ANSWERS FOR SELECTED ODD-NUMBERED PROBLEMS

EXERCISES 1.1 (PAGE 10)

1. linear, second order **3.** linear, fourth order
5. nonlinear, second order **7.** linear, third order
9. linear in x but nonlinear in y
15. domain of function is $[-2, \infty)$; largest interval of definition for solution is $-2, \infty)$
17. domain of function is the set of real numbers except $x = 2$ and $x = -2$; largest intervals of definition fo solution are $(-\infty, -2)$, $(-2, 2)$, or $(2, \infty)$
19. $X = \dfrac{e^t - 1}{e^t - 2}$ defined on $-\infty, \ln 2)$ or on $(\ln 2, \infty)$
27. $m = -2$ **29.** $m = 2, m = 3$ **31.** $m = 0, m = -1$
33. $y = 2$ **35.** no constant solutions

EXERCISES 1.2 (PAGE 17)

1. $y = 1/(1 - 4e^{-x})$
3. $y = 1/(x^2 - 1)$; $(1, \infty)$
5. $y = 1/(x^2 + 1)$; $(-\infty, \infty)$
7. $x = -\cos t + 8 \sin t$
9. $x = \frac{\sqrt{3}}{4} \cos t + \frac{1}{4} \sin t$ **11.** $y = \frac{3}{2} e^x - \frac{1}{2} e^{-x}$
13. $y = 5e^{-x-1}$ **15.** $y = 0, y = x^3$
17. half-planes defined by either $y > 0$ or $y < 0$
19. half-planes defined by either $x > 0$ or $x < 0$
21. the regions defined by $y > 2$, $y < -2$, or $-2 < y < 2$
23. any region not containing $(0, 0)$
25. yes
27. no
29. **(a)** $y = cx$
(b) any rectangular region not touching the y-axis
(c) No, the function is not differentiable at $x = 0$.
31. **(b)** $y = 1/(1 - x)$ on $(-\infty, 1)$;
$y = -1/(x + 1)$ on $(-1, \infty)$;
(c) $y = 0$ on $(-\infty, \infty)$
39. $y = 3 \sin 2x$
41. $y = 0$
43. no solution

EXERCISES 1.3 (PAGE 28)

1. $\dfrac{dP}{dt} = kP + r$; $\dfrac{dP}{dt} = kP - r$
3. $\dfrac{dP}{dt} = k_1 P - k_2 P^2$
7. $\dfrac{dx}{dt} = kx(1000 - x)$
9. $\dfrac{dA}{dt} + \dfrac{1}{100} A = 0$; $A(0) = 50$
11. $\dfrac{dA}{dt} + \dfrac{7}{600 - t} A = 6$ **13.** $\dfrac{dh}{dt} = -\dfrac{c\pi}{450} \sqrt{h}$
15. $L\dfrac{di}{dt} + Ri = E(t)$ **17.** $m\dfrac{dv}{dt} = mg - kv^2$
19. $m\dfrac{d^2x}{dt^2} = -kx$
21. $m\dfrac{dv}{dt} + v\dfrac{dm}{dt} + kv = -mg + R$
23. $\dfrac{d^2r}{dt^2} + \dfrac{gR^2}{r^2} = 0$ **25.** $\dfrac{dA}{dt} = k(M - A), k > 0$
27. $\dfrac{dx}{dt} + kx = r, k > 0$ **29.** $\dfrac{dy}{dx} = \dfrac{-x + \sqrt{x^2 + y^2}}{y}$

CHAPTER 1 IN REVIEW (PAGE 33)

1. $\dfrac{dy}{dx} = 10y$ **3.** $y'' + k^2 y = 0$
5. $y'' - 2y' + y = 0$ **7.** **(a), (d)**
9. **(b)** **11.** **(b)**
13. $y = c_1$ and $y = c_2 e^x$, c_1 and c_2 constants
15. $y' = x^2 + y^2$
17. **(a)** The domain is the set of all real numbers.
(b) either $(-\infty, 0)$ or $(0, \infty)$
19. For $x_0 = -1$ the interval is $(-\infty, 0)$, and for $x_0 = 2$ the interval is $(0, \infty)$.
21. **(c)** $y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \quad 0 \end{cases}$ **23.** $(-\infty, \infty)$
25. $(0, \infty)$ **31.** $y = \frac{1}{2} e^{3x} - \frac{1}{2} e^{-x} - 2x$
33. $y = \frac{3}{2} e^{3x-3} + \frac{9}{2} e^{-x+1} - 2x$.
35. $y_0 = -3, y_1 = 0$
37. $\dfrac{dP}{dt} = k(P - 200 + 10t)$

EXERCISES 2.1 (PAGE 43)

21. 0 is asymptotically stable (attractor); 3 is unstable (repeller).
23. 2 is semi-stable.
25. -2 is unstable (repeller); 0 is semi-stable; 2 is asymptotically stable (attractor).
27. -1 is asymptotically stable (attractor); 0 is unstable (repeller).
39. $0 < P_0 < h/k$
41. $\sqrt{mg/k}$

EXERCISES 2.2 (PAGE 51)

1. $y = -\frac{1}{5}\cos 5x + c$ **3.** $y = \frac{1}{3}e^{-3x} + c$

5. $y = cx^4$ **7.** $-3e^{-2y} = 2e^{3x} + c$

9. $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 = \frac{1}{2}y^2 + 2y + \ln|y| + c$

11. $4\cos y = 2x + \sin 2x + c$

13. $(e^x + 1)^{-2} + 2(e^y + 1)^{-1} = c$

15. $S = ce^{kr}$ **17.** $P = \dfrac{ce^t}{1 + ce^t}$

19. $(y + 3)^5 e^x = c(x + 4)^5 e^y$ **21.** $y = \sin\left(\frac{1}{2}x^2 + c\right)$

23. $x = \tan\left(4t - \frac{3}{4}\pi\right)$ **25.** $y = \dfrac{e^{-(1+1/x)}}{x}$

27. $y = \frac{1}{2}x + \frac{\sqrt{3}}{2}\sqrt{1 - x^2}$ **29.** $y = e^{\int_4^x e^{t^2} dt}$

31. $y = -\sqrt{x^2 + x - 1}$; $\left(-\infty, -\frac{1 + \sqrt{5}}{2}\right)$.

33. $y = -\ln(2 - e^x)$; $(-\infty, \ln 2)$

35. **(a)** $y = 2, y = -2, y = 2\dfrac{3 - e^{4x-1}}{3 + e^{4x-1}}$

37. $y = -1$ and $y = 1$ are singular solutions of Problem 21; $y = 0$ of Problem 22

39. $y = 1$

41. $y = 1 + \frac{1}{10}\tan\left(\frac{1}{10}x\right)$

45. $y = \tan x - \sec x + c$

47. $y = [-1 + c(1 + \sqrt{x})]^2$

49. $y = 2\sqrt{\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + 4}$

57. $y(x) = (4h/L^2)x^2 + a$

EXERCISES 2.3 (PAGE 61)

1. $y = ce^{5x}$, $(-\infty, \infty)$

3. $y = \frac{1}{4}e^{3x} + ce^{-x}$, $(-\infty, \infty)$; ce^{-x} is transient

5. $y = \frac{1}{3} + ce^{-x^3}$, $(-\infty, \infty)$; ce^{-x^3} is transient

7. $y = x^{-1}\ln x + cx^{-1}$, $(0, \infty)$; solution is transient

9. $y = cx - x\cos x$, $(0, \infty)$

11. $y = \frac{1}{7}x^3 - \frac{1}{5}x + cx^{-4}$, $(0, \infty)$; cx^{-4} is transient

13. $y = \frac{1}{2}x^{-2}e^x + cx^{-2}e^{-x}$, $(0, \infty)$; $cx^{-2}e^{-x}$ is transient

15. $x = 2y^6 + cy^4$, $(0, \infty)$

17. $y = \sin x + c\cos x$, $(-\pi/2, \pi/2)$

19. $(x + 1)e^x y = x^2 + c$, $(-1, \infty)$; solution is transient

21. $(\sec\theta + \tan\theta)r = \theta - \cos\theta + c$, $(-\pi/2, \pi/2)$

23. $y = e^{-3x} + cx^{-1}e^{-3x}$, $(0, \infty)$; solution is transient

25. $y = -\frac{1}{5}x - \frac{1}{25} + \frac{76}{25}e^{5x}$; $(-\infty, \infty)$

27. $y = x^{-1}e^x + (2 - e)x^{-1}$; $(0, \infty)$

29. $i = \dfrac{E}{R} + \left(i_0 - \dfrac{E}{R}\right)e^{-Rt/L}$; $(-\infty, \infty)$

31. $y = 2x + 1 + 5/x$; $(0, \infty)$

33. $(x + 1)y = x\ln x - x + 21$; $(0, \infty)$

35. $y = -2 + 3e^{-\cos x}$; $(-\infty, \infty)$

37. $y = \begin{cases} \frac{1}{2}(1 - e^{-2x}), & 0 \le x \le 3 \\ \frac{1}{2}(e^6 - 1)e^{-2x}, & x > 3 \end{cases}$

39. $y = \begin{cases} \frac{1}{2} + \frac{3}{2}e^{-x^2}, & 0 \le x < 1 \\ \left(\frac{1}{2}e + \frac{3}{2}\right)e^{-x^2}, & x \quad 1 \end{cases}$

41. $y = \begin{cases} 2x - 1 + 4e^{-2x}, & 0 \le x \le 1 \\ 4x^2\ln x + (1 + 4e^{-2})x^2, & x > 1 \end{cases}$

43. $y = e^{x^2 - 1} + \frac{1}{2}\sqrt{\pi}e^{x^2}(\text{erf}(x) - \text{erf}(1))$

53. $E(t) = E_0 e^{-(t-4)/RC}$

EXERCISES 2.4 (PAGE 69)

1. $x^2 - x + \frac{3}{2}y^2 + 7y = c$ **3.** $\frac{5}{2}x^2 + 4xy - 2y^4 = c$

5. $x^2y^2 - 3x + 4y = c$ **7.** not exact

9. $xy^3 + y^2\cos x - \frac{1}{2}x^2 = c$

11. not exact

13. $xy - 2xe^x + 2e^x - 2x^3 = c$

15. $x^3y^3 - \tan^{-1}3x = c$

17. $-\ln|\cos x| + \cos x \sin y = c$

19. $t^4y - 5t^3 - ty + y^3 = c$

21. $\frac{1}{3}x^3 + x^2y + xy^2 - y = \frac{4}{3}$

23. $4ty + t^2 - 5t + 3y^2 - y = 8$

25. $y^2\sin x - x^3y - x^2 + y\ln y - y = 0$

27. $k = 10$ **29.** $x^2y^2\cos x = c$

31. $x^2y^2 + x^3 = c$ **33.** $3x^2y^3 + y^4 = c$

35. $-2ye^{3x} + \frac{10}{3}e^{3x} + x = c$

37. $e^{y^2}(x^2 + 4) = 20$

39. **(c)** $y_1(x) = -x^2 - \sqrt{x^4 - x^3 + 4}$

$y_2(x) = -x^2 + \sqrt{x^4 - x^3 + 4}$

45. **(a)** $v(x) = 8\sqrt{\dfrac{x}{3} - \dfrac{9}{x^2}}$ **(b)** 12.7 ft/s

EXERCISES 2.5 (PAGE 74)

1. $y + x\ln|x| = cx$

3. $(x - y)\ln|x - y| = y + c(x - y)$

5. $x + y\ln|x| = cy$

7. $\ln(x^2 + y^2) + 2\tan^{-1}(y/x) = c$

9. $4x = y(\ln|y| - c)^2$ **11.** $y^3 + 3x^3\ln|x| = 8x^3$

13. $\ln|x| = e^{y/x} - 1$ **15.** $y^3 = 1 + cx^{-3}$

17. $y^{-3} = x + \frac{1}{3} + ce^{3x}$ **19.** $e^{t/y} = ct$

21. $y^{-3} = -\frac{9}{5}x^{-1} + \frac{49}{5}x^{-6}$

23. $y = -x - 1 + \tan(x + c)$

25. $2y - 2x + \sin 2(x + y) = c$
27. $4(y - 2x + 3) = (x + c)^2$
29. $-\cot(x + y) + \csc(x + y) = x + \sqrt{2} - 1$
35. **(b)** $y = \dfrac{2}{x} + \left(-\frac{1}{4}x + cx^{-3}\right)^{-1}$

EXERCISES 2.6 (PAGE 79)

1. $y_2 = 2.9800, \quad y_4 = 3.1151$
3. $y_{10} = 2.5937, \quad y_{20} = 2.6533; y = e^x$
5. $y_5 = 0.4198, \quad y_{10} = 0.4124$
7. $y_5 = 0.5639, \quad y_{10} = 0.5565$
9. $y_5 = 1.2194, \quad y_{10} = 1.2696$
13. Euler: $y_{10} = 3.8191, \quad y_{20} = 5.9363$
RK4: $y_{10} = 42.9931, \quad y_{20} = 84.0132$

CHAPTER 2 IN REVIEW (PAGE 80)

1. $-A/k$, a repeller for $k > 0$, an attractor for $k < 0$
3. true
5. $\dfrac{d^3y}{dx^3} = x\sin y$
7. true
9. $y = c_1 e^{e^x}$
11. $\dfrac{dy}{dx} + (\sin x)y = x$
13. $\dfrac{dy}{dx} = (y - 1)^2(y - 3)^2$
15. semi-stable for n even and unstable for n odd; semi-stable for n even and asymptotically stable for n odd.
19. $2x + \sin 2x = 2\ln(y^2 + 1) + c$
21. $(6x + 1)y^3 = -3x^3 + c$
23. $Q = ct^{-1} + \frac{1}{25}t^4(-1 + 5\ln t)$
25. $y = \frac{1}{4} + c(x^2 + 4)^{-4}$
27. $y = \csc x, (\pi, 2\pi)$
29. **(b)** $y = \frac{1}{4}(x + 2\sqrt{y_0} - x_0)^2, (x_0 - 2\sqrt{y_0}, \infty)$

EXERCISES 3.1 (PAGE 90)

1. 7.9 yr; 10 yr
3. 760; approximately 11 persons/yr
5. 11 h
7. 136.5 h
9. $I(15) = 0.00098I_0$ or approximately 0.1% of I_0
11. 15,600 years
13. $T(1) = 36.67°$ F; approximately 3.06 min
15. approximately 82.1 s; approximately 145.7 s
17. 390°
19. about 1.6 hours prior to the discovery of the body
21. $A(t) = 200 - 170e^{-t/50}$
23. $A(t) = 1000 - 1000e^{-t/100}$
25. $A(t) = 1000 - 10t - \frac{1}{10}(100 - t)^2$; 100 min
27. 64.38 lb
29. $i(t) = \frac{3}{5} - \frac{3}{5}e^{-500t}$; $i \to \frac{3}{5}$ as $t \to \infty$
31. $q(t) = \frac{1}{100} - \frac{1}{100}e^{-50t}$; $i(t) = \frac{1}{2}e^{-50t}$
33. $i(t) = \begin{cases} 60 - 60e^{-t/10}, & 0 \le t \le 20 \\ 60(e^2 - 1)e^{-t/10}, & t > 20 \end{cases}$
35. **(a)** $v(t) = \dfrac{mg}{k} + \left(v_0 - \dfrac{mg}{k}\right)e^{-kt/m}$
(b) $v \to \dfrac{mg}{k}$ as $t \to \infty$
(c) $s(t) = \dfrac{mg}{k}t - \dfrac{m}{k}\left(v_0 - \dfrac{mg}{k}\right)e^{-kt/m} + \dfrac{m}{k}\left(v_0 - \dfrac{mg}{k}\right)$
39. **(a)** $v(t) = \dfrac{g}{4k}\left(\dfrac{k}{\rho}t + r_0\right) - \dfrac{g\rho r_0}{4k}\left(\dfrac{r_0}{\frac{k}{\rho}t + r_0}\right)^3$
(c) $33\frac{1}{3}$ seconds
41. **(a)** $P(t) = P_0 e^{(k_1 - k_2)t}$
43. **(a)** As $t \to \infty$, $x(t) \to r/k$.
(b) $x(t) = r/k - (r/k)e^{-kt}$; $(\ln 2)/k$
47. **(c)** 1.988 ft

EXERCISES 3.2 (PAGE 100)

1. **(a)** $N = 2000$
(b) $N(t) = \dfrac{2000e^t}{1999 + e^t}$; $N(10) = 1834$
3. 1,000,000; 5.29 mo
5. **(b)** $P(t) = \dfrac{4(P_0 - 1) - (P_0 - 4)e^{-3t}}{(P_0 - 1) - (P_0 - 4)e^{-3t}}$
(c) For $0 < P_0 < 1$, time of extinction is
$$t = -\frac{1}{3}\ln\frac{4(P_0 - 1)}{P_0 - 4}.$$
7. $P(t) = \dfrac{5}{2} + \dfrac{\sqrt{3}}{2}\tan\left[-\dfrac{\sqrt{3}}{2}t + \tan^{-1}\left(\dfrac{2P_0 - 5}{\sqrt{3}}\right)\right]$; time of extinction is
$$t = \frac{2}{\sqrt{3}}\left[\tan^{-1}\frac{5}{\sqrt{3}} + \tan^{-1}\left(\frac{2P_0 - 5}{\sqrt{3}}\right)\right]$$
9. 29.3 g; $X \to 60$ as $t \to \infty$; 0 g of A and 30 g of B
11. **(a)** $h(t) = \left(\sqrt{H} - \dfrac{4A_h}{A_w}t\right)^2$; I is $0 \le t \le \sqrt{H}A_w/4A_h$
(b) $576\sqrt{10}$ s or 30.36 min
13. **(a)** approximately 858.65 s or 14.31 min
(b) 243 s or 4.05 min

15. **(a)** $v(t) = \sqrt{\dfrac{mg}{k}}\tanh\left(\sqrt{\dfrac{kg}{m}}\,t + c_1\right)$

where $c_1 = \tanh^{-1}\left(\sqrt{\dfrac{k}{mg}}\,v_0\right)$

(b) $\sqrt{\dfrac{mg}{k}}$

(c) $s(t) = \dfrac{m}{k}\ln\cosh\left(\sqrt{\dfrac{kg}{m}}\,t + c_1\right) + c_2,$

where $c_2 = -(m/k)\ln\cosh c_1$

17. **(a)** $m\dfrac{dv}{dt} = mg - kv^2 - \ \ V,$

where ρ is the weight density of water

(b) $v(t) = \sqrt{\dfrac{mg - \ \ V}{k}}\tanh\left(\dfrac{\sqrt{kmg - k\ \ V}}{m}\,t + c_1\right)$

(c) $\sqrt{\dfrac{mg - \ \ V}{k}}$

19. **(a)** $W = 0$ and $W = 2$

(b) $W(x) = 2\,\text{sech}^2(x - c_1)$

(c) $W(x) = 2\,\text{sech}^2 x$

21. **(a)** $P(t) = \dfrac{1}{(-0.001350t + 10^{-0.01})^{100}}$

(b) approximately 724 months

(b) approximately 12,839 and 28,630,966

EXERCISES 3.3 (PAGE 110)

1. $x(t) = x_0\, e^{-\lambda_1 t}$

$y(t) = \dfrac{x_0\lambda_1}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$

$z(t) = x_0\left(1 - \dfrac{\lambda_2}{\lambda_2 - \lambda_1}e^{-\lambda_1 t} + \dfrac{\lambda_1}{\lambda_2 - \lambda_1}e^{-\lambda_2 t}\right)$

3. 5, 20, 147 days. The time when $y(t)$ and $z(t)$ are the same makes sense because most of A and half of B are gone, so half of C should have been formed.

5. $\dfrac{dx_1}{dt} = 6 - \frac{2}{25}x_1 + \frac{1}{50}x_2$

$\dfrac{dx_2}{dt} = \frac{2}{25}x_1 - \frac{2}{25}x_2$

7. **(a)** $\dfrac{dx_1}{dt} = 3\dfrac{x_2}{100 - t} - 2\dfrac{x_1}{100 + t}$

$\dfrac{dx_2}{dt} = 2\dfrac{x_1}{100 + t} - 3\dfrac{x_2}{100 - t}$

(b) $x_1(t) + x_2(t) = 150;\quad x_2(30)\ \ 47.4$ lb

13. $L_1\dfrac{di_2}{dt} + (R_1 + R_2)i_2 + R_1 i_3 = E(t)$

$L_2\dfrac{di_3}{dt} + R_1 i_2 + (R_1 + R_3)\,i_3 = E(t)$

15. $i(0) = i_0,\ s(0) = n - i_0,\ r(0) = 0$

CHAPTER 3 IN REVIEW (PAGE 113)

1. $dP/dt = 0.15P$

3. $P(45) = 8.99$ billion

5. $x = 10\ln\left(\dfrac{10 + \sqrt{100 - y^2}}{y}\right) - \sqrt{100 - y^2}$

7. **(a)** $\dfrac{BT_1 + T_2}{1 + B},\ \dfrac{BT_1 + T_2}{1 + B}$

(b) $T(t) = \dfrac{BT_1 + T_2}{1 + B} + \dfrac{T_1 - T_2}{1 + B}\,e^{k(1+B)t}$

9. $i(t) = \begin{cases} 4t - \frac{1}{5}t^2, & 0 \le t < 10 \\ 20, & t\ \ 10 \end{cases}$

11. $x(t) = \dfrac{\alpha c_1 e^{\alpha k_1 t}}{1 + c_1 e^{\alpha k_1 t}};\quad y(t) = c_2(1 + c_1 e^{\alpha k_1 t})^{k_2/k_1}$

13. $x = -y + 1 + c_2 e^{-y}$

15. **(a)** $K(t) = K_0 e^{-(\lambda_1 + \lambda_2)t},$

$C(t) = \dfrac{\lambda_1}{\lambda_1 + \lambda_2}K_0\left[1 - e^{-(\lambda_1 + \lambda_2)t}\right],$

$A(t) = \dfrac{\lambda_2}{\lambda_1 + \lambda_2}K_0\left[1 - e^{-(\lambda_1 + \lambda_2)t}\right]$

(b) 1.3×10^9 years

(c) 89%, 11%

EXERCISES 4.1 (PAGE 127)

1. $y = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

3. $y = 3x - 4x\ln x$

9. $(-\infty, 2)$

11. **(a)** $y = \dfrac{e}{e^2 - 1}(e^x - e^{-x})$ **(b)** $y = \dfrac{\sinh x}{\sinh 1}$

13. **(a)** $y = e^x\cos x - e^x\sin x$

(b) no solution

(c) $y = e^x\cos x + e^{-\pi/2}e^x\sin x$

(d) $y = c_2 e^x\sin x$, where c_2 is arbitrary

15. dependent

17. dependent

19. dependent

21. independent

23. The functions satisfy the DE and are linearly independent on the interval since $W(e^{-3x}, e^{4x}) = 7e^x \neq 0$; $y = c_1 e^{-3x} + c_2 e^{4x}$.

25. The functions satisfy the DE and are linearly independent on the interval since $W(e^x\cos 2x, e^x\sin 2x) = 2e^{2x} \neq 0$; $y = c_1 e^x\cos 2x + c_2 e^x\sin 2x$.

27. The functions satisfy the DE and are linearly independent on the interval since $W(x^3, x^4) = x^6 \neq 0$; $y = c_1 x^3 + c_2 x^4$.

29. The functions satisfy the DE and are linearly independent on the interval since $W(x, x^{-2}, x^{-2}\ln x) = 9x^{-6} \neq 0$; $y = c_1 x + c_2 x^{-2} + c_3 x^{-2}\ln x$.

35. **(b)** $y_p = x^2 + 3x + 3e^{2x};\quad y_p = -2x^2 - 6x - \frac{1}{3}e^{2x}$

EXERCISES 4.2 (PAGE 131)

1. $y_2 = xe^{2x}$
3. $y_2 = \sin 4x$
5. $y_2 = \sinh x$
7. $y_2 = xe^{2x/3}$
9. $y_2 = x^4 \ln|x|$
11. $y_2 = 1$
13. $y_2 = x\cos(\ln x)$
15. $y_2 = x^2 + x + 2$
17. $y_2 = e^{2x}$, $y_p = -\frac{1}{2}$
19. $y_2 = e^{2x}$, $y_p = \frac{5}{2}e^{3x}$

EXERCISES 4.3 (PAGE 137)

1. $y = c_1 + c_2e^{-x/4}$
3. $y = c_1e^{3x} + c_2e^{-2x}$
5. $y = c_1e^{-4x} + c_2xe^{-4x}$
7. $y = c_1e^{2x/3} + c_2e^{-x/4}$
9. $y = c_1\cos 3x + c_2\sin 3x$
11. $y = e^{2x}(c_1\cos x + c_2\sin x)$
13. $y = e^{-x/3}\left(c_1\cos\frac{1}{3}\sqrt{2}\,x + c_2\sin\frac{1}{3}\sqrt{2}\,x\right)$
15. $y = c_1 + c_2e^{-x} + c_3e^{5x}$
17. $y = c_1e^{-x} + c_2e^{3x} + c_3xe^{3x}$
19. $u = c_1e^{t} + e^{-t}(c_2\cos t + c_3\sin t)$
21. $y = c_1e^{-x} + c_2xe^{-x} + c_3x^2e^{-x}$
23. $y = c_1 + c_2x + e^{-x/2}\left(c_3\cos\frac{1}{2}\sqrt{3}\,x + c_4\sin\frac{1}{2}\sqrt{3}\,x\right)$
25. $y = c_1\cos\frac{1}{2}\sqrt{3}\,x + c_2\sin\frac{1}{2}\sqrt{3}\,x$
$\quad + c_3x\cos\frac{1}{2}\sqrt{3}\,x + c_4x\sin\frac{1}{2}\sqrt{3}\,x$
27. $u = c_1e^{r} + c_2re^{r} + c_3e^{-r} + c_4re^{-r} + c_5e^{-5r}$
29. $y = 2\cos 4x - \frac{1}{2}\sin 4x$
31. $y = -\frac{1}{3}e^{-(t-1)} + \frac{1}{3}e^{5(t-1)}$
33. $y = 0$
35. $y = \frac{5}{36} - \frac{5}{36}e^{-6x} + \frac{1}{6}xe^{-6x}$
37. $y = e^{5x} - xe^{5x}$
39. $y = 0$
41. $y = \dfrac{1}{2}\left(1 - \dfrac{5}{\sqrt{3}}\right)e^{-\sqrt{3}x} + \dfrac{1}{2}\left(1 + \dfrac{5}{\sqrt{3}}\right)e^{\sqrt{3}x}$;
$\quad y = \cosh\sqrt{3}x + \dfrac{5}{\sqrt{3}}\sinh\sqrt{3}x$
49. $y'' - 6y' + 5y = 0$
51. $y'' - 2y' = 0$
53. $y'' + 9y = 0$
55. $y'' + 2y' + 2y = 0$
57. $y''' - 8y'' = 0$

EXERCISES 4.4 (PAGE 147)

1. $y = c_1e^{-x} + c_2e^{-2x} + 3$
3. $y = c_1e^{5x} + c_2xe^{5x} + \frac{6}{5}x + \frac{3}{5}$
5. $y = c_1e^{-2x} + c_2xe^{-2x} + x^2 - 4x + \frac{7}{2}$
7. $y = c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x + \left(-4x^2 + 4x - \frac{4}{3}\right)e^{3x}$
9. $y = c_1 + c_2e^{x} + 3x$
11. $y = c_1e^{x/2} + c_2xe^{x/2} + 12 + \frac{1}{2}x^2e^{x/2}$
13. $y = c_1\cos 2x + c_2\sin 2x - \frac{3}{4}x\cos 2x$
15. $y = c_1\cos x + c_2\sin x - \frac{1}{2}x^2\cos x + \frac{1}{2}x\sin x$
17. $y = c_1e^{x}\cos 2x + c_2e^{x}\sin 2x + \frac{1}{4}xe^{x}\sin 2x$
19. $y = c_1e^{-x} + c_2xe^{-x} - \frac{1}{2}\cos x$
$\quad + \frac{12}{25}\sin 2x - \frac{9}{25}\cos 2x$
21. $y = c_1 + c_2x + c_3e^{6x} - \frac{1}{4}x^2 - \frac{6}{37}\cos x + \frac{1}{37}\sin x$
23. $y = c_1e^{x} + c_2xe^{x} + c_3x^2e^{x} - x - 3 - \frac{2}{3}x^3e^{x}$
25. $y = c_1\cos x + c_2\sin x + c_3x\cos x + c_4x\sin x$
$\quad + x^2 - 2x - 3$
27. $y = \sqrt{2}\sin 2x - \frac{1}{2}$
29. $y = -200 + 200e^{-x/5} - 3x^2 + 30x$
31. $y = -10e^{-2x}\cos x + 9e^{-2x}\sin x + 7e^{-4x}$
33. $x = \dfrac{F_0}{2\omega^2}\sin\omega t - \dfrac{F_0}{2\omega}t\cos\omega t$
35. $y = 11 - 11e^{x} + 9xe^{x} + 2x - 12x^2e^{x} + \frac{1}{2}e^{5x}$
37. $y = 6\cos x - 6(\cot 1)\sin x + x^2 - 1$
39. $y = \dfrac{-4\sin\sqrt{3}x}{\sin\sqrt{3} + \sqrt{3}\cos\sqrt{3}} + 2x$
41. $y = \begin{cases}\cos 2x + \frac{5}{6}\sin 2x + \frac{1}{3}\sin x, & 0 \le x \le \pi/2\\ \frac{2}{3}\cos 2x + \frac{5}{6}\sin 2x, & x > \pi/2\end{cases}$

EXERCISES 4.5 (PAGE 155)

1. $(3D - 2)(3D + 2)y = \sin x$
3. $(D - 6)(D + 2)y = x - 6$
5. $D(D + 5)^2y = e^{x}$
7. $(D - 1)(D - 2)(D + 5)y = xe^{-x}$
9. $D(D + 2)(D^2 - 2D + 4)y = 4$
15. D^4
17. $D(D - 2)$
19. $D^2 + 4$
21. $D^3(D^2 + 16)$
23. $(D + 1)(D - 1)^3$
25. $D(D^2 - 2D + 5)$
27. $1, x, x^2, x^3, x^4$
29. $e^{6x}, e^{-3x/2}$
31. $\cos\sqrt{5}x, \sin\sqrt{5}x$
33. $1, e^{5x}, xe^{5x}$
35. $y = c_1e^{-3x} + c_2e^{3x} - 6$
37. $y = c_1 + c_2e^{-x} + 3x$
39. $y = c_1e^{-2x} + c_2xe^{-2x} + \frac{1}{2}x + 1$
41. $y = c_1 + c_2x + c_3e^{-x} + \frac{2}{3}x^4 - \frac{8}{3}x^3 + 8x^2$
43. $y = c_1e^{-3x} + c_2e^{4x} + \frac{1}{7}xe^{4x}$
45. $y = c_1e^{-x} + c_2e^{3x} - e^{x} + 3$
47. $y = c_1\cos 5x + c_2\sin 5x + \frac{1}{4}\sin x$
49. $y = c_1e^{-3x} + c_2xe^{-3x} - \frac{1}{49}xe^{4x} + \frac{2}{343}e^{4x}$
51. $y = c_1e^{-x} + c_2e^{x} + \frac{1}{6}x^3e^{x} - \frac{1}{4}x^2e^{x} + \frac{1}{4}xe^{x} - 5$
53. $y = e^{x}(c_1\cos 2x + c_2\sin 2x) + \frac{1}{3}e^{x}\sin x$
55. $y = c_1\cos 5x + c_2\sin 5x - 2x\cos 5x$
57. $y = e^{-x/2}\left(c_1\cos\dfrac{\sqrt{3}}{2}x + c_2\sin\dfrac{\sqrt{3}}{2}x\right)$
$\quad + \sin x + 2\cos x - x\cos x$
59. $y = c_1 + c_2x + c_3e^{-8x} + \frac{11}{256}x^2 + \frac{7}{32}x^3 - \frac{1}{16}x^4$
61. $y = c_1e^{x} + c_2xe^{x} + c_3x^2e^{x} + \frac{1}{6}x^3e^{x} + x - 13$
63. $y = c_1 + c_2x + c_3e^{x} + c_4xe^{x} + \frac{1}{2}x^2e^{x} + \frac{1}{2}x^2$
65. $y = \frac{5}{8}e^{-8x} + \frac{5}{8}e^{8x} - \frac{1}{4}$
67. $y = -\frac{41}{125} + \frac{41}{125}e^{5x} - \frac{1}{10}x^2 + \frac{9}{25}x$
69. $y = -\pi\cos x - \frac{11}{3}\sin x - \frac{8}{3}\cos 2x + 2x\cos x$
71. $y = 2e^{2x}\cos 2x - \frac{3}{64}e^{2x}\sin 2x + \frac{1}{8}x^3 + \frac{3}{16}x^2 + \frac{3}{32}x$

EXERCISES 4.6 (PAGE 161)

1. $y = c_1 \cos x + c_2 \sin x + x \sin x + \cos x \ln|\cos x|$
3. $y = c_1 \cos x + c_2 \sin x - \frac{1}{2} x \cos x$
5. $y = c_1 \cos x + c_2 \sin x + \frac{1}{2} - \frac{1}{6} \cos 2x$
7. $y = c_1 e^x + c_2 e^{-x} + \frac{1}{2} x \sinh x$
9. $y = c_1 e^{2x} + c_2 e^{-2x} + \frac{1}{4}\left(e^{2x} \ln|x| - e^{-2x} \int_{x_0}^{x} \frac{e^{4t}}{t}\, dt\right)$, $x_0 > 0$
11. $y = c_1 e^{-x} + c_2 e^{-2x} + (e^{-x} + e^{-2x}) \ln(1 + e^x)$
13. $y = c_1 e^{-2x} + c_2 e^{-x} - e^{-2x} \sin e^x$
15. $y = c_1 e^{-t} + c_2 t e^{-t} + \frac{1}{2} t^2 e^{-t} \ln t - \frac{3}{4} t^2 e^{-t}$
17. $y = c_1 e^x \sin x + c_2 e^x \cos x + \frac{1}{3} x e^x \sin x + \frac{1}{3} e^x \cos x \ln|\cos x|$
19. $y = \frac{1}{4} e^{-x/2} + \frac{3}{4} e^{x/2} + \frac{1}{8} x^2 e^{x/2} - \frac{1}{4} x e^{x/2}$
21. $y = \frac{4}{9} e^{-4x} + \frac{25}{36} e^{2x} - \frac{1}{4} e^{-2x} + \frac{1}{9} e^{-x}$
23. $y = c_1 x^{-1/2} \cos x + c_2 x^{-1/2} \sin x + x^{-1/2}$
25. $y = c_1 + c_2 \cos x + c_3 \sin x - \ln|\cos x| - \sin x \ln|\sec x + \tan x|$
27. $y = c_1 e^x + c_2 e^{-x} + c_3 e^{2x} + \frac{1}{30} e^{4x}$

EXERCISES 4.7 (PAGE 168)

1. $y = c_1 x^{-1} + c_2 x^2$
3. $y = c_1 + c_2 \ln x$
5. $y = c_1 \cos(2 \ln x) + c_2 \sin(2 \ln x)$
7. $y = c_1 x^{(2-\sqrt{6})} + c_2 x^{(2+\sqrt{6})}$
9. $y = c_1 \cos\left(\frac{1}{5} \ln x\right) + c_2 \sin\left(\frac{1}{5} \ln x\right)$
11. $y = c_1 x^{-2} + c_2 x^{-2} \ln x$
13. $y = x^{-1/2}\left[c_1 \cos\left(\frac{1}{6}\sqrt{3} \ln x\right) + c_2 \sin\left(\frac{1}{6}\sqrt{3} \ln x\right)\right]$
15. $y = c_1 x^3 + c_2 \cos\left(\sqrt{2} \ln x\right) + c_3 \sin\left(\sqrt{2} \ln x\right)$
17. $y = c_1 + c_2 x + c_3 x^2 + c_4 x^{-3}$
19. $y = c_1 + c_2 x^5 + \frac{1}{5} x^5 \ln x$
21. $y = c_1 x + c_2 x \ln x + x(\ln x)^2$
23. $y = c_1 x^{-1} + c_2 x - \ln x$
25. $y = 2 - 2x^{-2}$
27. $y = \cos(\ln x) + 2 \sin(\ln x)$
29. $y = \frac{3}{4} - \ln x + \frac{1}{4} x^2$
31. $y = c_1 x^{-10} + c_2 x^2$
33. $y = c_1 x^{-1} + c_2 x^{-8} + \frac{1}{30} x^2$
35. $y = x^2[c_1 \cos(3 \ln x) + c_2 \sin(3 \ln x)] + \frac{4}{13} + \frac{3}{10} x$
37. $y = 2(-x)^{1/2} - 5(-x)^{1/2} \ln(-x)$, $x < 0$
39. $y = c_1(x + 3)^2 + c_2(x + 3)^7$
41. $y = c_1 \cos[\ln(x + 2)] + c_2 \sin[\ln(x + 2)]$

EXERCISES 4.8 (PAGE 179)

1. $y_p(x) = \frac{1}{4} \int_{x_0}^{x} \sinh 4(x - t) f(t)\, dt$
3. $y_p(x) = \int_{x_0}^{x} (x - t) e^{-(x-t)} f(t)\, dt$
5. $y_p(x) = \frac{1}{3} \int_{x_0}^{x} \sin 3(x - t) f(t)\, dt$
7. $y = c_1 e^{-4x} + c_2 e^{4x} + \frac{1}{4} \int_{x_0}^{x} \sinh 4(x - t)\, t e^{-2t}\, dt$
9. $y = c_1 e^{-x} + c_2 x e^{-x} + \int_{x_0}^{x} (x - t) e^{-(x-t)} e^{-t}\, dt$
11. $y = c_1 \cos 3x + c_2 \sin 3x + \frac{1}{3} \int_{x_0}^{x} \sin 3(x - t)(t + \sin t)\, dt$
13. $y_p(x) = \frac{1}{4} x e^{2x} - \frac{1}{16} e^{2x} + \frac{1}{16} e^{-2x}$
15. $y_p(x) = \frac{1}{2} x^2 e^{5x}$
17. $y_p(x) = -\cos x + \frac{\pi}{2} \sin x - x \sin x - \cos x \ln|\sin x|$
19. $y = \frac{25}{16} e^{-2x} - \frac{9}{16} e^{2x} + \frac{1}{4} x e^{2x}$
21. $y = -e^{5x} + 6x e^{5x} + \frac{1}{2} x^2 e^{5x}$
23. $y = -x \sin x - \cos x \ln|\sin x|$
25. $y = (\cos 1 - 2) e^{-x} + (1 + \sin 1 - \cos 1) e^{-2x} - e^{-2x} \sin e^x$
27. $y = 4x - 2x^2 - x \ln x$
29. $y = \frac{46}{45} x^3 - \frac{1}{20} x^{-2} + \frac{1}{36} - \frac{1}{6} \ln x$
31. $y(x) = 5e^x + 3e^{-x} + y_p(x)$,
    where $y_p(x) = \begin{cases} 1 - \cosh x, & x < 0 \\ -1 + \cosh x, & x \quad 0 \end{cases}$
33. $y = \cos x - \sin x + y_p(x)$,
    where $y_p(x) = \begin{cases} 0, & x < 0 \\ 10 - 10 \cos x, & 0 \le x \le 3\pi \\ -20 \cos x, & x > 3\pi \end{cases}$
35. $y_p(x) = (x - 1) \int_0^x t f(t)\, dt + x \int_x^1 (t - 1) f(t)\, dt$
37. $y_p(x) = \frac{1}{2} x^2 - \frac{1}{2} x$
39. $y_p(x) = \frac{\sin(x - 1)}{\sin 1} - \frac{\sin x}{\sin 1} + 1$
41. $y_p(x) = -e^x \cos x - e^x \sin x + e^x$
43. $y_p(x) = \frac{1}{2} (\ln x)^2 + \frac{1}{2} \ln x$

EXERCISES 4.9 (PAGE 184)

1. $x = c_1 e^t + c_2 t e^t$
   $y = (c_1 - c_2) e^t + c_2 t e^t$
3. $x = c_1 \cos t + c_2 \sin t + t + 1$
   $y = c_1 \sin t - c_2 \cos t + t - 1$

5. $x = \frac{1}{2}c_1 \sin t + \frac{1}{2}c_2 \cos t - 2c_3 \sin\sqrt{6}t - 2c_4 \cos\sqrt{6}t$
$y = c_1 \sin t + c_2 \cos t + c_3 \sin\sqrt{6}t + c_4 \cos\sqrt{6}t$

7. $x = c_1e^{2t} + c_2e^{-2t} + c_3 \sin 2t + c_4 \cos 2t + \frac{1}{5}e^t$
$y = c_1e^{2t} + c_2e^{-2t} - c_3 \sin 2t - c_4 \cos 2t - \frac{1}{5}e^t$

9. $x = c_1 - c_2 \cos t + c_3 \sin t + \frac{17}{15}e^{3t}$
$y = c_1 + c_2 \sin t + c_3 \cos t - \frac{4}{15}e^{3t}$

11. $x = c_1e^t + c_2e^{-t/2}\cos\frac{1}{2}\sqrt{3}t + c_3e^{-t/2}\sin\frac{1}{2}\sqrt{3}t$
$y = \left(-\frac{3}{2}c_2 - \frac{1}{2}\sqrt{3}c_3\right)e^{-t/2}\cos\frac{1}{2}\sqrt{3}t$
$+ \left(\frac{1}{2}\sqrt{3}c_2 - \frac{3}{2}c_3\right)e^{-t/2}\sin\frac{1}{2}\sqrt{3}t$

13. $x = c_1e^{4t} + \frac{4}{3}e^t$
$y = -\frac{3}{4}c_1e^{4t} + c_2 + 5e^t$

15. $x = c_1 + c_2t + c_3e^t + c_4e^{-t} - \frac{1}{2}t^2$
$y = (c_1 - c_2 + 2) + (c_2 + 1)t + c_4e^{-t} - \frac{1}{2}t^2$

17. $x = c_1e^t + c_2e^{-t/2}\sin\frac{1}{2}\sqrt{3}t + c_3e^{-t/2}\cos\frac{1}{2}\sqrt{3}t$
$y = c_1e^t + \left(-\frac{1}{2}c_2 - \frac{1}{2}\sqrt{3}c_3\right)e^{-t/2}\sin\frac{1}{2}\sqrt{3}t$
$+ \left(\frac{1}{2}\sqrt{3}c_2 - \frac{1}{2}c_3\right)e^{-t/2}\cos\frac{1}{2}\sqrt{3}t$
$z = c_1e^t + \left(-\frac{1}{2}c_2 + \frac{1}{2}\sqrt{3}c_3\right)e^{-t/2}\sin\frac{1}{2}\sqrt{3}t$
$+ \left(-\frac{1}{2}\sqrt{3}c_2 - \frac{1}{2}c_3\right)e^{-t/2}\cos\frac{1}{2}\sqrt{3}t$

19. $x = -6c_1e^{-t} - 3c_2e^{-2t} + 2c_3e^{3t}$
$y = c_1e^{-t} + c_2e^{-2t} + c_3e^{3t}$
$z = 5c_1e^{-t} + c_2e^{-2t} + c_3e^{3t}$

21. $x = e^{-3t+3} - te^{-3t+3}$
$y = -e^{-3t+3} + 2te^{-3t+3}$

23. $mx'' = 0$
$my'' = -mg;$
$x = c_1t + c_2$
$y = -\frac{1}{2}gt^2 + c_3t + c_4$

EXERCISES 4.10 (PAGE 189)

3. $y = \ln|\cos(c_1 - x)| + c_2$

5. $y = \dfrac{1}{c_1^2}\ln|c_1x + 1| - \dfrac{1}{c_1}x + c_2$

7. $\frac{1}{3}y^3 - c_1y = x + c_2$

9. $y = \frac{2}{3}(x + 1)^{3/2} + \frac{4}{3}$

11. $y = \tan\left(\frac{1}{4}\pi - \frac{1}{2}x\right), -\frac{1}{2}\pi < x < \frac{3}{2}\pi$

13. $y = -\dfrac{1}{c_1}\sqrt{1 - c_1^2x^2} + c_2$

15. $y = 1 + x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4 + \frac{1}{10}x^5 + \cdots$

17. $y = 1 + x - \frac{1}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4 + \frac{7}{60}x^5 + \cdots$

19. $y = -\sqrt{1 - x^2}$

CHAPTER 4 IN REVIEW (PAGE 190)

1. $y = 0$
3. false
5. $y = c_1\cos 5x + c_2\sin 5x$
7. $x^2y'' - 3xy' + 4y = 0$
9. $y_p = x^2 + x - 2$
11. $(-\infty, 0); (0, \infty)$
13. $y = c_1e^{3x} + c_2e^{-5x} + c_3xe^{-5x} + c_4e^x + c_5xe^x + c_6x^2e^x;$
$y = c_1x^3 + c_2x^{-5} + c_3x^{-5}\ln x + c_4x + c_5x\ln x$
$+ c_6x(\ln x)^2$

15. $y = c_1e^{(1+\sqrt{3})x} + c_2e^{(1-\sqrt{3})x}$

17. $y = c_1 + c_2e^{-5x} + c_3xe^{-5x}$

19. $y = c_1e^{-x/3} + e^{-3x/2}\left(c_2\cos\frac{1}{2}\sqrt{7}x + c_3\sin\frac{1}{2}\sqrt{7}x\right)$

21. $y = e^{3x/2}\left(c_2\cos\frac{1}{2}\sqrt{11}x + c_3\sin\frac{1}{2}\sqrt{11}x\right) + \frac{4}{5}x^3 + \frac{36}{25}x^2$
$+ \frac{46}{125}x - \frac{222}{625}$

23. $y = c_1 + c_2e^{2x} + c_3e^{3x} + \frac{1}{5}\sin x - \frac{1}{5}\cos x + \frac{4}{3}x$

25. $y = e^x(c_1\cos x + c_2\sin x)$
$- e^x\cos x\ln|\sec x + \tan x|$

27. $y = c_1x^{-1/3} + c_2x^{1/2}$
29. $y = c_1x^2 + c_2x^3 + x^4 - x^2\ln x$
31. (a) $y = c_1\cos\omega x + c_2\sin\omega x + A\cos\alpha x$
$+ B\sin\alpha x, \quad \omega \neq \alpha;$
$y = c_1\cos\omega x + c_2\sin\omega x + Ax\cos\omega x$
$+ Bx\sin\omega x, \quad \omega = \alpha$
(b) $y = c_1e^{-\omega x} + c_2e^{\omega x} + Ae^{\alpha x}, \omega \neq \alpha;$
$y = c_1e^{-\omega x} + c_2e^{\omega x} + Axe^{\omega x}, \omega = \alpha$
33. (a) $y = c_1\cosh x + c_2\sinh x + c_3x\cosh x$
$+ c_4x\sinh x$
(b) $y_p = Ax^2\cosh x + Bx^2\sinh x$
35. $y = e^{x-\pi}\cos x$
37. $y = \frac{13}{4}e^x - \frac{5}{4}e^{-x} - x - \frac{1}{2}\sin x$
39. $y = x^2 + 4$
43. $x = -c_1e^t - \frac{3}{2}c_2e^{2t} + \frac{5}{2}$
$y = c_1e^t + c_2e^{2t} - 3$
45. $x = c_1e^t + c_2e^{5t} + te^t$
$y = -c_1e^t + 3c_2e^{5t} - te^t + 2e^t$

EXERCISES 5.1 (PAGE 205)

1. $\dfrac{\sqrt{2}\,\pi}{8}$

3. $x(t) = -\frac{1}{4}\cos 4\sqrt{6}t$

5. (a) $x\left(\frac{\pi}{12}\right) = -\frac{1}{4}; x\left(\frac{\pi}{8}\right) = -\frac{1}{2}; x\left(\frac{\pi}{6}\right) = -\frac{1}{4};$
$x\left(\frac{\pi}{4}\right) = \frac{1}{2}; x\left(\frac{9\pi}{32}\right) = \frac{\sqrt{2}}{4}$
(b) 4 ft/s; downward
(c) $t = \dfrac{(2n + 1)\pi}{16}, n = 0, 1, 2, \ldots$

7. **(a)** the 20-kg mass
(b) the 20-kg mass; the 50-kg mass
(c) $t = n\pi$, $n = 0, 1, 2, \ldots$; at the equilibrium position; the 50-kg mass is moving upward whereas the 20-kg mass is moving upward when n is even and downward when n is odd.

9. **(a)** $x(t) = \frac{1}{2}\cos 2t + \frac{3}{4}\sin 2t$

(b) $x(t) = \frac{\sqrt{13}}{4}\sin(2t + 0.588)$

(c) $x(t) = \frac{\sqrt{13}}{4}\cos(2t - 0.983)$

11. **(a)** $x(t) = -\frac{2}{3}\cos 10t + \frac{1}{2}\sin 10t$
$= \frac{5}{6}\sin(10t - 0.927)$

(b) $\dfrac{5}{6}$ ft; $\dfrac{\pi}{5}$
(c) 15 cycles
(d) 0.721 s
(e) $\dfrac{(2n+1)\pi}{20} + 0.0927$, $n = 0, 1, 2, \ldots$
(f) $x(3) = -0.597$ ft
(g) $x'(3) = -5.814$ ft/s
(h) $x''(3) = 59.702$ ft/s^2
(i) $\pm 8\frac{1}{3}$ ft/s
(j) $0.1451 + \dfrac{n\pi}{5}$; $0.3545 + \dfrac{n\pi}{5}$, $n = 0, 1, 2, \ldots$
(k) $0.3545 + \dfrac{n\pi}{5}$, $n = 0, 1, 2, \ldots$

13. 120 lb/ft; $x(t) = \frac{\sqrt{3}}{12}\sin 8\sqrt{3}\,t$

17. **(a)** above
(b) heading upward

19. **(a)** below
(b) heading upward

21. $\frac{1}{2}$ s; $\frac{1}{2}$ s, $x\left(\frac{1}{2}\right) = e^{-2}$; that is, the weight is approximately 0.14 ft below the equilibrium position.

23. **(a)** $x(t) = \frac{4}{3}e^{-2t} - \frac{1}{3}e^{-8t}$
(b) $x(t) = -\frac{2}{3}e^{-2t} + \frac{5}{3}e^{-8t}$

25. **(a)** $x(t) = e^{-2t}\left(-\cos 4t - \frac{1}{2}\sin 4t\right)$

(b) $x(t) = \frac{\sqrt{5}}{2}e^{-2t}\sin\left(4t + 4.249\right)$

(c) $t = 1.294$ s

27. **(a)** $\beta > \frac{5}{2}$ **(b)** $\beta = \frac{5}{2}$ **(c)** $0 < \beta < \frac{5}{2}$

29. $x(t) = e^{-t/2}\left(-\frac{4}{3}\cos\frac{\sqrt{47}}{2}t - \frac{64}{3\sqrt{47}}\sin\frac{\sqrt{47}}{2}t\right)$
$+ \frac{10}{3}(\cos 3t + \sin 3t)$

31. $x(t) = \frac{1}{4}e^{-4t} + te^{-4t} - \frac{1}{4}\cos 4t$

33. $x(t) = -\frac{1}{2}\cos 4t + \frac{9}{4}\sin 4t + \frac{1}{2}e^{-2t}\cos 4t$
$- 2e^{-2t}\sin 4t$

35. **(a)** $m\dfrac{d^2x}{dt^2} = -k(x - h) - \beta\dfrac{dx}{dt}$ or

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega^2 x = \omega^2 h(t),$$

where $2\lambda = \beta/m$ and $\omega^2 = k/m$

(b) $x(t) = e^{-2t}\left(-\frac{56}{13}\cos 2t - \frac{72}{13}\sin 2t\right) + \frac{56}{13}\cos t$
$+ \frac{32}{13}\sin t$

37. $x(t) = -\cos 2t - \frac{1}{8}\sin 2t + \frac{3}{4}t\sin 2t + \frac{5}{4}t\cos 2t$

39. **(b)** $\dfrac{F_0}{2\omega}t\sin\omega t$

45. 4.568 C; 0.0509 s

47. $q(t) = 10 - 10e^{-3t}(\cos 3t + \sin 3t)$
$i(t) = 60e^{-3t}\sin 3t$; 10.432 C

49. $q_p = \frac{100}{13}\sin t + \frac{150}{13}\cos t$
$i_p = \frac{100}{13}\cos t - \frac{150}{13}\sin t$

53. $q(t) = -\frac{1}{2}e^{-10t}(\cos 10t + \sin 10t) + \frac{3}{2}$; $\frac{3}{2}$ C

57.
$$q(t) = \left(q_0 - \frac{E_0C}{1 - \gamma^2LC}\right)\cos\frac{t}{\sqrt{LC}} + \sqrt{LC}\,i_0\sin\frac{t}{\sqrt{LC}} + \frac{E_0C}{1 - \gamma^2LC}\cos\gamma t$$

$$i(t) = i_0\cos\frac{t}{\sqrt{LC}} - \frac{1}{\sqrt{LC}}\left(q_0 - \frac{E_0C}{1 - \gamma^2LC}\right)\sin\frac{t}{\sqrt{LC}} - \frac{E_0C\gamma}{1 - \gamma^2LC}\sin\gamma t$$

EXERCISES 5.2 (PAGE 215)

1. **(a)** $y(x) = \dfrac{w_0}{24EI}(6L^2x^2 - 4Lx^3 + x^4)$

3. **(a)** $y(x) = \dfrac{w_0}{48EI}(3L^2x^2 - 5Lx^3 + 2x^4)$

5. **(a)** $y(x) = \dfrac{w_0}{360EI}(7L^4x - 10L^2x^3 + 3x^5)$
(c) x 0.51933, y_{max} 0.234799

7.
$$y(x) = -\frac{w_0EI}{P^2}\cosh\sqrt{\frac{P}{EI}}x + \left(\frac{w_0EI}{P^2}\sinh\sqrt{\frac{P}{EI}}L - \frac{w_0L\sqrt{EI}}{P\sqrt{P}}\right)\frac{\sinh\sqrt{\frac{P}{EI}}x}{\cosh\sqrt{\frac{P}{EI}}L} + \frac{w_0}{2P}x^2 + \frac{w_0EI}{P^2}$$

9. $\lambda_n = n^2$, $n = 1, 2, 3, \ldots$; $y = \sin nx$

11. $\lambda_n = \dfrac{(2n-1)^2\pi^2}{4L^2}$, $n = 1, 2, 3, \ldots$;
$y = \cos\dfrac{(2n-1)\pi x}{2L}$

13. $\lambda_n = n^2, n = 0, 1, 2, \ldots; \quad y = \cos nx$

15. $\lambda_n = \dfrac{n^2\pi^2}{25}, n = 1, 2, 3, \ldots; \quad y = e^{-x} \sin\dfrac{n\pi x}{5}$

17. $\lambda_n = n^2, n = 1, 2, 3, \ldots; \quad y = \sin(n \ln x)$

19. $\lambda_n = n^4\pi^4, \quad n = 1, 2, 3, \ldots; \quad y = \sin n\pi x$

21. $x = L/4, x = L/2, x = 3\,L/4$

25. $\omega_n = \dfrac{n\pi\sqrt{T}}{L\sqrt{\ }}, n = 1, 2, 3, \ldots; \quad y = \sin\dfrac{n\pi x}{L}$

27. $u(r) = \left(\dfrac{u_0 - u_1}{b - a}\right)\dfrac{ab}{r} + \dfrac{u_1 b - u_0 a}{b - a}$

EXERCISE 5.3 (PAGE 224)

7. $\dfrac{d^2x}{dt^2} + x = 0$

15. **(a)** 5 ft **(b)** $4\sqrt{10}$ ft/s **(c)** $0 \le t \le \frac{3}{8}\sqrt{10}$; 7.5 ft

17. **(a)** $xy'' = r\sqrt{1 + (y')^2}$.
    When $t = 0, x = a, y = 0, dy/dx = 0$.
    (b) When $r \neq 1$,
    $$y(x) = \frac{a}{2}\left[\frac{1}{1+r}\left(\frac{x}{a}\right)^{1+r} - \frac{1}{1-r}\left(\frac{x}{a}\right)^{1-r}\right] + \frac{ar}{1-r^2}$$
    When $r = 1$,
    $$y(x) = \frac{1}{2}\left[\frac{1}{2a}(x^2 - a^2) + \frac{1}{a}\ln\frac{a}{x}\right]$$
    (c) The paths intersect when $r < 1$.

19. **(a)** $\theta(t) = \omega_0\sqrt{\dfrac{l}{g}}\ \sin\sqrt{\dfrac{g}{l}}\,t$
    (b) use at θ_{max}, $\sin\sqrt{g/l}\,t = 1$
    (c) use $\cos\theta_{max}$ $\quad 1 - \frac{1}{2}\theta^2_{max}$
    (d) v_b $\quad$ 21,797 cm/s

CHAPTER 5 IN REVIEW (PAGE 228)

1. 8 ft
3. $\frac{5}{4}$ m
5. False; there could be an impressed force driving the system.
7. overdamped
9. $y = 0$ since $\lambda = 8$ is not an eigenvalue
11. 14.4 lb
13. $x(t) = -\frac{2}{3}e^{-2t} + \frac{1}{3}e^{-4t}$
15. $0 < m \le 2$
17. $\gamma = \frac{8}{3}\sqrt{3}$
19. $x(t) = e^{-4t}\left(\frac{26}{17}\cos 2\sqrt{2}\,t + \frac{28}{17}\sqrt{2}\sin 2\sqrt{2}\,t\right) + \frac{8}{17}e^{-t}$

21. **(a)** $q(t) = -\frac{1}{150}\sin 100t + \frac{1}{75}\sin 50t$
    (b) $i(t) = -\frac{2}{3}\cos 100t + \frac{2}{3}\cos 50t$
    (c) $t = \dfrac{n\pi}{50}, n = 0, 1, 2, \ldots$

25. $m\dfrac{d^2x}{dt^2} + kx = 0$

27. $mx'' + f_k\,\mathrm{sgn}(x') + kx = 0$

EXERCISES 6.1 (PAGE 237)

1. $(-1, 1], R = 1$
3. $[-\frac{1}{2}, \frac{1}{2}), R = \frac{1}{2}$
5. $(-5, 15), R = 10$
7. $[0, \frac{2}{3}], R = \frac{1}{3}$
9. $(-\frac{75}{32}, \frac{75}{32}), R = \frac{75}{32}$
11. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{n!2^n}x^n$
13. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{n+1}}x^n$
15. $\displaystyle\sum_{n=1}^{\infty}\frac{-1}{n}x^n$
17. $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}(x - 2\pi)^{2n+1}$
19. $x - \frac{2}{3}x^3 + \frac{2}{15}x^5 - \frac{4}{315}x^7 + \cdots$
21. $1 + \frac{1}{2}x^2 + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \cdots; (-\pi/2, \pi/2)$
23. $\displaystyle\sum_{k=3}^{\infty}(k-2)c_{k-2}x^k$
25. $\displaystyle\sum_{k=0}^{\infty}[(k+1)c_{k+1} - c_k]x^k$
27. $\displaystyle 2c_1 + \sum_{k=1}^{\infty}[2(k+1)c_{k+1} + 6c_{k-1}]x^k$
29. $\displaystyle c_0 + 2c_2 + \sum_{k=1}^{\infty}[(k+2)(k+1)c_{k+2} - (2k-1)c_k]x^k$
35. $\displaystyle y = c_0\sum_{k=0}^{\infty}\frac{1}{k!}(5x)^k$
37. $\displaystyle y = c_0\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{x^2}{2}\right)^k$

EXERCISES 6.2 (PAGE 246)

1. 5; 4

3. $y_1(x) = c_0\left[1 - \dfrac{1}{2!}x^2 + \dfrac{1}{4!}x^4 - \dfrac{1}{6!}x^6 + \cdots\right]$
   $y_2(x) = c_1\left[x - \dfrac{1}{3!}x^3 + \dfrac{1}{5!}x^5 - \dfrac{1}{7!}x^7 + \cdots\right]$

5. $y_1(x) = c_0$
   $y_2(x) = c_1\left[x + \dfrac{1}{2!}x^2 + \dfrac{1}{3!}x^3 + \dfrac{1}{4!}x^4 + \cdots\right]$

7. $$y_1(x) = c_0\left[1 + \frac{1}{3\cdot 2}x^3 + \frac{1}{6\cdot 5\cdot 3\cdot 2}x^6 + \frac{1}{9\cdot 8\cdot 6\cdot 5\cdot 3\cdot 2}x^9 + \cdots\right]$$
   $$y_2(x) = c_1\left[x + \frac{1}{4\cdot 3}x^4 + \frac{1}{7\cdot 6\cdot 4\cdot 3}x^7 + \frac{1}{10\cdot 9\cdot 7\cdot 6\cdot 4\cdot 3}x^{10} + \cdots\right]$$

9. $y_1(x) = c_0\left[1 - \frac{1}{2!}x^2 - \frac{3}{4!}x^4 - \frac{21}{6!}x^6 - \cdots\right]$

$y_2(x) = c_1\left[x + \frac{1}{3!}x^3 + \frac{5}{5!}x^5 + \frac{45}{7!}x^7 + \cdots\right]$

11. $y_1(x) = c_0\left[1 - \frac{1}{3!}x^3 + \frac{4^2}{6!}x^6 - \frac{7^2 \cdot 4^2}{9!}x^9 + \cdots\right]$

$y_2(x) = c_1\left[x - \frac{2^2}{4!}x^4 + \frac{5^2 \cdot 2^2}{7!}x^7 - \frac{8^2 \cdot 5^2 \cdot 2^2}{10!}x^{10} + \cdots\right]$

13. $y_1(x) = c_0$; $y_2(x) = c_1\sum_{n=1}^{\infty}\frac{1}{n}x^n$

15. $y_1(x) = c_0\left[1 + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{8}x^4 + \cdots\right]$

$y_2(x) = c_1\left[x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{1}{4}x^4 + \cdots\right]$

17. $y_1(x) = c_0\left[1 + \frac{1}{4}x^2 - \frac{7}{4 \cdot 4!}x^4 + \frac{23 \cdot 7}{8 \cdot 6!}x^6 - \cdots\right]$

$y_2(x) = c_1\left[x - \frac{1}{6}x^3 + \frac{14}{2 \cdot 5!}x^5 - \frac{34 \cdot 14}{4 \cdot 7!}x^7 - \cdots\right]$

19. $y(x) = -2\left[1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots\right] + 6x$

$= 8x - 2e^x$

21. $y(x) = 3 - 12x^2 + 4x^4$

23. $y_1(x) = c_0\left[1 - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \cdots\right]$

$y_2(x) = c_1\left[x - \frac{1}{12}x^4 + \frac{1}{180}x^6 + \cdots\right]$

EXERCISES 6.3 (PAGE 255)

1. $x = 0$, irregular singular point

3. $x = -3$, regular singular point; $x = 3$, irregular singular point

5. $x = 0, 2i, -2i$, regular singular points

7. $x = -3, 2$, regular singular points

9. $x = 0$, irregular singular point; $x = -5, 5, 2$, regular singular points

11. for $x = 1$: $p(x) = 5$, $q(x) = \frac{x(x-1)^2}{x+1}$

for $x = -1$: $p(x) = \frac{5(x+1)}{x-1}$, $q(x) = x^2 + x$

13. $r_1 = \frac{1}{3}$, $r_2 = -1$

15. $r_1 = \frac{3}{2}$, $r_2 = 0$

$y(x) = C_1x^{3/2}\left[1 - \frac{2}{5}x + \frac{2^2}{7 \cdot 5 \cdot 2}x^2 - \frac{2^3}{9 \cdot 7 \cdot 5 \cdot 3!}x^3 + \cdots\right]$

$+ C_2\left[1 + 2x - 2x^2 + \frac{2^3}{3 \cdot 3!}x^3 - \cdots\right]$

17. $r_1 = \frac{7}{8}$, $r_2 = 0$

$y(x) = c_1x^{7/8}\left[1 - \frac{2}{15}x + \frac{2^2}{23 \cdot 15 \cdot 2}x^2 - \frac{2^3}{31 \cdot 23 \cdot 15 \cdot 3!}x^3 + \cdots\right]$

$+ c_2\left[1 - 2x + \frac{2^2}{9 \cdot 2}x^2 - \frac{2^3}{17 \cdot 9 \cdot 3!}x^3 + \cdots\right]$

19. $r_1 = \frac{1}{3}$, $r_2 = 0$

$y(x) = C_1x^{1/3}\left[1 + \frac{1}{3}x + \frac{1}{3^2 \cdot 2}x^2 + \frac{1}{3^3 \cdot 3!}x^3 + \cdots\right]$

$+ C_2\left[1 + \frac{1}{2}x + \frac{1}{5 \cdot 2}x^2 + \frac{1}{8 \cdot 5 \cdot 2}x^3 + \cdots\right]$

21. $r_1 = \frac{5}{2}$, $r_2 = 0$

$y(x) = C_1x^{5/2}\left[1 + \frac{2 \cdot 2}{7}x + \frac{2^2 \cdot 3}{9 \cdot 7}x^2 + \frac{2^3 \cdot 4}{11 \cdot 9 \cdot 7}x^3 + \cdots\right]$

$+ C_2\left[1 + \frac{1}{3}x - \frac{1}{6}x^2 - \frac{1}{6}x^3 - \cdots\right]$

23. $r_1 = \frac{2}{3}$, $r_2 = \frac{1}{3}$

$y(x) = C_1x^{2/3}\left[1 - \frac{1}{2}x + \frac{5}{28}x^2 - \frac{1}{21}x^3 + \cdots\right]$

$+ C_2x^{1/3}\left[1 - \frac{1}{2}x + \frac{1}{5}x^2 - \frac{7}{120}x^3 + \cdots\right]$

25. $r_1 = 0$, $r_2 = -1$

$y(x) = C_1\sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n} + C_2x^{-1}\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n}$

$= C_1x^{-1}\sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n+1} + C_2x^{-1}\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n}$

$= \frac{1}{x}[C_1\sinh x + C_2\cosh x]$

27. $r_1 = 1$, $r_2 = 0$

$y(x) = C_1x + C_2\left[x\ln x - 1 + \frac{1}{2}x^2 + \frac{1}{12}x^3 + \frac{1}{72}x^4 + \cdots\right]$

29. $r_1 = r_2 = 0$

$y(x) = C_1y(x) + C_2\left[y_1(x)\ln x + y_1(x)\left(-x + \frac{1}{4}x^2 - \frac{1}{3 \cdot 3!}x^3 + \frac{1}{4 \cdot 4!}x^4 - \cdots\right)\right]$

where $y_1(x) = \sum_{n=0}^{\infty}\frac{1}{n!}x^n = e^x$

33. **(b)** $y_1(t) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\left(\sqrt{\lambda}\, t\right)^{2n} = \frac{\sin\left(\sqrt{\lambda}\, t\right)}{\sqrt{\lambda}\, t}$

$y_2(t) = t^{-1} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\left(\sqrt{\lambda}\, t\right)^{2n} = \frac{\cos\left(\sqrt{\lambda}\, t\right)}{t}$

(c) $y = C_1 x \sin\left(\frac{\sqrt{\lambda}}{x}\right) + C_2 x \cos\left(\frac{\sqrt{\lambda}}{x}\right)$

EXERCISES 6.4 (PAGE 267)

1. $y = c_1 J_{1/3}(x) + c_2 J_{-1/3}(x)$
3. $y = c_1 J_{5/2}(x) + c_2 J_{-5/2}(x)$
5. $y = c_1 J_0(x) + c_2 Y_0(x)$
7. $y = c_1 J_2(3x) + c_2 Y_2(3x)$
9. $y = c_1 J_{2/3}(5x) + c_2 J_{-2/3}(5x)$
11. $y = c_1 x^{-1/2} J_{1/2}(\alpha x) + c_2 x^{-1/2} J_{-1/2}(\alpha x)$
13. $y = x^{-1/2}\,[c_1 J_1(4x^{1/2}) + c_2 Y_1(4x^{1/2})]$
15. $y = x\,[c_1 J_1(x) + c_2 Y_1(x)]$
17. $y = x^{1/2}\,[c_1 J_{3/2}(x) + c_2 Y_{3/2}(x)]$
19. $y = x^{-1}\left[c_1 J_{1/2}\left(\tfrac{1}{2}x^2\right) + c_2 J_{-1/2}\left(\tfrac{1}{2}x^2\right)\right]$
23. $y = x^{1/2}\,[c_1 J_{1/2}(x) + c_2 J_{-1/2}(x)]$
$= C_1 \sin x + C_2 \cos x$
25. $y = x^{-1/2}\left[c_1 J_{1/2}\left(\tfrac{1}{8}x^2\right) + c_2 J_{-1/2}\left(\tfrac{1}{8}x^2\right)\right]$
$= C_1 x^{-3/2} \sin\left(\tfrac{1}{8}x^2\right) + C_2 x^{-3/2} \cos\left(\tfrac{1}{8}x^2\right)$
35. $y = c_1 x^{1/2} J_{1/3}\left(\tfrac{2}{3}\alpha x^{3/2}\right) + c_2 x^{1/2} J_{-1/3}\left(\tfrac{2}{3}\alpha x^{3/2}\right)$
45. $P_2(x)$, $P_3(x)$, $P_4(x)$, and $P_5(x)$ are given in the text,
$P_6(x) = \frac{1}{16}(231x^6 - 315x^4 + 105x^2 - 5)$,
$P_7(x) = \frac{1}{16}(429x^7 - 693x^5 + 315x^3 - 35x)$
47. $\lambda_1 = 2$, $\lambda_2 = 12$, $\lambda_3 = 30$
53. $y = x - 4x^3 + \frac{16}{5}x^5$

CHAPTER 6 IN REVIEW (PAGE 271)

1. False
3. $[-\frac{1}{2}, \frac{1}{2}]$
7. $x^2(x-1)y'' + y' + y = 0$
9. $r_1 = \frac{1}{2}$, $r_2 = 0$
$y_1(x) = C_1 x^{1/2}\left[1 - \frac{1}{3}x + \frac{1}{30}x^2 - \frac{1}{630}x^3 + \cdots\right]$
$y_2(x) = C_2\left[1 - x + \frac{1}{6}x^2 - \frac{1}{90}x^3 + \cdots\right]$
11. $y_1(x) = c_0\left[1 + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{5}{8}x^4 + \cdots\right]$
$y_2(x) = c_1\left[x + \frac{1}{2}x^3 + \frac{1}{4}x^4 + \cdots\right]$
13. $r_1 = 3$, $r_2 = 0$
$y_1(x) = C_1 x^3\left[1 + \frac{1}{4}x + \frac{1}{20}x^2 + \frac{1}{120}x^3 + \cdots\right]$
$y_2(x) = C_2\left[1 + x + \frac{1}{2}x^2\right]$
15. $y(x) = 3\left[1 - x^2 + \frac{1}{3}x^4 - \frac{1}{15}x^6 + \cdots\right]$
$-2\left[x - \frac{1}{2}x^3 + \frac{1}{8}x^5 - \frac{1}{48}x^7 + \cdots\right]$
17. $\frac{1}{6}\pi$

19. $x = 0$ is an ordinary point

21. $y(x) = c_0\left[1 - \frac{1}{3}x^3 + \frac{1}{3^2 \cdot 2!}x^6 - \frac{1}{3^3 \cdot 3!}x^9 + \cdots\right]$
$+ c_1\left[x - \frac{1}{4}x^4 + \frac{1}{4 \cdot 7}x^7\right.$
$\left. - \frac{1}{4 \cdot 7 \cdot 10}x^{10} + \cdots\right] + \left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right.$
$\left. + \frac{1}{3^2 \cdot 2!}x^6 - \frac{1}{3^3 \cdot 3!}x^9 + \cdots\right]$

EXERCISES 7.1 (PAGE 280)

1. $\frac{2}{s}e^{-s} - \frac{1}{s}$
3. $\frac{1}{s^2} - \frac{1}{s^2}e^{-s}$
5. $\frac{1 + e^{-\pi s}}{s^2 + 1}$
7. $\frac{1}{s}e^{-s} + \frac{1}{s^2}e^{-s}$
9. $\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s^2}e^{-s}$
11. $\frac{e^7}{s - 1}$
13. $\frac{1}{(s-4)^2}$
15. $\frac{1}{s^2 + 2s + 2}$
17. $\frac{s^2 - 1}{(s^2 + 1)^2}$
19. $\frac{48}{s^5}$
21. $\frac{4}{s^2} - \frac{10}{s}$
23. $\frac{2}{s^3} + \frac{6}{s^2} - \frac{3}{s}$
25. $\frac{6}{s^4} + \frac{6}{s^3} + \frac{3}{s^2} + \frac{1}{s}$
27. $\frac{1}{s} + \frac{1}{s - 4}$
29. $\frac{1}{s} + \frac{2}{s - 2} + \frac{1}{s - 4}$
31. $\frac{8}{s^3} - \frac{15}{s^2 + 9}$
33. Use $\sinh kt = \dfrac{e^{kt} - e^{-kt}}{2}$ and linearity to show that

$$\mathscr{L}\{\sinh kt\} = \frac{k}{s^2 - k^2}.$$

35. $\frac{1}{2(s-2)} - \frac{1}{2s}$
37. $\frac{2}{s^2 + 16}$
39. $\frac{4\cos 5 + (\sin 5)s}{s^2 + 16}$
43. $\frac{\sqrt{\pi}}{s^{1/2}}$
45. $\frac{3\sqrt{\pi}}{4s^{5/2}}$

EXERCISES 7.2 (PAGE 288)

1. $\frac{1}{2}t^2$
3. $t - 2t^4$
5. $1 + 3t + \frac{3}{2}t^2 + \frac{1}{6}t^3$
7. $t - 1 + e^{2t}$
9. $\frac{1}{4}e^{-t/4}$
11. $\frac{5}{7}\sin 7t$
13. $\cos\frac{t}{2}$
15. $2\cos 3t - 2\sin 3t$
17. $\frac{1}{3} - \frac{1}{3}e^{-3t}$
19. $\frac{3}{4}e^{-3t} + \frac{1}{4}e^{t}$
21. $0.3e^{0.1t} + 0.6e^{-0.2t}$
23. $\frac{1}{2}e^{2t} - e^{3t} + \frac{1}{2}e^{6t}$
25. $\frac{1}{5} - \frac{1}{5}\cos\sqrt{5}\,t$
27. $-4 + 3e^{-t} + \cos t + 3\sin t$

29. $\frac{1}{3}\sin t - \frac{1}{6}\sin 2t$ **31.** $y = -1 + e^t$

33. $y = \frac{1}{10}e^{4t} + \frac{19}{10}e^{-6t}$ **35.** $y = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t}$

37. $y = 10\cos t + 2\sin t - \sqrt{2}\sin\sqrt{2}\,t$

39. $y = -\frac{8}{9}e^{-t/2} + \frac{1}{9}e^{-2t} + \frac{5}{18}e^{t} + \frac{1}{2}e^{-t}$

41. $y = \frac{1}{4}e^{-t} - \frac{1}{4}e^{-3t}\cos 2t + \frac{1}{4}e^{-3t}\sin 2t$

EXERCISES 7.3 (PAGE 297)

1. $\dfrac{1}{(s-10)^2}$ **3.** $\dfrac{6}{(s+2)^4}$

5. $\dfrac{1}{(s-2)^2} + \dfrac{2}{(s-3)^2} + \dfrac{1}{(s-4)^2}$ **7.** $\dfrac{3}{(s-1)^2+9}$

9. $\dfrac{s}{s^2+25} - \dfrac{s-1}{(s-1)^2+25} + 3\dfrac{s+4}{(s+4)^2+25}$

11. $\frac{1}{2}t^2e^{-2t}$ **13.** $e^{3t}\sin t$

15. $e^{-2t}\cos t - 2e^{-2t}\sin t$ **17.** $e^{-t} - te^{-t}$

19. $5 - t - 5e^{-t} - 4te^{-t} - \frac{3}{2}t^2e^{-t}$

21. $y = te^{-4t} + 2e^{-4t}$ **23.** $y = e^{-t} + 2te^{-t}$

25. $y = \frac{1}{9}t + \frac{2}{27} - \frac{2}{27}e^{3t} + \frac{10}{9}te^{3t}$ **27.** $y = -\frac{3}{2}e^{3t}\sin 2t$

29. $y = \frac{1}{2} - \frac{1}{2}e^{t}\cos t + \frac{1}{2}e^{t}\sin t$

31. $y = (e+1)te^{-t} + (e-1)e^{-t}$

33. $x(t) = -\frac{3}{2}e^{-7t/2}\cos\frac{\sqrt{15}}{2}t - \frac{7\sqrt{15}}{10}e^{-7t/2}\sin\frac{\sqrt{15}}{2}t$

37. $\dfrac{e^{-s}}{s^2}$ **39.** $\dfrac{e^{-2s}}{s^2} + 2\dfrac{e^{-2s}}{s}$

41. $\dfrac{s}{s^2+4}e^{-\pi s}$ **43.** $\frac{1}{2}(t-2)^2\,\mathscr{U}(t-2)$

45. $-\sin t\,\mathscr{U}(t-\pi)$ **47.** $\mathscr{U}(t-1) - e^{-(t-1)}\,\mathscr{U}(t-1)$

49. (c) **51.** (f)

53. (a)

55. $f(t) = 2 - 4\mathscr{U}(t-3)$; $\mathscr{L}\{f(t)\} = \dfrac{2}{s} - \dfrac{4}{s}e^{-3s}$

57. $f(t) = t^2\,\mathscr{U}(t-1)$; $\mathscr{L}\{f(t)\} = 2\dfrac{e^{-s}}{s^3} + 2\dfrac{e^{-s}}{s^2} + \dfrac{e^{-s}}{s}$

59. $f(t) = t - t\,\mathscr{U}(t-2)$; $\mathscr{L}\{f(t)\} = \dfrac{1}{s^2} - \dfrac{e^{-2s}}{s^2} - 2\dfrac{e^{-2s}}{s}$

61. $f(t) = \mathscr{U}(t-a) - \mathscr{U}(t-b)$; $\mathscr{L}\{f(t)\} = \dfrac{e^{-as}}{s} - \dfrac{e^{-bs}}{s}$

63. $y = [5 - 5e^{-(t-1)}]\,\mathscr{U}(t-1)$

65. $y = -\frac{1}{4} + \frac{1}{2}t + \frac{1}{4}e^{-2t} - \frac{1}{4}\mathscr{U}(t-1)$
$\quad - \frac{1}{2}(t-1)\,\mathscr{U}(t-1) + \frac{1}{4}e^{-2(t-1)}\,\mathscr{U}(t-1)$

67. $y = \cos 2t - \frac{1}{6}\sin 2(t-2\pi)\,\mathscr{U}(t-2\pi)$
$\quad + \frac{1}{3}\sin(t-2\pi)\,\mathscr{U}(t-2\pi)$

69. $y = \sin t + [1 - \cos(t-\pi)]\mathscr{U}(t-\pi)$
$\quad - [1 - \cos(t-2\pi)]\,\mathscr{U}(t-2\pi)$

71. $x(t) = \frac{5}{4}t - \frac{5}{16}\sin 4t - \frac{5}{4}(t-5)\,\mathscr{U}(t-5)$
$\quad + \frac{5}{16}\sin 4(t-5)\,\mathscr{U}(t-5) - \frac{25}{4}\,\mathscr{U}(t-5)$
$\quad + \frac{25}{4}\cos 4(t-5)\,\mathscr{U}(t-5)$

73. $q(t) = \frac{2}{5}\,\mathscr{U}(t-3) - \frac{2}{5}e^{-5(t-3)}\,\mathscr{U}(t-3)$

75. **(a)** $i(t) = \dfrac{1}{101}e^{-10t} - \dfrac{1}{101}\cos t + \dfrac{10}{101}\sin t$

$$- \frac{10}{101}e^{-10(t-3\pi/2)}\,\mathscr{U}\left(t - \frac{3\pi}{2}\right)$$
$$+ \frac{10}{101}\cos\left(t - \frac{3\pi}{2}\right)\mathscr{U}\left(t - \frac{3\pi}{2}\right)$$
$$+ \frac{1}{101}\sin\left(t - \frac{3\pi}{2}\right)\mathscr{U}\left(t - \frac{3\pi}{2}\right)$$

(b) $i_{\max}$ 0.1 at t 1.7, $i_{\min}$ −0.1 at t 4.7

77. $y(x) = \dfrac{w_0L^2}{16EI}x^2 - \dfrac{w_0L}{12EI}x^3 + \dfrac{w_0}{24EI}x^4$

$$- \frac{w_0}{24EI}\left(x - \frac{L}{2}\right)^4\mathscr{U}\left(x - \frac{L}{2}\right)$$

79. $y(x) = \dfrac{w_0L^2}{48EI}x^2 - \dfrac{w_0L}{24EI}x^3$

$$+ \frac{w_0}{60EIL}\left[\frac{5L}{2}x^4 - x^5 + \left(x - \frac{L}{2}\right)^5\mathscr{U}\left(x - \frac{L}{2}\right)\right]$$

81. **(a)** $\dfrac{dT}{dt} = k[T - 70 - 57.5t - (230 - 57.5t)\mathscr{U}(t-4)]$

EXERCISES 7.4 (PAGE 309)

1. $\dfrac{1}{(s+10)^2}$ **3.** $\dfrac{s^2-4}{(s^2+4)^2}$

5. $\dfrac{6s^2+2}{(s^2-1)^3}$ **7.** $\dfrac{12s-24}{[(s-2)^2+36]^2}$

9. $y = -\frac{1}{2}e^{-t} + \frac{1}{2}\cos t - \frac{1}{2}t\cos t + \frac{1}{2}t\sin t$

11. $y = 2\cos 3t + \frac{5}{3}\sin 3t + \frac{1}{6}t\sin 3t$

13. $y = \frac{1}{4}\sin 4t + \frac{1}{8}t\sin 4t$
$\quad - \frac{1}{8}(t-\pi)\sin 4(t-\pi)\mathscr{U}(t-\pi)$

17. $y = \frac{2}{3}t^3 + c_1t^2$ **19.** $\dfrac{6}{s^5}$

21. $\dfrac{s-1}{(s+1)[(s-1)^2+1]}$ **23.** $\dfrac{1}{s(s-1)}$

25. $\dfrac{s+1}{s[(s+1)^2+1]}$ **27.** $\dfrac{1}{s^2(s-1)}$

29. $\dfrac{3s^2+1}{s^2(s^2+1)^2}$ **31.** $e^t - 1$

33. $e^t - \frac{1}{2}t^2 - t - 1$

37. $f(t) = \sin t$

39. $f(t) = -\frac{1}{8}e^{-t} + \frac{1}{8}e^t + \frac{3}{4}te^t + \frac{1}{2}t^2e^t$

41. $f(t) = e^{-t}$

43. $f(t) = \frac{3}{8}e^{2t} + \frac{1}{8}e^{-2t} + \frac{1}{2}\cos 2t + \frac{1}{4}\sin 2t$

45. $y(t) = \sin t - \frac{1}{2}t\sin t$

47. $i(t) = 100[e^{-10(t-1)} - e^{-20(t-1)}]\mathcal{U}(t-1) - 100[e^{-10(t-2)} - e^{-20(t-2)}]\mathcal{U}(t-2)$

49. $\dfrac{1-e^{-as}}{s(1+e^{-as})}$

51. $\dfrac{a}{s}\left(\dfrac{1}{bs} - \dfrac{1}{e^{bs}-1}\right)$

53. $\dfrac{\coth(\pi s/2)}{s^2+1}$

55. $i(t) = \dfrac{1}{R}\left(1 - e^{-Rt/L}\right) + \dfrac{2}{R}\displaystyle\sum_{n=1}^{\infty}(-1)^n\left(1 - e^{-R(t-n)/L}\right)\mathcal{U}(t-n)$

57. $x(t) = 2(1 - e^{-t}\cos 3t - \frac{1}{3}e^{-t}\sin 3t) + 4\displaystyle\sum_{n=1}^{\infty}(-1)^n\left[1 - e^{-(t-n\pi)}\cos 3(t-n\pi) - \frac{1}{3}e^{-(t-n\pi)}\sin 3(t-n\pi)\right]\mathcal{U}(t-n\pi)$

EXERCISES 7.5 (PAGE 315)

1. $y = e^{3(t-2)}\,\mathcal{U}(t-2)$

3. $y = \sin t + \sin t\,\mathcal{U}(t-2\pi)$

5. $y = -\cos t\,\mathcal{U}\left(t - \frac{\pi}{2}\right) + \cos t\,\mathcal{U}\left(t - \frac{3\pi}{2}\right)$

7. $y = \frac{1}{2} - \frac{1}{2}e^{-2t} + \left[\frac{1}{2} - \frac{1}{2}e^{-2(t-1)}\right]\mathcal{U}(t-1)$

9. $y = e^{-2(t-2\pi)}\sin t\,\mathcal{U}(t-2\pi)$

11. $y = e^{-2t}\cos 3t + \frac{2}{3}e^{-2t}\sin 3t + \frac{1}{3}e^{-2(t-\pi)}\sin 3(t-\pi)\,\mathcal{U}(t-\pi) + \frac{1}{3}e^{-2(t-3\pi)}\sin 3(t-3\pi)\,\mathcal{U}(t-3\pi)$

13. $y(x) = \begin{cases} \dfrac{P_0}{EI}\left(\dfrac{L}{4}x^2 - \dfrac{1}{6}x^3\right), & 0 \le x < \dfrac{L}{2} \\ \dfrac{P_0L^2}{4EI}\left(\dfrac{1}{2}x - \dfrac{L}{12}\right), & \dfrac{L}{2} \le x \le L \end{cases}$

EXERCISES 7.6 (PAGE 319)

1. $x = -\frac{1}{3}e^{-2t} + \frac{1}{3}e^t$
$y = \frac{1}{3}e^{-2t} + \frac{2}{3}e^t$

3. $x = -\cos 3t - \frac{5}{3}\sin 3t$
$y = 2\cos 3t - \frac{7}{3}\sin 3t$

5. $x = -2e^{3t} + \frac{5}{2}e^{2t} - \frac{1}{2}$
$y = \frac{8}{3}e^{3t} - \frac{5}{2}e^{2t} - \frac{1}{6}$

7. $x = -\frac{1}{2}t - \frac{3}{4}\sqrt{2}\sin\sqrt{2}t$
$y = -\frac{1}{2}t + \frac{3}{4}\sqrt{2}\sin\sqrt{2}t$

9. $x = 8 + \dfrac{2}{3!}t^3 + \dfrac{1}{4!}t^4$
$y = -\dfrac{2}{3!}t^3 + \dfrac{1}{4!}t^4$

11. $x = \frac{1}{2}t^2 + t + 1 - e^{-t}$
$y = -\frac{1}{3} + \frac{1}{3}e^{-t} + \frac{1}{3}te^{-t}$

13. $x_1 = \dfrac{1}{5}\sin t + \dfrac{2\sqrt{6}}{15}\sin\sqrt{6}t + \dfrac{2}{5}\cos t - \dfrac{2}{5}\cos\sqrt{6}t$
$x_2 = \dfrac{2}{5}\sin t - \dfrac{\sqrt{6}}{15}\sin\sqrt{6}t + \dfrac{4}{5}\cos t + \dfrac{1}{5}\cos\sqrt{6}t$

15. **(b)** $i_2 = \frac{100}{9} - \frac{100}{9}e^{-900t}$
$i_3 = \frac{80}{9} - \frac{80}{9}e^{-900t}$
(c) $i_1 = 20 - 20e^{-900t}$

17. $i_2 = -\frac{20}{13}e^{-2t} + \frac{375}{1469}e^{-15t} + \frac{145}{113}\cos t + \frac{85}{113}\sin t$
$i_3 = \frac{30}{13}e^{-2t} + \frac{250}{1469}e^{-15t} - \frac{280}{113}\cos t + \frac{810}{113}\sin t$

19. $i_1 = \dfrac{6}{5} - \dfrac{6}{5}e^{-100t}\cosh 50\sqrt{2}t - \dfrac{9\sqrt{2}}{10}e^{-100t}\sinh 50\sqrt{2}t$
$i_2 = \dfrac{6}{5} - \dfrac{6}{5}e^{-100t}\cosh 50\sqrt{2}t - \dfrac{6\sqrt{2}}{5}e^{-100t}\sinh 50\sqrt{2}t$

CHAPTER 7 IN REVIEW (PAGE 320)

1. $\dfrac{1}{s^2} - \dfrac{2}{s^2}e^{-s}$

3. false

5. true

7. $\dfrac{1}{s+7}$

9. $\dfrac{2}{s^2+4}$

11. $\dfrac{4s}{(s^2+4)^2}$

13. $\frac{1}{6}t^5$

15. $\frac{1}{2}t^2e^{5t}$

17. $e^{5t}\cos 2t + \frac{5}{2}e^{5t}\sin 2t$

19. $\cos\pi(t-1)\mathcal{U}(t-1) + \sin\pi(t-1)\mathcal{U}(t-1)$

21. -5

23. $e^{-k(s-a)}F(s-a)$

25. $f(t)\mathcal{U}(t-t_0)$

27. $f(t-t_0)\mathcal{U}(t-t_0)$

29. $f(t) = t - (t-1)\mathcal{U}(t-1) - \mathcal{U}(t-4)$;
$\mathscr{L}\{f(t)\} = \dfrac{1}{s^2} - \dfrac{1}{s^2}e^{-s} - \dfrac{1}{s}e^{-4s}$;
$\mathscr{L}\{e^tf(t)\} = \dfrac{1}{(s-1)^2} - \dfrac{1}{(s-1)^2}e^{-(s-1)} - \dfrac{1}{s-1}e^{-4(s-1)}$

31. $f(t) = 2 + (t-2)\,\mathcal{U}(t-2)$;
$\mathscr{L}\{f(t)\} = \dfrac{2}{s} + \dfrac{1}{s^2}e^{-2s}$;
$\mathscr{L}\{e^tf(t)\} = \dfrac{2}{s-1} + \dfrac{1}{(s-1)^2}e^{-2(s-1)}$

33. $y = 5te^t + \frac{1}{2}t^2e^t$

35. $y = -\frac{6}{25} + \frac{1}{5}t + \frac{3}{2}e^{-t} - \frac{13}{50}e^{-5t} - \frac{4}{25}\,\mathcal{U}(t-2) - \frac{1}{5}(t-2)\,\mathcal{U}(t-2) + \frac{1}{4}e^{-(t-2)}\,\mathcal{U}(t-2) - \frac{9}{100}e^{-5(t-2)}\,\mathcal{U}(t-2)$

37. $y(t) = e^{-2t} + \left[-\frac{1}{4} + \frac{1}{2}(t-1) + \frac{1}{4}e^{-2(t-1)}\right]\mathscr{U}(t-1)$
$- 2\left[-\frac{1}{4} + \frac{1}{2}(t-2) + \frac{1}{4}e^{-2(t-2)}\right]\mathscr{U}(t-2)$
$+ \left[-\frac{1}{4} + \frac{1}{2}(t-3) + \frac{1}{4}e^{-2(t-3)}\right]\mathscr{U}(t-3)$

39. $y = 1 + t + \frac{1}{2}t^2$

41. $x = -\frac{1}{4} + \frac{9}{8}e^{-2t} + \frac{1}{8}e^{2t}$
$y = t + \frac{9}{4}e^{-2t} - \frac{1}{4}e^{2t}$

43. $i(t) = -9 + 2t + 9e^{-t/5}$

45. $$y(x) = \frac{w_0}{12EIL}\left[-\frac{1}{5}x^5 + \frac{L}{2}x^4 - \frac{L^2}{2}x^3 + \frac{L^3}{4}x^2 + \frac{1}{5}\left(x - \frac{L}{2}\right)^5\mathscr{U}\left(x - \frac{L}{2}\right)\right]$$

47. (a) $$\theta_1(t) = \frac{\theta_0 + \psi_0}{2}\cos\omega t + \frac{\theta_0 - \psi_0}{2}\cos\sqrt{\omega^2 + 2K}\,t$$
$$\theta_2(t) = \frac{\theta_0 + \psi_0}{2}\cos\omega t - \frac{\theta_0 - \psi_0}{2}\cos\sqrt{\omega^2 + 2K}\,t$$

49. (a) $x(t) = (v_0\cos\theta)\,t, \quad y(t) = -\frac{1}{2}gt^2 + (v_0\sin\theta)t$

(b) $y(x) = -\dfrac{g}{2v_0^2\cos^2\theta}x^2 + \dfrac{\sin\theta}{\cos\theta}x$; solve $y(x) = 0$ and use the double-angle formula for $\sin 2\theta$

(d) approximately 2729 ft; approximately 11.54 s

EXERCISES 8.1 (PAGE 332)

1. $\mathbf{X}' = \begin{pmatrix} 3 & -5 \\ 4 & 8 \end{pmatrix}\mathbf{X}$, where $\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}$

3. $\mathbf{X}' = \begin{pmatrix} -3 & 4 & -9 \\ 6 & -1 & 0 \\ 10 & 4 & 3 \end{pmatrix}\mathbf{X}$, where $\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

5. $\mathbf{X}' = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix}\mathbf{X} + \begin{pmatrix} 0 \\ -3t^2 \\ t^2 \end{pmatrix} + \begin{pmatrix} t \\ 0 \\ -t \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$,
where $\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

7. $\dfrac{dx}{dt} = 4x + 2y + e^t$
$\dfrac{dy}{dt} = -x + 3y - e^t$

9. $\dfrac{dx}{dt} = x - y + 2z + e^{-t} - 3t$
$\dfrac{dy}{dt} = 3x - 4y + z + 2e^{-t} + t$
$\dfrac{dz}{dt} = -2x + 5y + 6z + 2e^{-t} - t$

17. Yes; $W(\mathbf{X}_1, \mathbf{X}_2) = -2e^{-8t} \neq 0$ implies that $\mathbf{X}_1$ and $\mathbf{X}_2$ are linearly independent on $(-\infty, \infty)$.

19. No; $W(\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3) = 0$ for every t. The solution vectors are linearly dependent on $(-\infty, \infty)$. Note that $\mathbf{X}_3 = 2\mathbf{X}_1 + \mathbf{X}_2$.

EXERCISES 8.2 (PAGE 346)

1. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 2 \end{pmatrix}e^{5t} + c_2\begin{pmatrix} -1 \\ 1 \end{pmatrix}e^{-t}$

3. $\mathbf{X} = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix}e^{-3t} + c_2\begin{pmatrix} 2 \\ 5 \end{pmatrix}e^{t}$

5. $\mathbf{X} = c_1\begin{pmatrix} 5 \\ 2 \end{pmatrix}e^{8t} + c_2\begin{pmatrix} 1 \\ 4 \end{pmatrix}e^{-10t}$

7. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}e^{t} + c_2\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}e^{2t} + c_3\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}e^{-t}$

9. $\mathbf{X} = c_1\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}e^{-t} + c_2\begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}e^{3t} + c_3\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}e^{-2t}$

11. $\mathbf{X} = c_1\begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}e^{-t} + c_2\begin{pmatrix} -12 \\ 6 \\ 5 \end{pmatrix}e^{-t/2} + c_3\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}e^{-3t/2}$

13. $\mathbf{X} = 3\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{t/2} + 2\begin{pmatrix} 0 \\ 1 \end{pmatrix}e^{-t/2}$

19. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 3 \end{pmatrix} + c_2\left[\begin{pmatrix} 1 \\ 3 \end{pmatrix}t + \begin{pmatrix} \frac{1}{4} \\ -\frac{1}{4} \end{pmatrix}\right]$

21. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{2t} + c_2\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}te^{2t} + \begin{pmatrix} -\frac{1}{3} \\ 0 \end{pmatrix}e^{2t}\right]$

23. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}e^{t} + c_2\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}e^{2t} + c_3\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}e^{2t}$

25. $\mathbf{X} = c_1\begin{pmatrix} -4 \\ -5 \\ 2 \end{pmatrix} + c_2\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}e^{5t} + c_3\left[\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}te^{5t} + \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ -1 \end{pmatrix}e^{5t}\right]$

27. $\mathbf{X} = c_1\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}e^{t} + c_2\left[\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}te^{t} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}e^{t}\right] + c_3\left[\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\frac{t^2}{2}e^{t} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}te^{t} + \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \end{pmatrix}e^{t}\right]$

29. $\mathbf{X} = -7\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t} + 13\begin{pmatrix} 2t+1 \\ t+1 \end{pmatrix} e^{4t}$

31. Corresponding to the eigenvalue $\lambda_1 = 2$ of multiplicity five, the eigenvectors ar

$$\mathbf{K}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{K}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{K}_3 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

33. $\mathbf{X} = c_1\begin{pmatrix} \cos t \\ 2\cos t + \sin t \end{pmatrix} e^{4t} + c_2\begin{pmatrix} \sin t \\ 2\sin t - \cos t \end{pmatrix} e^{4t}$

35. $\mathbf{X} = c_1\begin{pmatrix} \cos t \\ -\cos t - \sin t \end{pmatrix} e^{4t} + c_2\begin{pmatrix} \sin t \\ -\sin t + \cos t \end{pmatrix} e^{4t}$

37. $\mathbf{X} = c_1\begin{pmatrix} 5\cos 3t \\ 4\cos 3t + 3\sin 3t \end{pmatrix} + c_2\begin{pmatrix} 5\sin 3t \\ 4\sin 3t - 3\cos 3t \end{pmatrix}$

39. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + c_2\begin{pmatrix} -\cos t \\ \cos t \\ \sin t \end{pmatrix} + c_3\begin{pmatrix} \sin t \\ -\sin t \\ \cos t \end{pmatrix}$

41. $\mathbf{X} = c_1\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} e^{t} + c_2\begin{pmatrix} \sin t \\ \cos t \\ \cos t \end{pmatrix} e^{t} + c_3\begin{pmatrix} \cos t \\ -\sin t \\ -\sin t \end{pmatrix} e^{t}$

43. $\mathbf{X} = c_1\begin{pmatrix} 28 \\ -5 \\ 25 \end{pmatrix} e^{2t} + c_2\begin{pmatrix} 4\cos 3t - 3\sin 3t \\ -5\cos 3t \\ 0 \end{pmatrix} e^{-2t}$

$+ c_3\begin{pmatrix} 3\cos 3t + 4\sin 3t \\ -5\sin 3t \\ 0 \end{pmatrix} e^{-2t}$

45. $\mathbf{X} = -\begin{pmatrix} 25 \\ -7 \\ 6 \end{pmatrix} e^{t} - \begin{pmatrix} \cos 5t - 5\sin 5t \\ \cos 5t \\ \cos 5t \end{pmatrix}$

$+ 6\begin{pmatrix} 5\cos 5t + \sin 5t \\ \sin 5t \\ \sin 5t \end{pmatrix}$

EXERCISES 8.3 (PAGE 354)

1. $\mathbf{X} = c_1\begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{-t} + c_2\begin{pmatrix} -3 \\ 1 \end{pmatrix} e^{t} + \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

3. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t} + c_2\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t} + \begin{pmatrix} -\frac{1}{4} \\ \frac{3}{4} \end{pmatrix} t^2$

$+ \begin{pmatrix} \frac{1}{4} \\ -\frac{1}{4} \end{pmatrix} t + \begin{pmatrix} -2 \\ \frac{3}{4} \end{pmatrix}$

5. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ -3 \end{pmatrix} e^{3t} + c_2\begin{pmatrix} 1 \\ 9 \end{pmatrix} e^{7t} + \begin{pmatrix} \frac{55}{36} \\ -\frac{19}{4} \end{pmatrix} e^{t}$

7. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{t} + c_2\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} e^{2t} + c_3\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} e^{5t} - \begin{pmatrix} \frac{3}{2} \\ \frac{7}{2} \\ 2 \end{pmatrix} e^{4t}$

9. $\mathbf{X} = 13\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t} + 2\begin{pmatrix} -4 \\ 6 \end{pmatrix} e^{2t} + \begin{pmatrix} -9 \\ 6 \end{pmatrix}$

11. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2\begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{t} - \begin{pmatrix} 11 \\ 11 \end{pmatrix} t - \begin{pmatrix} 15 \\ 10 \end{pmatrix}$

13. $\mathbf{X} = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{t/2} + c_2\begin{pmatrix} 10 \\ 3 \end{pmatrix} e^{3t/2} - \begin{pmatrix} \frac{13}{2} \\ \frac{13}{4} \end{pmatrix} te^{t/2} - \begin{pmatrix} \frac{15}{2} \\ \frac{9}{4} \end{pmatrix} e^{t/2}$

15. $\mathbf{X} = c_1\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{t} + c_2\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + \begin{pmatrix} 3 \\ 3 \end{pmatrix} e^{t} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} te^{t}$

17. $\mathbf{X} = c_1\begin{pmatrix} 4 \\ 1 \end{pmatrix} e^{3t} + c_2\begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-3t} + \begin{pmatrix} -12 \\ 0 \end{pmatrix} t - \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \end{pmatrix}$

19. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t} + c_2\begin{pmatrix} t \\ \frac{1}{2} - t \end{pmatrix} e^{t} + \begin{pmatrix} \frac{1}{2} \\ -2 \end{pmatrix} e^{-t}$

21. $\mathbf{X} = c_1\begin{pmatrix} \cos t \\ \sin t \end{pmatrix} + c_2\begin{pmatrix} \sin t \\ -\cos t \end{pmatrix} + \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} t$

$+ \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} \ln|\cos t|$

23. $\mathbf{X} = c_1\begin{pmatrix} \cos t \\ \sin t \end{pmatrix} e^{t} + c_2\begin{pmatrix} \sin t \\ -\cos t \end{pmatrix} e^{t} + \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} te^{t}$

25. $\mathbf{X} = c_1\begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} + c_2\begin{pmatrix} \sin t \\ \cos t \end{pmatrix} + \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} t$

$+ \begin{pmatrix} -\sin t \\ \sin t \tan t \end{pmatrix} - \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} \ln|\cos t|$

27. $\mathbf{X} = c_1\begin{pmatrix} 2\sin t \\ \cos t \end{pmatrix} e^{t} + c_2\begin{pmatrix} 2\cos t \\ -\sin t \end{pmatrix} e^{t} + \begin{pmatrix} 3\sin t \\ \frac{3}{2}\cos t \end{pmatrix} te^{t}$

$+ \begin{pmatrix} \cos t \\ -\frac{1}{2}\sin t \end{pmatrix} e^{t} \ln|\sin t| + \begin{pmatrix} 2\cos t \\ -\sin t \end{pmatrix} e^{t} \ln|\cos t|$

29. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} e^{2t} + c_3\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} e^{3t}$

$+ \begin{pmatrix} -\frac{1}{2}e^{2t} + \frac{1}{2}te^{2t} \\ -e^{t} + \frac{1}{4}e^{2t} + \frac{1}{2}te^{2t} \\ \frac{1}{2}t^2 e^{3t} \end{pmatrix}$

31. $\mathbf{X} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} te^{2t} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{2t} + \begin{pmatrix} -2 \\ 2 \end{pmatrix} te^{4t} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} e^{4t}$

33. $\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-2t} + \frac{6}{29}\begin{pmatrix} 3 \\ -1 \end{pmatrix} e^{-12t} - \frac{4}{29}\begin{pmatrix} 19 \\ 42 \end{pmatrix} \cos t$

$+ \frac{4}{29}\begin{pmatrix} 83 \\ 69 \end{pmatrix} \sin t$

EXERCISES 8.4 (PAGE 359)

1. $e^{\mathbf{A}t} = \begin{pmatrix} e^t & 0 \\ 0 & e^{2t} \end{pmatrix}; \quad e^{-\mathbf{A}t} = \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{pmatrix}$

3. $e^{\mathbf{A}t} = \begin{pmatrix} t+1 & t & t \\ t & t+1 & t \\ -2t & -2t & -2t+1 \end{pmatrix}$

5. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 0 \end{pmatrix}e^t + c_2\begin{pmatrix} 0 \\ 1 \end{pmatrix}e^{2t}$

7. $\mathbf{X} = c_1\begin{pmatrix} t+1 \\ t \\ -2t \end{pmatrix} + c_2\begin{pmatrix} t \\ t+1 \\ -2t \end{pmatrix} + c_3\begin{pmatrix} t \\ t \\ -2t+1 \end{pmatrix}$

9. $\mathbf{X} = c_3\begin{pmatrix} 1 \\ 0 \end{pmatrix}e^t + c_4\begin{pmatrix} 0 \\ 1 \end{pmatrix}e^{2t} + \begin{pmatrix} -3 \\ \frac{1}{2} \end{pmatrix}$

11. $\mathbf{X} = c_1\begin{pmatrix} \cosh t \\ \sinh t \end{pmatrix} + c_2\begin{pmatrix} \sinh t \\ \cosh t \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

13. $\mathbf{X} = \begin{pmatrix} t+1 \\ t \\ -2t \end{pmatrix} - 4\begin{pmatrix} t \\ t+1 \\ -2t \end{pmatrix} + 6\begin{pmatrix} t \\ t \\ -2t+1 \end{pmatrix}$

15. $e^{\mathbf{A}t} = \begin{pmatrix} \frac{3}{2}e^{2t} - \frac{1}{2}e^{-2t} & \frac{3}{4}e^{2t} - \frac{3}{4}e^{-2t} \\ -e^{2t} + e^{-2t} & -\frac{1}{2}e^{2t} + \frac{3}{2}e^{-2t} \end{pmatrix};$

$\mathbf{X} = c_1\begin{pmatrix} \frac{3}{2}e^{2t} - \frac{1}{2}e^{-2t} \\ -e^{2t} + e^{-2t} \end{pmatrix} + c_2\begin{pmatrix} \frac{3}{4}e^{2t} - \frac{3}{4}e^{-2t} \\ -\frac{1}{2}e^{2t} + \frac{3}{2}e^{-2t} \end{pmatrix}$ or

$\mathbf{X} = c_3\begin{pmatrix} 3 \\ -2 \end{pmatrix}e^{2t} + c_4\begin{pmatrix} 1 \\ -2 \end{pmatrix}e^{-2t}$

17. $e^{\mathbf{A}t} = \begin{pmatrix} e^{2t} + 3te^{2t} & -9te^{2t} \\ te^{2t} & e^{2t} - 3te^{2t} \end{pmatrix};$

$\mathbf{X} = c_1\begin{pmatrix} 1+3t \\ t \end{pmatrix}e^{2t} + c_2\begin{pmatrix} -9t \\ 1-3t \end{pmatrix}e^{2t}$

23. $\mathbf{X} = c_1\begin{pmatrix} \frac{3}{2}e^{3t} - \frac{1}{2}e^{5t} \\ \frac{3}{2}e^{3t} - \frac{3}{2}e^{5t} \end{pmatrix} + c_2\begin{pmatrix} -\frac{1}{2}e^{3t} + \frac{1}{2}e^{5t} \\ -\frac{1}{2}e^{3t} + \frac{3}{2}e^{5t} \end{pmatrix}$ or

$\mathbf{X} = c_3\begin{pmatrix} 1 \\ 1 \end{pmatrix}e^{3t} + c_4\begin{pmatrix} 1 \\ 3 \end{pmatrix}e^{5t}$

CHAPTER 8 IN REVIEW (PAGE 360)

1. $k = \frac{1}{3}$

5. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^t + c_2\left[\begin{pmatrix} 1 \\ -1 \end{pmatrix}te^t + \begin{pmatrix} 0 \\ 1 \end{pmatrix}e^t\right]$

7. $\mathbf{X} = c_1\begin{pmatrix} \cos 2t \\ -\sin 2t \end{pmatrix}e^t + c_2\begin{pmatrix} \sin 2t \\ \cos 2t \end{pmatrix}e^t$

9. $\mathbf{X} = c_1\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}e^{2t} + c_2\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}e^{4t} + c_3\begin{pmatrix} 7 \\ 12 \\ -16 \end{pmatrix}e^{-3t}$

11. $\mathbf{X} = c_1\begin{pmatrix} 1 \\ 0 \end{pmatrix}e^{2t} + c_2\begin{pmatrix} 4 \\ 1 \end{pmatrix}e^{4t} + \begin{pmatrix} 16 \\ -4 \end{pmatrix}t + \begin{pmatrix} 11 \\ -1 \end{pmatrix}$

13. $\mathbf{X} = c_1\begin{pmatrix} \cos t \\ \cos t - \sin t \end{pmatrix} + c_2\begin{pmatrix} \sin t \\ \sin t + \cos t \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$+ \begin{pmatrix} \sin t \\ \sin t + \cos t \end{pmatrix}\ln|\csc t - \cot t|$

15. (b) $\mathbf{X} = c_1\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + c_2\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + c_3\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}e^{3t}$

EXERCISES 9.1 (PAGE 367)

1. for $h = 0.1$, $y_5 = 2.0801$; for $h = 0.05$, $y_{10} = 2.0592$

3. for $h = 0.1$, $y_5 = 0.5470$; for $h = 0.05$, $y_{10} = 0.5465$

5. for $h = 0.1$, $y_5 = 0.4053$; for $h = 0.05$, $y_{10} = 0.4054$

7. for $h = 0.1$, $y_5 = 0.5503$; for $h = 0.05$, $y_{10} = 0.5495$

9. for $h = 0.1$, $y_5 = 1.3260$; for $h = 0.05$, $y_{10} = 1.3315$

11. for $h = 0.1$, $y_5 = 3.8254$; for $h = 0.05$, $y_{10} = 3.8840$; at $x = 0.5$ the actual value is $y(0.5) = 3.9082$

13. (a) $y_1 = 1.2$

(b) $y''(c)\dfrac{h^2}{2} = 4e^{2c}\dfrac{(0.1)^2}{2} = 0.02e^{2c} \le 0.02e^{0.2} = 0.0244$

(c) Actual value is $y(0.1) = 1.2214$. Error is 0.0214.

(d) If $h = 0.05$, $y_2 = 1.21$.

(e) Error with $h = 0.1$ is 0.0214. Error with $h = 0.05$ is 0.0114.

15. (a) $y_1 = 0.8$

(b) $y''(c)\dfrac{h^2}{2} = 5e^{-2c}\dfrac{(0.1)^2}{2} = 0.025e^{-2c} \le 0.025$ for $0 \le c \le 0.1$.

(c) Actual value is $y(0.1) = 0.8234$. Error is 0.0234.

(d) If $h = 0.05$, $y_2 = 0.8125$.

(e) Error with $h = 0.1$ is 0.0234. Error with $h = 0.05$ is 0.0109.

17. (a) Error is $19h^2e^{-3(c-1)}$.

(b) $y''(c)\dfrac{h^2}{2} \le 19(0.1)^2(1) = 0.19$

(c) If $h = 0.1$, $y_5 = 1.8207$. If $h = 0.05$, $y_{10} = 1.9424$.

(d) Error with $h = 0.1$ is 0.2325. Error with $h = 0.05$ is 0.1109.

19. (a) Error is $\frac{1}{(c+1)^2}\frac{h^2}{2}$.

(b) $\left|y''(c)\frac{h^2}{2}\right| \le (1)\frac{(0.1)^2}{2} = 0.005$

(c) If $h = 0.1$, $y_5 = 0.4198$. If $h = 0.05$, $y_{10} = 0.4124$.

(d) Error with $h = 0.1$ is 0.0143. Error with $h = 0.05$ is 0.0069.

EXERCISES 9.2 (PAGE 371)

1. $y_5 = 3.9078$; actual value is $y(0.5) = 3.9082$

3. $y_5 = 2.0533$

5. $y_5 = 0.5463$

7. $y_5 = 0.4055$

9. $y_5 = 0.5493$

11. $y_5 = 1.3333$

13. (a) 35.7130

(c) $v(t) = \sqrt{\frac{mg}{k}}\tanh\sqrt{\frac{kg}{m}}\,t$; $v(5) = 35.7678$

15. (a) for $h = 0.1$, $y_4 = 903.0282$;
for $h = 0.05$, $y_8 = 1.1 \times 10^{15}$

17. (a) $y_1 = 0.82341667$

(b) $y^{(5)}(c)\frac{h^5}{5!} = 40e^{-2c}\frac{h^5}{5!} \le 40e^{2(0)}\frac{(0.1)^5}{5!}$
$= 3.333 \times 10^{-6}$

(c) Actual value is $y(0.1) = 0.8234134413$. Error is $3.225 \times 10^{-6} \le 3.333 \times 10^{-6}$.

(d) If $h = 0.05$, $y_2 = 0.82341363$.

(e) Error with $h = 0.1$ is 3.225×10^{-6}. Error with $h = 0.05$ is 1.854×10^{-7}.

19. (a) $y^{(5)}(c)\frac{h^5}{5!} = \frac{24}{(c+1)^5}\frac{h^5}{5!}$

(b) $\frac{24}{(c+1)^5}\frac{h^5}{5!} \le 24\frac{(0.1)^5}{5!} = 2.0000 \times 10^{-6}$

(c) From calculation with $h = 0.1$, $y_5 = 0.40546517$.
From calculation with $h = 0.05$, $y_{10} = 0.40546511$.

EXERCISES 9.3 (PAGE 375)

1. $y(x) = -x + e^x$; actual values are $y(0.2) = 1.0214$, $y(0.4) = 1.0918$, $y(0.6) = 1.2221$, $y(0.8) = 1.4255$; approximations are given in Example 1.

3. $y_4 = 0.7232$

5. for $h = 0.2$, $y_5 = 1.5569$; for $h = 0.1$, $y_{10} = 1.5576$

7. for $h = 0.2$, $y_5 = 0.2385$; for $h = 0.1$, $y_{10} = 0.2384$

EXERCISES 9.4 (PAGE 379)

1. $y(x) = -2e^{2x} + 5xe^{2x}$; $y(0.2) = -1.4918$, $y_2 = -1.6800$

3. $y_1 = -1.4928$, $y_2 = -1.4919$

5. $y_1 = 1.4640$, $y_2 = 1.4640$

7. $x_1 = 8.3055$, $y_1 = 3.4199$;
$x_2 = 8.3055$, $y_2 = 3.4199$

9. $x_1 = -3.9123$, $y_1 = 4.2857$;
$x_2 = -3.9123$, $y_2 = 4.2857$

11. $x_1 = 0.4179$, $y_1 = -2.1824$;
$x_2 = 0.4173$, $y_2 = -2.1821$

EXERCISES 9.5 (PAGE 383)

1. $y_1 = -5.6774$, $y_2 = -2.5807$, $y_3 = 6.3226$

3. $y_1 = -0.2259$, $y_2 = -0.3356$, $y_3 = -0.3308$, $y_4 = -0.2167$

5. $y_1 = 3.3751$, $y_2 = 3.6306$, $y_3 = 3.6448$, $y_4 = 3.2355$, $y_5 = 2.1411$

7. $y_1 = 3.8842$, $y_2 = 2.9640$, $y_3 = 2.2064$, $y_4 = 1.5826$, $y_5 = 1.0681$, $y_6 = 0.6430$, $y_7 = 0.2913$

9. $y_1 = 0.2660$, $y_2 = 0.5097$, $y_3 = 0.7357$, $y_4 = 0.9471$, $y_5 = 1.1465$, $y_6 = 1.3353$, $y_7 = 1.5149$, $y_8 = 1.6855$, $y_9 = 1.8474$

11. $y_1 = 0.3492$, $y_2 = 0.7202$, $y_3 = 1.1363$, $y_4 = 1.6233$, $y_5 = 2.2118$, $y_6 = 2.9386$, $y_7 = 3.8490$

13. (c) $y_0 = -2.2755$, $y_1 = -2.0755$, $y_2 = -1.8589$, $y_3 = -1.6126$, $y_4 = -1.3275$

CHAPTER 9 IN REVIEW (PAGE 384)

1. Comparison of numerical methods with $h = 0.1$:

x_n	Euler	Improved Euler	RK4
1.10	2.1386	2.1549	2.1556
1.20	2.3097	2.3439	2.3454
1.30	2.5136	2.5672	2.5695
1.40	2.7504	2.8246	2.8278
1.50	3.0201	3.1157	3.1197

Comparison of numerical methods with $h = 0.05$:

x_n	Euler	Improved Euler	RK4
1.10	2.1469	2.1554	2.1556
1.20	2.3272	2.3450	2.3454
1.30	2.5409	2.5689	2.5695
1.40	2.7883	2.8269	2.8278
1.50	3.0690	3.1187	3.1197

3. Comparison of numerical methods with $h = 0.1$:

x_n	Euler	Improved Euler	RK4
0.60	0.6000	0.6048	0.6049
0.70	0.7095	0.7191	0.7194
0.80	0.8283	0.8427	0.8431
0.90	0.9559	0.9752	0.9757
1.00	1.0921	1.1163	1.1169

Comparison of numerical methods with $h = 0.05$:

x_n	Euler	Improved Euler	RK4
0.60	0.6024	0.6049	0.6049
0.70	0.7144	0.7193	0.7194
0.80	0.8356	0.8430	0.8431
0.90	0.9657	0.9755	0.9757
1.00	1.1044	1.1168	1.1169

5. $h = 0.2$: $y(0.2)$ 3.2; $h = 0.1$: $y(0.2)$ 3.23

7. $x(0.2)$ 1.62, $y(0.2)$ 1.84

EXERCISES FOR APPENDIX I (PAGE APP-2)

1. **(a)** 24 **(b)** 720 **(c)** $\frac{4\sqrt{\pi}}{3}$ **(d)** $-\frac{8\sqrt{\pi}}{15}$

3. 0.297

EXERCISES FOR APPENDIX II (PAGE APP-18)

1. **(a)** $\begin{pmatrix} 2 & 11 \\ 2 & -1 \end{pmatrix}$ **(b)** $\begin{pmatrix} -6 & 1 \\ 14 & -19 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & 28 \\ 12 & -12 \end{pmatrix}$

3. **(a)** $\begin{pmatrix} -11 & 6 \\ 17 & -22 \end{pmatrix}$ **(b)** $\begin{pmatrix} -32 & 27 \\ -4 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} 19 & -18 \\ -30 & 31 \end{pmatrix}$ **(d)** $\begin{pmatrix} 19 & 6 \\ 3 & 22 \end{pmatrix}$

5. **(a)** $\begin{pmatrix} 9 & 24 \\ 3 & 8 \end{pmatrix}$ **(b)** $\begin{pmatrix} 3 & 8 \\ -6 & -16 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ **(d)** $\begin{pmatrix} -4 & -5 \\ 8 & 10 \end{pmatrix}$

7. **(a)** 180 **(b)** $\begin{pmatrix} 4 & 8 & 10 \\ 8 & 16 & 20 \\ 10 & 20 & 25 \end{pmatrix}$ **(c)** $\begin{pmatrix} 6 \\ 12 \\ -5 \end{pmatrix}$

9. **(a)** $\begin{pmatrix} 7 & 38 \\ 10 & 75 \end{pmatrix}$ **(b)** $\begin{pmatrix} 7 & 38 \\ 10 & 75 \end{pmatrix}$

11. $\begin{pmatrix} -14 \\ 1 \end{pmatrix}$

13. $\begin{pmatrix} -38 \\ -2 \end{pmatrix}$

15. singular

17. nonsingular; $\mathbf{A}^{-1} = \frac{1}{4}\begin{pmatrix} -5 & -8 \\ 3 & 4 \end{pmatrix}$

19. nonsingular; $\mathbf{A}^{-1} = \frac{1}{2}\begin{pmatrix} 0 & -1 & 1 \\ 2 & 2 & -2 \\ -4 & -3 & 5 \end{pmatrix}$

21. nonsingular; $\mathbf{A}^{-1} = -\frac{1}{9}\begin{pmatrix} -2 & -2 & -1 \\ -13 & 5 & 7 \\ 8 & -1 & -5 \end{pmatrix}$

23. $\mathbf{A}^{-1}(t) = \frac{1}{2e^{3t}}\begin{pmatrix} 3e^{4t} & -e^{4t} \\ -4e^{-t} & 2e^{-t} \end{pmatrix}$

25. $\frac{d\mathbf{X}}{dt} = \begin{pmatrix} -5e^{-t} \\ -2e^{-t} \\ 7e^{-t} \end{pmatrix}$

27. $\frac{d\mathbf{X}}{dt} = 4\begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{2t} - 12\begin{pmatrix} 2 \\ 1 \end{pmatrix}e^{-3t}$

29. **(a)** $\begin{pmatrix} 4e^{4t} & -\pi \sin \pi t \\ 2 & 6t \end{pmatrix}$ **(b)** $\begin{pmatrix} \frac{1}{2}e^{8} - \frac{1}{2} & 0 \\ 4 & 6 \end{pmatrix}$

(c) $\begin{pmatrix} \frac{1}{4}e^{4t} - \frac{1}{4} & (1/\pi)\sin \pi t \\ t^2 & t^3 - t \end{pmatrix}$

31. $x = 3, y = 1, z = -5$

33. $x = 2 + 4t, y = -5 - t, z = t$

35. $x = -\frac{1}{2}, y = \frac{3}{2}, z = \frac{7}{2}$

37. $x_1 = 1, x_2 = 0, x_3 = 2, x_4 = 0$

41. $\mathbf{A}^{-1} = \begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & -\frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} & 0 \end{pmatrix}$

43. $\mathbf{A}^{-1} = \begin{pmatrix} 5 & 6 & -3 \\ 2 & 2 & -1 \\ -1 & -1 & 1 \end{pmatrix}$

45. $\mathbf{A}^{-1} = \begin{pmatrix} -\frac{1}{2} & -\frac{2}{3} & -\frac{1}{6} & \frac{7}{6} \\ 1 & \frac{1}{3} & \frac{1}{3} & -\frac{4}{3} \\ 0 & -\frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

47. $\lambda_1 = 6, \lambda_2 = 1, \mathbf{K}_1 = \begin{pmatrix} 2 \\ 7 \end{pmatrix}, \mathbf{K}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

49. $\lambda_1 = \lambda_2 = -4, \mathbf{K}_1 = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

51. $\lambda_1 = 0, \lambda_2 = 4, \lambda_3 = -4,$

$\mathbf{K}_1 = \begin{pmatrix} 9 \\ 45 \\ 25 \end{pmatrix}, \mathbf{K}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{K}_3 = \begin{pmatrix} 1 \\ 9 \\ 1 \end{pmatrix}$

53. $\lambda_1 = \lambda_2 = \lambda_3 = -2,$

$\mathbf{K}_1 = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}, \mathbf{K}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

55. $\lambda_1 = 3i, \lambda_2 = -3i,$

$\mathbf{K}_1 = \begin{pmatrix} 1 - 3i \\ 5 \end{pmatrix}, \mathbf{K}_2 = \begin{pmatrix} 1 + 3i \\ 5 \end{pmatrix}$