \sin 330°)$

15.



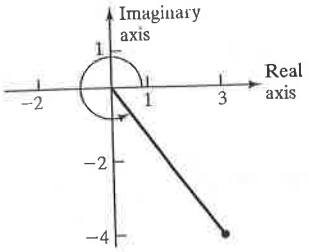
$3(\cos 270° + i \sin 270°)$

17.



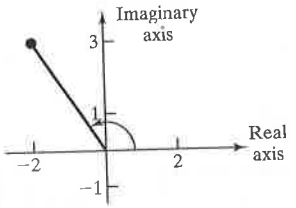
$4\sqrt{2}(\cos 315° + i \sin 315°)$

19.



$5(\cos 306.9° + i \sin 306.9°)$

21.



$\sqrt{13}(\cos 123.7° + i \sin 123.7°)$

23. $-1 + \sqrt{3}i$ 25. $2\sqrt{2} - 2\sqrt{2}i$ 27. $-3i$ 29. $-0.035 + 0.197i$ 31. $1.970 + 0.347i$

33. $zw = 8(\cos 60° + i \sin 60°); \frac{z}{w} = \frac{1}{2}(\cos 20° + i \sin 20°)$ 35. $zw = 12(\cos 40° + i \sin 40°); \frac{z}{w} = \frac{3}{4}(\cos 220° + i \sin 220°)$

37. $zw = 4\left(\cos\frac{9\pi}{40} + i \sin\frac{9\pi}{40}\right); \frac{z}{w} = \cos\frac{\pi}{40} + i \sin\frac{\pi}{40}$ 39. $zw = 4\sqrt{2}(\cos 15° + i \sin 15°); \frac{z}{w} = \sqrt{2}(\cos 75° + i \sin 75°)$ 41. $-32 + 32\sqrt{3}i$

43. $32i$ 45. $\frac{27}{2} + \frac{27\sqrt{3}}{2}i$ 47. $-\frac{25\sqrt{2}}{2} + \frac{25\sqrt{2}}{2}i$ 49. $-4 + 4i$ 51. $-23 + 14.142i$

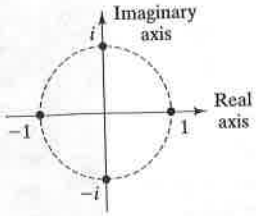
53. $\sqrt[6]{2}(\cos 15° + i \sin 15°), \sqrt[6]{2}(\cos 135° + i \sin 135°), \sqrt[6]{2}(\cos 255° + i \sin 255°)$

55. $\sqrt[4]{8}(\cos 75° + i \sin 75°), \sqrt[4]{8}(\cos 165° + i \sin 165°), \sqrt[4]{8}(\cos 255° + i \sin 255°), \sqrt[4]{8}(\cos 345° + i \sin 345°)$

57. $2(\cos 67.5° + i \sin 67.5°), 2(\cos 157.5° + i \sin 157.5°), 2(\cos 247.5° + i \sin 247.5°), 2(\cos 337.5° + i \sin 337.5°)$

59. $\cos 18° + i \sin 18°, \cos 90° + i \sin 90°, \cos 162° + i \sin 162°, \cos 234° + i \sin 234°, \cos 306° + i \sin 306°$

61. $1, i, -1, -i$



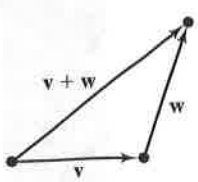
63. Look at formula (8); $|z_k| = \sqrt[n]{r}$ for all k.

65. Look at formula (8). The z_k are spaced apart by an angle of $\frac{2\pi}{n}$.

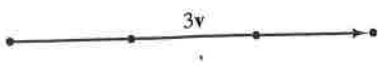
5.4 Assess Your Understanding *(page 341)*

1. unit 2. scalar 3. horizontal; vertical 4. T 5. T 6. F

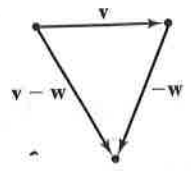
7.



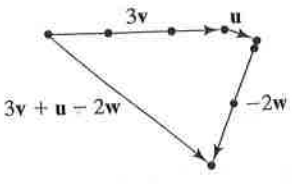
9.



11.



13.



15. T 17. F 19. F 21. T 23. 12 25. $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j}$ 27. $\mathbf{v} = 2\mathbf{i} + 4\mathbf{j}$ 29. $\mathbf{v} = 8\mathbf{i} - \mathbf{j}$ 31. $\mathbf{v} = -\mathbf{i} + \mathbf{j}$ 33. 5 35. $\sqrt{2}$ 37. $\sqrt{13}$ 39. $-\mathbf{j}$