2.3 Exercises

1–22 Differentiate the function.

1. $f(x) = 2^{40}$
2. $f(x) = \pi^2$
3. $f(t) = 2 - \frac{2}{3}t$
4. $F(x) = \frac{3}{4}x^8$
5. $f(x) = x^3 - 4x + 6$
6. $f(t) = \frac{1}{2}t^6 - 3t^4 + t$
7. $g(x) = x^2(1 - 2x)$
8. $h(x) = (x - 2)(2x + 3)$
9. $g(t) = 2t^{-3/4}$
10. $B(y) = cy^{-6}$
11. $A(s) = -\dfrac{12}{s^5}$
12. $y = x^{5/3} - x^{2/3}$
13. $S(p) = \sqrt{p} - p$
14. $y = \sqrt{x}\,(x - 1)$
15. $R(a) = (3a + 1)^2$
16. $S(R) = 4\pi R^2$
17. $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$
18. $y = \dfrac{\sqrt{x} + x}{x^2}$
19. $H(x) = (x + x^{-1})^3$
20. $g(u) = \sqrt{2}\,u + \sqrt{3u}$
21. $u = \sqrt[5]{t} + 4\sqrt{t^5}$
22. $v = \left(\sqrt{x} + \dfrac{1}{\sqrt[3]{x}}\right)^2$

23. Find the derivative of $f(x) = (1 + 2x^2)(x - x^2)$ in two ways: by using the Product Rule and by performing the multiplication first. Do your answers agree?

24. Find the derivative of the function

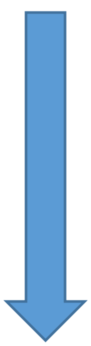
$$F(x) = \frac{x^4 - 5x^3 + \sqrt{x}}{x^2}$$

in two ways: by using the Quotient Rule and by simplifying first. Show that your answers are equivalent. Which method do you prefer?

25–44 Differentiate.

25. $V(x) = (2x^3 + 3)(x^4 - 2x)$
26. $L(x) = (1 + x + x^2)(2 - x^4)$
27. $F(y) = \left(\dfrac{1}{y^2} - \dfrac{3}{y^4}\right)(y + 5y^3)$
28. $J(v) = (v^3 - 2v)(v^{-4} + v^{-2})$
29. $g(x) = \dfrac{1 + 2x}{3 - 4x}$
30. $f(x) = \dfrac{x - 3}{x + 3}$

Graphing calculator or computer required   1. Homework Hints available at stewartcalculus.com



31. $y = \dfrac{x^3}{1 - x^2}$

32. $y = \dfrac{x + 1}{x^3 + x - 2}$

33. $y = \dfrac{v^3 - 2v\sqrt{v}}{v}$

34. $y = \dfrac{t}{(t - 1)^2}$

35. $y = \dfrac{t^2 + 2}{t^4 - 3t^2 + 1}$

36. $g(t) = \dfrac{t - \sqrt{t}}{t^{1/3}}$

37. $y = ax^2 + bx + c$

38. $y = A + \dfrac{B}{x} + \dfrac{C}{x^2}$

39. $f(t) = \dfrac{2t}{2 + \sqrt{t}}$

40. $y = \dfrac{cx}{1 + cx}$

41. $y = \sqrt[5]{t}\,(t^2 + t + t^{-1})$

42. $y = \dfrac{u^6 - 2u^3 + 5}{u^2}$

43. $f(x) = \dfrac{x}{x + \dfrac{c}{x}}$

44. $f(x) = \dfrac{ax + b}{cx + d}$

45. The general polynomial of degree n has the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where $a_n \neq 0$. Find the derivative of P.

46–48 Find $f'(x)$. Compare the graphs of f and f' and use them to explain why your answer is reasonable.

46. $f(x) = x/(x^2 - 1)$

47. $f(x) = 3x^{15} - 5x^3 + 3$

48. $f(x) = x + \dfrac{1}{x}$

49. (a) Use a graphing calculator or computer to graph the function $f(x) = x^4 - 3x^3 - 6x^2 + 7x + 30$ in the viewing rectangle $[-3, 5]$ by $[-10, 50]$.
(b) Using the graph in part (a) to estimate slopes, make a rough sketch, by hand, of the graph of f'. (See Example 1 in Section 2.2.)
(c) Calculate $f'(x)$ and use this expression, with a graphing device, to graph f'. Compare with your sketch in part (b).

50. (a) Use a graphing calculator or computer to graph the function $g(x) = x^2/(x^2 + 1)$ in the viewing rectangle $[-4, 4]$ by $[-1, 1.5]$.
(b) Using the graph in part (a) to estimate slopes, make a rough sketch, by hand, of the graph of g'. (See Example 1 in Section 2.2.)
(c) Calculate $g'(x)$ and use this expression, with a graphing device, to graph g'. Compare with your sketch in part (b).

51–52 Find an equation of the tangent line to the curve at the given point.

51. $y = \dfrac{2x}{x + 1}$, $(1, 1)$

52. $y = x^4 + 2x^2 - x$, $(1, 2)$

53. (a) The curve $y = 1/(1 + x^2)$ is called a **witch of Maria Agnesi**. Find an equation of the tangent line to this curve at the point $\left(-1, \frac{1}{2}\right)$.
(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

54. (a) The curve $y = x/(1 + x^2)$ is called a **serpentine**. Find an equation of the tangent line to this curve at the point $(3, 0.3)$.
(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

55–58 Find equations of the tangent line and normal line to the curve at the given point.

55. $y = x + \sqrt{x}$, $(1, 2)$

56. $y = (1 + 2x)^2$, $(1, 9)$

57. $y = \dfrac{3x + 1}{x^2 + 1}$, $(1, 2)$

58. $y = \dfrac{\sqrt{x}}{x + 1}$, $(4, 0.4)$

59–62 Find the first and second derivatives of the function.

59. $f(x) = x^4 - 3x^3 + 16x$

60. $G(r) = \sqrt{r} + \sqrt[3]{r}$

61. $f(x) = \dfrac{x^2}{1 + 2x}$

62. $f(x) = \dfrac{1}{3 - x}$

63. The equation of motion of a particle is $s = t^3 - 3t$, where s is in meters and t is in seconds. Find
(a) the velocity and acceleration as functions of t,
(b) the acceleration after 2 s, and
(c) the acceleration when the velocity is 0.

64. The equation of motion of a particle is

$$s = t^4 - 2t^3 + t^2 - t$$

where s is in meters and t is in seconds.
(a) Find the velocity and acceleration as functions of t.
(b) Find the acceleration after 1 s.
(c) Graph the position, velocity, and acceleration functions on the same screen.

65. Boyle's Law states that when a sample of gas is compressed at a constant pressure, the pressure P of the gas is inversely proportional to the volume V of the gas.

(a) Suppose that the pressure of a sample of air that occupies $0.106\ \text{m}^3$ at 25°C is 50 kPa. Write V as a function of P.

(b) Calculate dV/dP when $P = 50$ kPa. What is the meaning of the derivative? What are its units?

66. Car tires need to be inflated properly because overinflation or underinflation can cause premature treadware. The data in the table show tire life L (in thousands of miles) for a certain type of tire at various pressures P (in lb/in^2).

P	26	28	31	35	38	42	45
L	50	66	78	81	74	70	59

(a) Use a graphing calculator or computer to model tire life with a quadratic function of the pressure.

(b) Use the model to estimate dL/dP when $P = 30$ and when $P = 40$. What is the meaning of the derivative? What are the units? What is the significance of the signs of the derivatives?

67. Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$. Find the following values.

(a) $(fg)'(5)$ (b) $(f/g)'(5)$

(c) $(g/f)'(5)$

68. Find $h'(2)$, given that $f(2) = -3$, $g(2) = 4$, $f'(2) = -2$, and $g'(2) = 7$.

(a) $h(x) = 5f(x) - 4g(x)$ (b) $h(x) = f(x)g(x)$

(c) $h(x) = \dfrac{f(x)}{g(x)}$ (d) $h(x) = \dfrac{g(x)}{1 + f(x)}$

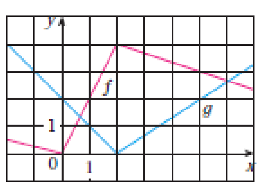
69. If $f(x) = \sqrt{x}\,g(x)$, where $g(4) = 8$ and $g'(4) = 7$, find $f'(4)$.

70. If $h(2) = 4$ and $h'(2) = -3$, find

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\bigg|_{x=2}$$

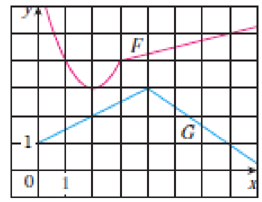
71. If f and g are the functions whose graphs are shown, let $u(x) = f(x)g(x)$ and $v(x) = f(x)/g(x)$.

(a) Find $u'(1)$. (b) Find $v'(5)$.



72. Let $P(x) = F(x)G(x)$ and $Q(x) = F(x)/G(x)$, where F and G are the functions whose graphs are shown.

(a) Find $P'(2)$. (b) Find $Q'(7)$.



73. If g is a differentiable function, find an expression for the derivative of each of the following functions.

(a) $y = xg(x)$ (b) $y = \dfrac{x}{g(x)}$ (c) $y = \dfrac{g(x)}{x}$

74. If f is a differentiable function, find an expression for the derivative of each of the following functions.

(a) $y = x^2 f(x)$ (b) $y = \dfrac{f(x)}{x^2}$

(c) $y = \dfrac{x^2}{f(x)}$ (d) $y = \dfrac{1 + xf(x)}{\sqrt{x}}$

75. Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 1$ where the tangent is horizontal.

76. For what values of x does the graph of $f(x) = x^3 + 3x^2 + x + 3$ have a horizontal tangent?

77. Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with slope 4.

78. Find an equation of the tangent line to the curve $y = x\sqrt{x}$ that is parallel to the line $y = 1 + 3x$.

79. Find equations of both lines that are tangent to the curve $y = 1 + x^3$ and are parallel to the line $12x - y = 1$.

80. Find equations of the tangent lines to the curve

$$y = \frac{x - 1}{x + 1}$$

that are parallel to the line $x - 2y = 2$.

81. Find an equation of the normal line to the parabola $y = x^2 - 5x + 4$ that is parallel to the line $x - 3y = 5$.

82. Where does the normal line to the parabola $y = x - x^2$ at the point $(1, 0)$ intersect the parabola a second time? Illustrate with a sketch.

83. Draw a diagram to show that there are two tangent lines to the parabola $y = x^2$ that pass through the point $(0, -4)$. Find the coordinates of the points where these tangent lines intersect the parabola.

84. (a) Find equations of both lines through the point $(2, -3)$ that are tangent to the parabola $y = x^2 + x$.

(b) Show that there is no line through the point (2, 7) that is tangent to the parabola. Then draw a diagram to see why.

85. (a) Use the Product Rule twice to prove that if f, g, and h are differentiable, then $(fgh)' = f'gh + fg'h + fgh'$.
(b) Taking $f = g = h$ in part (a), show that

$$\frac{d}{dx}[f(x)]^3 = 3[f(x)]^2 f'(x)$$

(c) Use part (b) to differentiate $y = (x^4 + 3x^3 + 17x + 82)^3$.

86. Find the nth derivative of each function by calculating the first few derivatives and observing the pattern that occurs.
(a) $f(x) = x^n$ (b) $f(x) = 1/x$

87. Find a second-degree polynomial P such that $P(2) = 5$, $P'(2) = 3$, and $P''(2) = 2$.

88. The equation $y'' + y' - 2y = x^2$ is called a **differential equation** because it involves an unknown function y and its derivatives y' and y''. Find constants A, B, and C such that the function $y = Ax^2 + Bx + C$ satisfies this equation. (Differential equations will be studied in detail in Chapter 9.)

89. Find a cubic function $y = ax^3 + bx^2 + cx + d$ whose graph has horizontal tangents at the points $(-2, 6)$ and $(2, 0)$.

90. Find a parabola with equation $y = ax^2 + bx + c$ that has slope 4 at $x = 1$, slope -8 at $x = -1$, and passes through the point (2, 15).

91. In this exercise we estimate the rate at which the total personal income is rising in the Richmond-Petersburg, Virginia, metropolitan area. In 1999, the population of this area was 961,400, and the population was increasing at roughly 9200 people per year. The average annual income was \$30,593 per capita, and this average was increasing at about \$1400 per year (a little above the national average of about \$1225 yearly). Use the Product Rule and these figures to estimate the rate at which total personal income was rising in the Richmond-Petersburg area in 1999. Explain the meaning of each term in the Product Rule.

92. A manufacturer produces bolts of a fabric with a fixed width. The quantity q of this fabric (measured in yards) that is sold is a function of the selling price p (in dollars per yard), so we can write $q = f(p)$. Then the total revenue earned with selling price p is $R(p) = pf(p)$.
(a) What does it mean to say that $f(20) = 10{,}000$ and $f'(20) = -350$?
(b) Assuming the values in part (a), find $R'(20)$ and interpret your answer.

93. Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ x + 1 & \text{if } x \geqslant 1 \end{cases}$$

Is f differentiable at 1? Sketch the graphs of f and f'.

94. At what numbers is the following function g differentiable?

$$g(x) = \begin{cases} 2x & \text{if } x \leqslant 0 \\ 2x - x^2 & \text{if } 0 < x < 2 \\ 2 - x & \text{if } x \geqslant 2 \end{cases}$$

Give a formula for g' and sketch the graphs of g and g'.

95. (a) For what values of x is the function $f(x) = |x^2 - 9|$ differentiable? Find a formula for f'.
(b) Sketch the graphs of f and f'.

96. Where is the function $h(x) = |x - 1| + |x + 2|$ differentiable? Give a formula for h' and sketch the graphs of h and h'.

97. For what values of a and b is the line $2x + y = b$ tangent to the parabola $y = ax^2$ when $x = 2$?

98. (a) If $F(x) = f(x)g(x)$, where f and g have derivatives of all orders, show that $F'' = f''g + 2f'g' + fg''$.
(b) Find similar formulas for F''' and $F^{(4)}$.
(c) Guess a formula for $F^{(n)}$.

99. Find the value of c such that the line $y = \frac{3}{2}x + 6$ is tangent to the curve $y = c\sqrt{x}$.

100. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \leqslant 2 \\ mx + b & \text{if } x > 2 \end{cases}$$

Find the values of m and b that make f differentiable everywhere.

101. An easy proof of the Quotient Rule can be given if we make the prior assumption that $F'(x)$ exists, where $F = f/g$. Write $f = Fg$; then differentiate using the Product Rule and solve the resulting equation for F'.

102. A tangent line is drawn to the hyperbola $xy = c$ at a point P.
(a) Show that the midpoint of the line segment cut from this tangent line by the coordinate axes is P.
(b) Show that the triangle formed by the tangent line and the coordinate axes always has the same area, no matter where P is located on the hyperbola.

103. Evaluate $\displaystyle\lim_{x \to 1} \frac{x^{1000} - 1}{x - 1}$.

104. Draw a diagram showing two perpendicular lines that intersect on the y-axis and are both tangent to the parabola $y = x^2$. Where do these lines intersect?

105. If $c > \frac{1}{2}$, how many lines through the point $(0, c)$ are normal lines to the parabola $y = x^2$? What if $c \leqslant \frac{1}{2}$?

106. Sketch the parabolas $y = x^2$ and $y = x^2 - 2x + 2$. Do you think there is a line that is tangent to both curves? If so, find its equation. If not, why not?