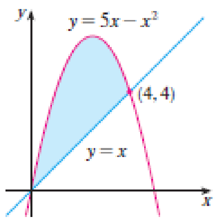
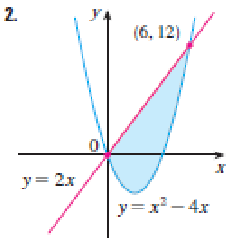
5.1 Exercises

1–4 Find the area of the shaded region.

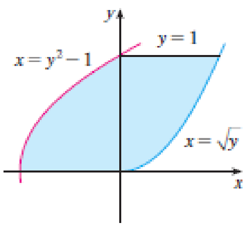
1.



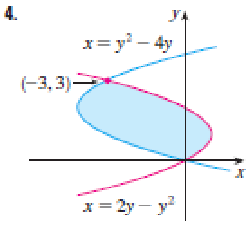
2.



3.



4.



5–12 Sketch the region enclosed by the given curves. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

5. $y = x + 1, \quad y = 9 - x^2, \quad x = -1, \quad x = 2$

6. $y = \sin x, \quad y = x, \quad x = \pi/2, \quad x = \pi$

7. $y = (x - 2)^2, \quad y = x$

8. $y = x^2 - 2x, \quad y = x + 4$

9. $y = \sqrt{x + 3}, \quad y = (x + 3)/2$

10. $y = \sin x, \quad y = 2x/\pi, \quad x \geqslant 0$

11. $x = 1 - y^2, \quad x = y^2 - 1$

12. $4x + y^2 = 12, \quad x = y$

13–28 Sketch the region enclosed by the given curves and find its area.

13. $y = 12 - x^2, \quad y = x^2 - 6$

14. $y = x^2, \quad y = 4x - x^2$

15. $y = \sec^2 x, \quad y = 8 \cos x, \quad -\pi/3 \leqslant x \leqslant \pi/3$

16. $y = \cos x, \quad y = 2 - \cos x, \quad 0 \leqslant x \leqslant 2\pi$

17. $x = 2y^2, \quad x = 4 + y^2$

18. $y = \sqrt{x - 1}, \quad x - y = 1$

19. $y = \cos \pi x, \quad y = 4x^2 - 1$

20. $x = y^4, \quad y = \sqrt{2 - x}, \quad y = 0$

21. $y = \cos x, \quad y = 1 - 2x/\pi$

22. $y = x^3, \quad y = x$

23. $y = \cos x, \quad y = \sin 2x, \quad x = 0, \quad x = \pi/2$

24. $y = \cos x, \quad y = 1 - \cos x, \quad 0 \leqslant x \leqslant \pi$

25. $y = \sqrt{x}, \quad y = \frac{1}{2}x, \quad x = 9$

26. $y = |x|, \quad y = x^2 - 2$

27. $y = 1/x^2, \quad y = x, \quad y = \frac{1}{8}x$

28. $y = \frac{1}{4}x^2, \quad y = 2x^2, \quad x + y = 3, \quad x \geqslant 0$

29–30 Use calculus to find the area of the triangle with the given vertices.

29. $(0, 0), \quad (3, 1), \quad (1, 2)$

30. $(2, 0), \quad (0, 2), \quad (-1, 1)$

31–32 Evaluate the integral and interpret it as the area of a region. Sketch the region.

31. $\int_0^{\pi/2} |\sin x - \cos 2x| \, dx$

32. $\int_0^4 |\sqrt{x + 2} - x| \, dx$

33–36 Use a graph to find approximate x-coordinates of the points of intersection of the given curves. Then find (approximately) the area of the region bounded by the curves.

33. $y = x \sin(x^2), \quad y = x^4, \quad x \geqslant 0$

34. $y = \dfrac{x}{(x^2 + 1)^2}, \quad y = x^5 - x, \quad x \geqslant 0$

35. $y = 3x^2 - 2x, \quad y = x^3 - 3x + 4$

36. $y = x^2 \cos(x^3), \quad y = x^{10}$

37–40 Graph the region between the curves and use your calculator to compute the area correct to five decimal places.

37. $y = \dfrac{2}{1 + x^4}, \quad y = x^2$

38. $y = x^6, \quad y = \sqrt{2 - x^4}$

39. $y = \tan^2 x, \quad y = \sqrt{x}$

40. $y = \cos x, \quad y = x + 2 \sin^4 x$

CAS **41.** Use a computer algebra system to find the exact area enclosed by the curves $y = x^5 - 6x^3 + 4x$ and $y = x$.

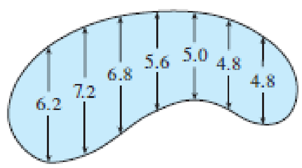
42. Sketch the region in the xy-plane defined by the inequalities $x - 2y^2 \geqslant 0$, $1 - x - |y| \geqslant 0$ and find its area.

43. Racing cars driven by Chris and Kelly are side by side at the start of a race. The table shows the velocities of each car (in miles per hour) during the first ten seconds of the race. Use the

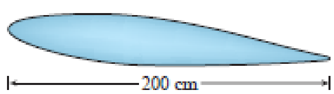
Midpoint Rule to estimate how much farther Kelly travels than Chris does during the first ten seconds.

t	v_C	v_K	t	v_C	v_K
0	0	0	6	69	80
1	20	22	7	75	86
2	32	37	8	81	93
3	46	52	9	86	98
4	54	61	10	90	102
5	62	71			

44. The widths (in meters) of a kidney-shaped swimming pool were measured at 2-meter intervals as indicated in the figure. Use the Midpoint Rule to estimate the area of the pool.

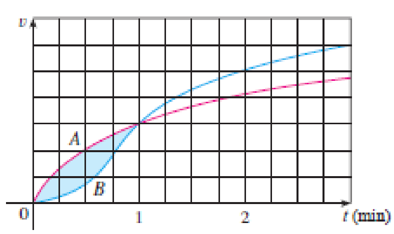


45. A cross-section of an airplane wing is shown. Measurements of the thickness of the wing, in centimeters, at 20-centimeter intervals are 5.8, 20.3, 26.7, 29.0, 27.6, 27.3, 23.8, 20.5, 15.1, 8.7, and 2.8. Use the Midpoint Rule to estimate the area of the wing's cross-section.

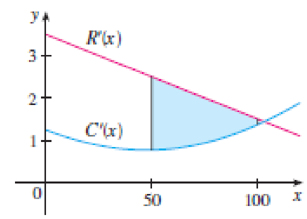


46. If the birth rate of a population is $b(t) = 2200 + 52.3t + 0.74t^2$ people per year and the death rate is $d(t) = 1460 + 28.8t$ people per year, find the area between these curves for $0 \leqslant t \leqslant 10$. What does this area represent?

47. Two cars, A and B, start side by side and accelerate from rest. The figure shows the graphs of their velocity functions.
(a) Which car is ahead after one minute? Explain.
(b) What is the meaning of the area of the shaded region?
(c) Which car is ahead after two minutes? Explain.
(d) Estimate the time at which the cars are again side by side.



48. The figure shows graphs of the marginal revenue function R' and the marginal cost function C' for a manufacturer. [Recall from Section 3.7 that $R(x)$ and $C(x)$ represent the revenue and cost when x units are manufactured. Assume that R and C are measured in thousands of dollars.] What is the meaning of the area of the shaded region? Use the Midpoint Rule to estimate the value of this quantity.



49. The curve with equation $y^2 = x^2(x + 3)$ is called **Tschirnhausen's cubic**. If you graph this curve you will see that part of the curve forms a loop. Find the area enclosed by the loop.

50. Find the area of the region bounded by the parabola $y = x^2$, the tangent line to this parabola at $(1, 1)$, and the x-axis.

51. Find the number b such that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two regions with equal area.

52. (a) Find the number a such that the line $x = a$ bisects the area under the curve $y = 1/x^2$, $1 \leqslant x \leqslant 4$.
(b) Find the number b such that the line $y = b$ bisects the area in part (a).

53. Find the values of c such that the area of the region bounded by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 576.

54. Suppose that $0 < c < \pi/2$. For what value of c is the area of the region enclosed by the curves $y = \cos x$, $y = \cos(x - c)$, and $x = 0$ equal to the area of the region enclosed by the curves $y = \cos(x - c)$, $x = \pi$, and $y = 0$?

The following exercises are intended only for those who have already covered Chapter 6.

55–57 Sketch the region bounded by the given curves and find the area of the region.

55. $y = 1/x, \quad y = 1/x^2, \quad x = 2$

56. $y = \sin x, \quad y = e^x, \quad x = 0, \quad x = \pi/2$

57. $y = \tan x, \quad y = 2 \sin x, \quad -\pi/3 \leqslant x \leqslant \pi/3$

58. For what values of m do the line $y = mx$ and the curve $y = x/(x^2 + 1)$ enclose a region? Find the area of the region.