Chapter 2

Figure 17

From equation (11), the linear speed v of the rock is

$$v = r\omega = 2 \text{ feet} \cdot 360\pi \frac{\text{radians}}{\text{minute}} = 720\pi \frac{\text{feet}}{\text{minute}} \approx 2262 \frac{\text{feet}}{\text{minute}}$$

The linear speed of the rock when it is released is 2262 ft/min ≈ 25.7 mi/hr. ■

Now Work PROBLEM 97

Historical Feature

Trigonometry was developed by Greek astronomers, who regarded the sky as the inside of a sphere, so it was natural that triangles on a sphere were investigated early (by Menelaus of Alexandria about AD 100) and that triangles in the plane were studied much later. The first book containing a systematic treatment of plane and spherical trigonometry was written by the Persian astronomer Nasir Eddin (about AD 1250).

Regiomontanus (1436–1476) is the person most responsible for moving trigonometry from astronomy into mathematics. His work was improved by Copernicus (1473–1543) and Copernicus's student Rhaeticus (1514–1576). Rhaeticus's book was the first to define the six trigonometric functions as ratios of sides of triangles, although he did not give the functions their present names. Credit for this is due to Thomas Finck (1583), but Finck's notation was by no means universally accepted at the time. The notation was finally stabilized by the textbooks of Leonhard Euler (1707–1783).

Trigonometry has since evolved from its use by surveyors, navigators, and engineers to present applications involving ocean tides, the rise and fall of food supplies in certain ecologies, brain wave patterns, and many other phenomena.

2.1 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. What is the formula for the circumference C of a circle of radius r? (pp. A15–A16)
2. What is the formula for the area A of a circle of radius r? (pp. A15–A16)

Concepts and Vocabulary

3. An angle θ is in _____ _____ if its vertex is at the origin of a rectangular coordinate system and its initial side coincides with the positive x-axis.
4. On a circle of radius r, a central angle of θ radians subtends an arc of length s = _____; the area of the sector formed by this angle θ is A = _____.
5. An object travels around a circle of radius r with constant speed. If s is the distance traveled in time t around the circle and θ is the central angle (in radians) swept out in time t, then the linear speed of the object is v = _____ and the angular speed of the object is ω = _____.
6. ***True or False*** $\pi = 180$.
7. ***True or False*** $180° = \pi$ radians.
8. ***True or False*** On the unit circle, if s is the length of the arc subtended by a central angle θ, measured in radians, then $s = \theta$.
9. ***True or False*** The area A of the sector of a circle of radius r formed by a central angle of θ degrees is $A = \frac{1}{2}r^2\theta$.
10. ***True or False*** For circular motion on a circle of radius r, linear speed equals angular speed divided by r.

Skill Building

In Problems 11–22, draw each angle.

11. $30°$ **12.** $60°$ **13.** $135°$ **14.** $-120°$ **15.** $450°$ **16.** $540°$

17. $\frac{3\pi}{4}$ **18.** $\frac{4\pi}{3}$ **19.** $-\frac{\pi}{6}$ **20.** $-\frac{2\pi}{3}$ **21.** $\frac{16\pi}{3}$ **22.** $\frac{21\pi}{4}$

In Problems 23–28, convert each angle to a decimal in degrees. Round your answer to two decimal places.

23. 40°10′25″ **24.** 61°42′21″ **25.** 1°2′3″ **26.** 73°40′40″ **27.** 9°9′9″ **28.** 98°22′45″

In Problems 29–34, convert each angle to D°M′S″ form. Round your answer to the nearest second.

29. 40.32° **30.** 61.24° **31.** 18.255° **32.** 29.411° **33.** 19.99° **34.** 44.01°

2.1

In Problems 35–46, convert each angle in degrees to radians. Express your answer as a multiple of π.

35. 30° **36.** 120° **37.** 240° **38.** 330° **39.** −60° **40.** −30°

41. 180° **42.** 270° **43.** −135° **44.** −225° **45.** −90° **46.** −180°

In Problems 47–58, convert each angle in radians to degrees.

47. $\frac{\pi}{3}$ **48.** $\frac{5\pi}{6}$ **49.** $-\frac{5\pi}{4}$ **50.** $-\frac{2\pi}{3}$ **51.** $\frac{\pi}{2}$ **52.** 4π

53. $\frac{\pi}{12}$ **54.** $\frac{5\pi}{12}$ **55.** $-\frac{\pi}{2}$ **56.** $-\pi$ **57.** $-\frac{\pi}{6}$ **58.** $-\frac{3\pi}{4}$

In Problems 59–64, convert each angle in degrees to radians. Express your answer in decimal form, rounded to two decimal places.

59. 17° **60.** 73° **61.** −40° **62.** −51° **63.** 125° **64.** 350°

In Problems 65–70, convert each angle in radians to degrees. Express your answer in decimal form, rounded to two decimal places.

65. 3.14 **66.** 0.75 **67.** 2 **68.** 3 **69.** 6.32 **70.** $\sqrt{2}$

In Problems 71–78, s denotes the length of the arc of a circle of radius r subtended by the central angle θ. *Find the missing quantity. Round answers to three decimal places.*

71. $r = 10$ meters, $\theta = \frac{1}{2}$ radian, $s = ?$

72. $r = 6$ feet, $\theta = 2$ radians, $s = ?$

73. $\theta = \frac{1}{3}$ radian, $s = 2$ feet, $r = ?$

74. $\theta = \frac{1}{4}$ radian, $s = 6$ centimeters, $r = ?$

75. $r = 5$ miles, $s = 3$ miles, $\theta = ?$

76. $r = 6$ meters, $s = 8$ meters, $\theta = ?$

77. $r = 2$ inches, $\theta = 30°$, $s = ?$

78. $r = 3$ meters, $\theta = 120°$, $s = ?$

In Problems 79–86, A denotes the area of the sector of a circle of radius r formed by the central angle θ. *Find the missing quantity. Round answers to three decimal places.*

79. $r = 10$ meters, $\theta = \frac{1}{2}$ radian, $A = ?$

80. $r = 6$ feet, $\theta = 2$ radians, $A = ?$

81. $\theta = \frac{1}{3}$ radian, $A = 2$ square feet, $r = ?$

82. $\theta = \frac{1}{4}$ radian, $A = 6$ square centimeters, $r = ?$

83. $r = 5$ miles, $A = 3$ square miles, $\theta = ?$

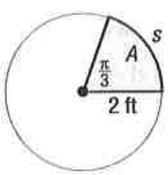
84. $r = 6$ meters, $A = 8$ square meters, $\theta = ?$

85. $r = 2$ inches, $\theta = 30°$, $A = ?$

86. $r = 3$ meters, $\theta = 120°$, $A = ?$

In Problems 87–90, find the length s and area A. Round answers to three decimal places.

87.



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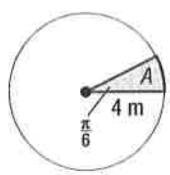

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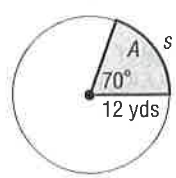

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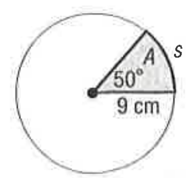


Applications and Extensions

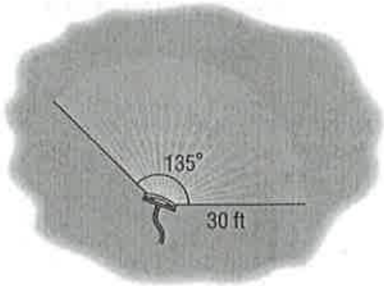
91. Movement of a Minute Hand The minute hand of a clock is 6 inches long. How far does the tip of the minute hand move in 15 minutes? How far does it move in 25 minutes? Round answers to two decimal places.

92. Movement of a Pendulum A pendulum swings through an angle of 20° each second. If the pendulum is 40 inches long, how far does its tip move each second? Round answers to two decimal places.

93. Area of a Sector Find the area of the sector of a circle of radius 4 meters formed by an angle of 45°. Round the answer to two decimal places.

94. Area of a Sector Find the area of the sector of a circle of radius 3 centimeters formed by an angle of 60°. Round the answer to two decimal places.

95. Watering a Lawn A water sprinkler sprays water over a distance of 30 feet while rotating through an angle of 135°. What area of lawn receives water?



96. Designing a Water Sprinkler An engineer is asked to design a water sprinkler that will cover a field of 100 square yards that is in the shape of a sector of a circle of radius 50 yards. Through what angle should the sprinkler rotate?

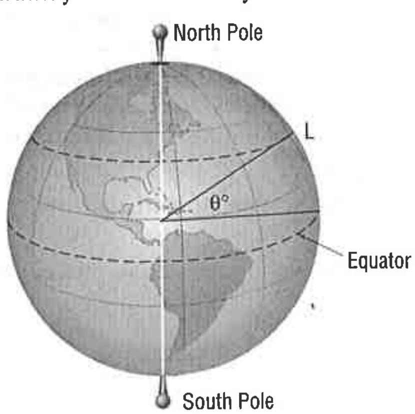
97. Motion on a Circle An object is traveling around a circle with a radius of 5 centimeters. If in 20 seconds a central angle of $\frac{1}{3}$ radian is swept out, what is the angular speed of the object? What is its linear speed?

98. Motion on a Circle An object is traveling around a circle with a radius of 2 meters. If in 20 seconds the object travels 5 meters, what is its angular speed? What is its linear speed?

99. Bicycle Wheels The diameter of each wheel of a bicycle is 26 inches. If you are traveling at a speed of 35 miles per hour on this bicycle, through how many revolutions per minute are the wheels turning?

100. Car Wheels The radius of each wheel of a car is 15 inches. If the wheels are turning at the rate of 3 revolutions per second, how fast is the car moving? Express your answer in inches per second and in miles per hour.

In Problems 101–104, the latitude of a location L is the angle formed by a ray drawn from the center of Earth to the Equator and a ray drawn from the center of Earth to L. See the figure.



101. Distance between Cities Memphis, Tennessee, is due north of New Orleans, Louisiana. Find the distance between Memphis (35°9′ north latitude) and New Orleans (29°57′ north latitude). Assume that the radius of Earth is 3960 miles.

102. Distance between Cities Charleston, West Virginia, is due north of Jacksonville, Florida. Find the distance between Charleston (38°21′ north latitude) and Jacksonville (30°20′ north latitude). Assume that the radius of Earth is 3960 miles.

103. Linear Speed on Earth Earth rotates on an axis through its poles. The distance from the axis to a location on Earth 30° north latitude is about 3429.5 miles. Therefore, a location on Earth at 30° north latitude is spinning on a circle of radius 3429.5 miles. Compute the linear speed on the surface of Earth at 30° north latitude.

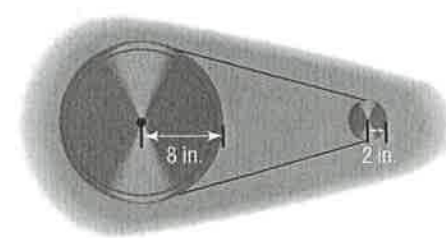
104. Linear Speed on Earth Earth rotates on an axis through its poles. The distance from the axis to a location on Earth 40° north latitude is about 3033.5 miles. Therefore, a location on Earth at 40° north latitude is spinning on a circle of radius 3033.5 miles. Compute the linear speed on the surface of Earth at 40° north latitude.

105. Speed of the Moon The mean distance of the Moon from Earth is 2.39×10^5 miles. Assuming that the orbit of the Moon around Earth is circular and that 1 revolution takes 27.3 days, find the linear speed of the Moon. Express your answer in miles per hour.

106. Speed of Earth The mean distance of Earth from the Sun is 9.29×10^7 miles. Assuming that the orbit of Earth around the Sun is circular and that 1 revolution takes 365 days, find the linear speed of Earth. Express your answer in miles per hour.

107. Pulleys Two pulleys, one with radius 2 inches and the other with radius 8 inches, are connected by a belt. (See the figure.) If the 2-inch pulley is caused to rotate at 3 revolutions per minute, determine the revolutions per minute of the 8-inch pulley.

[**Hint:** The linear speeds of the pulleys are the same; both equal the speed of the belt.]



108. Ferris Wheels A neighborhood carnival has a Ferris wheel whose radius is 30 feet. You measure the time it takes for one revolution to be 70 seconds. What is the linear speed (in feet per second) of this Ferris wheel? What is the angular speed in radians per second?

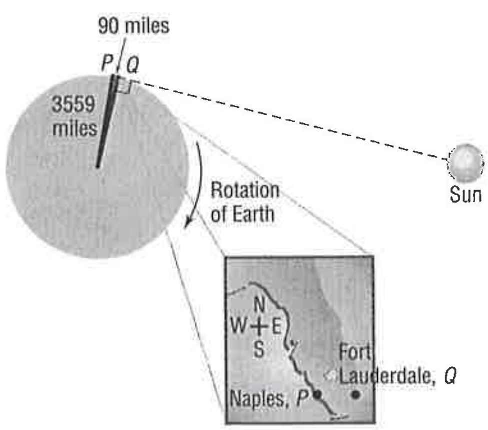
109. Computing the Speed of a River Current To approximate the speed of the current of a river, a circular paddle wheel with radius 4 feet is lowered into the water. If the current causes the wheel to rotate at a speed of 10 revolutions per minute, what is the speed of the current? Express your answer in miles per hour.

110. Spin Balancing Tires A spin balancer rotates the wheel of a car at 480 revolutions per minute. If the diameter of the wheel is 26 inches, what road speed is being tested? Express your answer in miles per hour. At how many revolutions per minute should the balancer be set to test a road speed of 80 miles per hour?

111. The Cable Cars of San Francisco At the Cable Car Museum you can see the four cable lines that are used to pull cable cars up and down the hills of San Francisco. Each cable travels at a speed of 9.55 miles per hour, caused by a rotating wheel whose diameter is 8.5 feet. How fast is the wheel rotating? Express your answer in revolutions per minute.

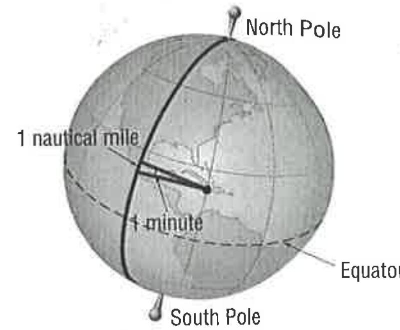
112. Difference in Time of Sunrise Naples, Florida, is approximately 90 miles due west of Ft. Lauderdale. How much sooner would a person in Ft. Lauderdale first see the rising Sun than a person in Naples? See the hint.

[**Hint:** Consult the figure. When a person at Q sees the first rays of the Sun, a person at P is still in the dark. The person at P sees the first rays after Earth has rotated so that P is at the location Q. Now use the fact that at the latitude of Ft. Lauderdale in 24 hours a length of arc of $2\pi(3559)$ miles is subtended.]

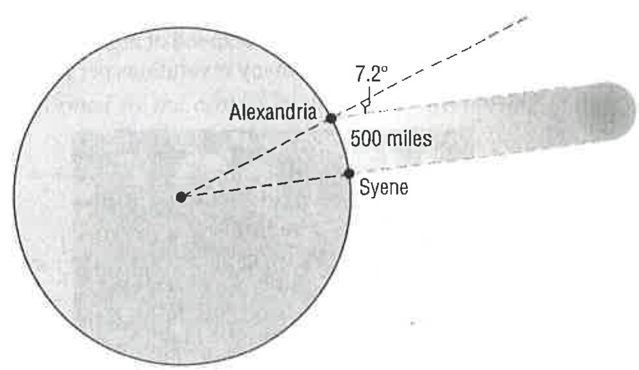


113. Keeping Up with the Sun How fast would you have to travel on the surface of Earth at the equator to keep up with the Sun (that is, so that the Sun would appear to remain in the same position in the sky)?

114. Nautical Miles A **nautical mile** equals the length of arc subtended by a central angle of 1 minute on a great circle* on the surface of Earth. (See the figure.) If the radius of Earth is taken as 3960 miles, express 1 nautical mile in terms of ordinary, or **statute,** miles.



115. Approximating the Circumference of Earth Eratosthenes of Cyrene (276–194 BC) was a Greek scholar who lived and worked in Cyrene and Alexandria. One day while visiting in Syene he noticed that the Sun's rays shone directly down a well. On this date 1 year later, in Alexandria, which is 500 miles due north of Syene he measured the angle of the Sun to be about 7.2 degrees. See the figure. Use this information to approximate the radius and circumference of Earth.



116. Designing a Little League Field For a 60-foot Little League Baseball field, the distance from home base to the nearest fence (or other obstruction) on fair territory should be a minimum of 200 feet. The commissioner of parks and recreation is making plans for a new 60-foot field. Because of limited ground availability, he will use the minimum required distance to the outfield fence. To increase safety, however, he plans to include a 10-foot wide warning track on the inside of the fence. To further increase safety, the fence and warning track will extend both directions into foul territory. In total the arc formed by the outfield fence (including the extensions into the foul territories) will be subtended by a central angle at home plate measuring 96°, as illustrated.

(a) Determine the length of the outfield fence.

(b) Determine the area of the warning track.

* Any circle drawn on the surface of Earth that divides Earth into two equal hemispheres.

Historical Feature

The name *sine* for the sine function is due to a medieval confusion. The name comes from the Sanskrit word *jiva* (meaning chord), first used in India by Araybhata the Elder (AD 510). He really meant half-chord, but abbreviated it. This was brought into Arabic as *jiba*, which was meaningless. Because the proper Arabic word *jaib* would be written the same way (short vowels are not written out in Arabic), *jiba* was pronounced as *jaib*, which meant bosom or hollow, and *jiba* remains as the Arabic word for sine to this day. Scholars translating the Arabic works into Latin found that the word *sinus* also meant bosom or hollow, and from *sinus* we get the word *sine*.

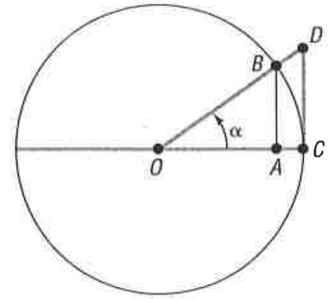
The name *tangent*, due to Thomas Finck (1583), can be understood by looking at Figure 34. The line segment $\overline{DC}$ is tangent to the circle at C. If $d(O, B) = d(O, C) = 1$, then the length of the line segment $\overline{DC}$ is

$$d(D, C) = \frac{d(D, C)}{1} = \frac{d(D, C)}{d(O, C)} = \tan\alpha$$

The old name for the tangent is *umbra versa* (meaning turned shadow), referring to the use of the tangent in solving height problems with shadows.

The names of the remaining functions came about as follows. If α and β are complementary angles, then $\cos\alpha = \sin\beta$. Because β is the complement of α, it was natural to write the cosine of α as *sin co* α. Probably for reasons involving ease of pronunciation, the *co* migrated to the front, and then cosine received a three-letter abbreviation to match sin, sec, and tan. The two other cofunctions were similarly treated, except that the long forms *cotan* and *cosec* survive to this day in some countries.

Figure 34



2.2 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. In a right triangle, with legs a and b and hypotenuse c, the Pythagorean Theorem states that ________. (p. A14)
2. The value of the function $f(x) = 3x - 7$ at 5 is ________. (pp. 22–31)
3. ***True or False*** For a function $y = f(x)$, for each x in the domain, there is exactly one element y in the range. (pp. 22–31)
4. If two triangles are similar, then corresponding angles are ________ and the lengths of corresponding sides are ________. (pp. A16–A19)
5. What point is symmetric with respect to the y-axis to the point $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$? (pp. 12–13)
6. If (x, y) is a point on the unit circle in quadrant IV and if $x = \frac{\sqrt{3}}{2}$, what is y? (p. 16)

Concepts and Vocabulary

7. $\tan\frac{\pi}{4} + \sin 30° =$ ________.
8. Using a calculator, $\sin 2 =$ ________, rounded to two decimal places.
9. ***True or False*** Exact values can be found for the trigonometric functions of 60°.
10. ***True or False*** Exact values can be found for the sine of any angle.

Skill Building

In Problems 11–18, t is a real number and P = (x, y) is the point on the unit circle that corresponds to t. Find the exact values of the six trigonometric functions of t.

11. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
12. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
13. $\left(-\frac{2}{5}, \frac{\sqrt{21}}{5}\right)$
14. $\left(-\frac{1}{5}, \frac{2\sqrt{6}}{5}\right)$
15. $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
16. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
17. $\left(\frac{2\sqrt{2}}{3}, -\frac{1}{3}\right)$
18. $\left(-\frac{\sqrt{5}}{3}, -\frac{2}{3}\right)$

In Problems 19–28, find the exact value. Do not use a calculator.

19. $\sin\frac{11\pi}{2}$
20. $\cos(7\pi)$
21. $\tan(6\pi)$
22. $\cot\frac{7\pi}{2}$
23. $\csc\frac{11\pi}{2}$
24. $\sec(8\pi)$
25. $\cos\left(-\frac{3\pi}{2}\right)$
26. $\sin(-3\pi)$
27. $\sec(-\pi)$
28. $\tan(-3\pi)$

2.2

In Problems 29–48, find the exact value of each expression. Do not use a calculator.

29. $\sin 45° + \cos 60°$
30. $\sin 30° - \cos 45°$
31. $\sin 90° + \tan 45°$
32. $\cos 180° - \sin 180°$
33. $\sin 45° \cos 45°$
34. $\tan 45° \cos 30°$
35. $\csc 45° \tan 60°$
36. $\sec 30° \cot 45°$
37. $4 \sin 90° - 3 \tan 180°$
38. $5 \cos 90° - 8 \sin 270°$
39. $2 \sin \frac{\pi}{3} - 3 \tan \frac{\pi}{6}$
40. $2 \sin \frac{\pi}{4} + 3 \tan \frac{\pi}{4}$
41. $\sin \frac{\pi}{4} - \cos \frac{\pi}{4}$
42. $\tan \frac{\pi}{3} + \cos \frac{\pi}{3}$
43. $2 \sec \frac{\pi}{4} + 4 \cot \frac{\pi}{3}$
44. $3 \csc \frac{\pi}{3} + \cot \frac{\pi}{4}$
45. $\tan \pi - \cos 0$
46. $\sin \frac{3\pi}{2} + \tan \pi$
47. $\csc \frac{\pi}{2} + \cot \frac{\pi}{2}$
48. $\sec \pi - \csc \frac{\pi}{2}$

In Problems 49–66, find the exact values of the six trigonometric functions of the given angle. If any are not defined, say "not defined Do not use a calculator.

49. $\frac{2\pi}{3}$
50. $\frac{5\pi}{6}$
51. $210°$
52. $240°$
53. $\frac{3\pi}{4}$
54. $\frac{11\pi}{4}$
55. $\frac{8\pi}{3}$
56. $\frac{13\pi}{6}$
57. $405°$
58. $390°$
59. $-\frac{\pi}{6}$
60. $-\frac{\pi}{3}$
61. $-45°$
62. $-60°$
63. $\frac{5\pi}{2}$
64. 5π
65. $720°$
66. $630°$

In Problems 67–82, use a calculator to find the approximate value of each expression rounded to two decimal places.

67. $\sin 28°$
68. $\cos 14°$
69. $\tan 21°$
70. $\cot 70°$
71. $\sec 41°$
72. $\csc 55°$
73. $\sin \frac{\pi}{10}$
74. $\cos \frac{\pi}{8}$
75. $\tan \frac{5\pi}{12}$
76. $\cot \frac{\pi}{18}$
77. $\sec \frac{\pi}{12}$
78. $\csc \frac{5\pi}{13}$
79. $\sin 1$
80. $\tan 1$
81. $\sin 1°$
82. $\tan 1°$

In Problems 83–92, a point on the terminal side of an angle θ in standard position is given. Find the exact values of the six trigonomet functions of θ.

83. $(-3, 4)$
84. $(5, -12)$
85. $(2, -3)$
86. $(-1, -2)$
87. $(-2, -2)$
88. $(1, -1)$
89. $(-3, -2)$
90. $(2, 2)$
91. $\left(\frac{1}{3}, -\frac{1}{4}\right)$
92. $(-0.3, -0.4)$

Applications and Extensions

93. Find the exact value of:
$$\sin 45° + \sin 135° + \sin 225° + \sin 315°$$

94. Find the exact value of:
$$\tan 60° + \tan 150°$$

95. Find the exact value of:
$$\sin 40° + \sin 130° + \sin 220° + \sin 310°.$$

96. Find the exact value of:
$$\tan 40° + \tan 140°.$$

97. If $f(\theta) = \sin \theta = 0.1$, find $f(\theta + \pi)$.

98. If $f(\theta) = \cos \theta = 0.3$, find $f(\theta + \pi)$.

99. If $f(\theta) = \tan \theta = 3$, find $f(\theta + \pi)$.

100. If $f(\theta) = \cot \theta = -2$, find $f(\theta + \pi)$.

101. If $\sin \theta = \frac{1}{5}$, find $\csc \theta$.

102. If $\cos \theta = \frac{2}{3}$, find $\sec \theta$.

In Problems 103–114, $f(\theta) = \sin \theta$ and $g(\theta) = \cos \theta$. Find the exact value of each function below if $\theta = 60°$. Do not use a calcula

103. $f(\theta)$
104. $g(\theta)$
105. $f\left(\frac{\theta}{2}\right)$
106. $g\left(\frac{\theta}{2}\right)$
107. $[f(\theta)]^2$
108. $[g(\theta)]^2$
109. $f(2\theta)$
110. $g(2\theta)$
111. $2f(\theta)$
112. $2g(\theta)$
113. $f(-\theta)$
114. $g(-\theta)$

115. Use a calculator in radian mode to complete the following table.

What can you conclude about the value of $f(\theta) = \frac{\sin \theta}{\theta}$ as θ approaches 0?

θ	0.5	0.4	0.2	0.1	0.01	0.001	0.0001	0.0000
$\sin \theta$								
$f(\theta) = \frac{\sin \theta}{\theta}$								

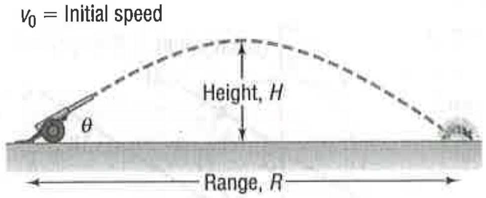
116. Use a calculator in radian mode to complete the following table.

What can you conclude about the value of $g(\theta) = \dfrac{\cos\theta - 1}{\theta}$ as θ approaches 0?

θ	0.5	0.4	0.2	0.1	0.01	0.001	0.0001	0.00001
$\cos\theta - 1$								
$g(\theta) = \dfrac{\cos\theta - 1}{\theta}$								

For Problems 117–120, use the following discussion.

Projectile Motion *The path of a projectile fired at an inclination θ to the horizontal with initial speed v_0 is a parabola (see the figure).*



The range R of the projectile, that is, the horizontal distance that the projectile travels, is found by using the function

$$R(\theta) = \frac{v_0^2 \sin(2\theta)}{g}$$

where $g \approx 32.2$ feet per second per second ≈ 9.8 meters per second per second is the acceleration due to gravity. The maximum height H of the projectile is given by the function

$$H(\theta) = \frac{v_0^2 (\sin\theta)^2}{2g}$$

In Problems 117–120, find the range R and maximum height H.

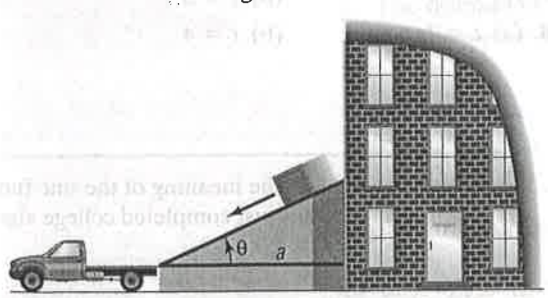
117. The projectile is fired at an angle of 45° to the horizontal with an initial speed of 100 feet per second.

118. The projectile is fired at an angle of 30° to the horizontal with an initial speed of 150 meters per second.

119. The projectile is fired at an angle of 25° to the horizontal with an initial speed of 500 meters per second.

120. The projectile is fired at an angle of 50° to the horizontal with an initial speed of 200 feet per second.

121. Inclined Plane See the figure.



If friction is ignored, the time t (in seconds) required for a block to slide down an inclined plane is given by the function

$$t(\theta) = \sqrt{\frac{2a}{g \sin\theta \cos\theta}}$$

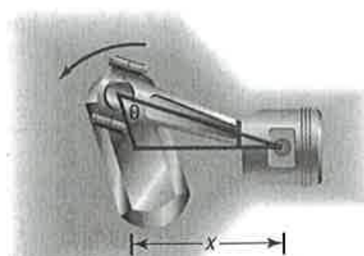
where a is the length (in feet) of the base and $g \approx 32$ feet per second per second is the acceleration due to gravity. How long does it take a block to slide down an inclined plane with base $a = 10$ feet when:

(a) $\theta = 30°$? (b) $\theta = 45°$? (c) $\theta = 60°$?

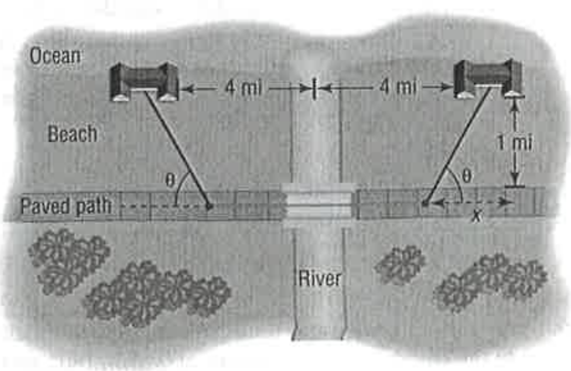
122. Piston Engines In a certain piston engine, the distance x (in centimeters) from the center of the drive shaft to the head of the piston is given by the function

$$x(\theta) = \cos\theta + \sqrt{16 + 0.5\cos(2\theta)}$$

where θ is the angle between the crank and the path of the piston head. See the figure. Find x when $\theta = 30°$ and when $\theta = 45°$.



123. Calculating the Time of a Trip Two oceanfront homes are located 8 miles apart on a straight stretch of beach, each a distance of 1 mile from a paved road that parallels the ocean. See the figure.



Sally can jog 8 miles per hour along the paved road, but only 3 miles per hour in the sand on the beach. Because of a river directly between the two houses, it is necessary to jog in the sand to the road, continue on the road, and then jog directly back in the sand to get from one house to the other. The

2.2

time T to get from one house to the other as a function of the angle θ shown in the illustration is

$$T(\theta) = 1 + \frac{2}{3\sin\theta} - \frac{1}{4\tan\theta}, \qquad 0° < \theta < 90°$$

(a) Calculate the time T for $\theta = 30°$. How long is Sally on the paved road?

(b) Calculate the time T for $\theta = 45°$. How long is Sally on the paved road?

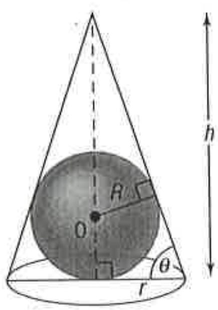
(c) Calculate the time T for $\theta = 60°$. How long is Sally on the paved road?

(d) Calculate the time T for $\theta = 90°$. Describe the path taken. Why can't the formula for T be used?

124. Designing Fine Decorative Pieces A designer of decorative art plans to market solid gold spheres encased in clear crystal cones. Each sphere is of fixed radius R and will be enclosed in a cone of height h and radius r. See the illustration. Many cones can be used to enclose the sphere, each having a different slant angle θ. The volume V of the cone can be expressed as a function of the slant angle θ of the cone as

$$V(\theta) = \frac{1}{3}\pi R^3 \frac{(1 + \sec\theta)^3}{(\tan\theta)^2}, \qquad 0° < \theta < 90°$$

What volume V is required to enclose a sphere of radius 2 centimeters in a cone whose slant angle θ is 30°? 45°? 60°?



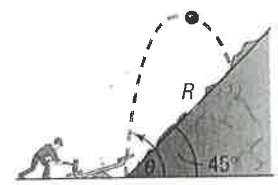
125. Projectile Motion An object is propelled upward at an angle θ, $45° < \theta < 90°$, to the horizontal with an initial velocity of v_0 feet per second from the base of an inclined plane that makes an angle of 45° with the horizontal. See the illustration. If air resistance is ignored, the distance R that it travels up the inclined plane as a function of θ is given by

$$R(\theta) = \frac{v_0^2\sqrt{2}}{32}[\sin(2\theta) - \cos(2\theta) - 1]$$

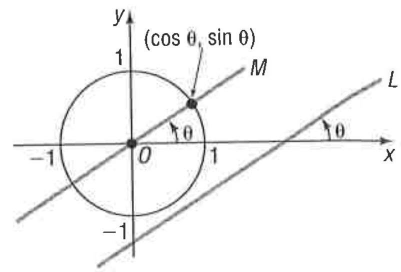
(a) Find the distance R that the object travels along the inclined plane if the initial velocity is 32 feet per second and $\theta = 60°$.

(b) Graph $R = R(\theta)$ if the initial velocity is 32 feet per second.

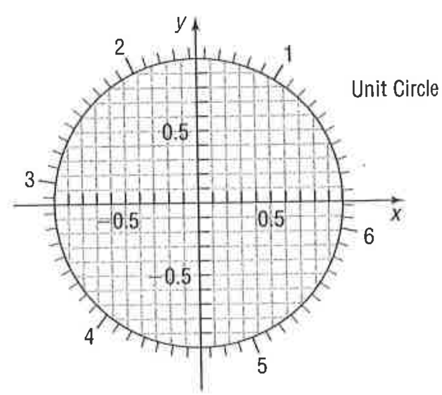
(c) What value of θ makes R largest?



126. If θ, $0 < \theta < \pi$, is the angle between the positive x-axis and a nonhorizontal, nonvertical line L, show that the slope m of L equals $\tan\theta$. The angle θ is called the **inclination** of L. [**Hint:** See the illustration, where we have drawn the line M parallel to L and passing through the origin. Use the fact that M intersects the unit circle at the point $(\cos\theta, \sin\theta)$.]



In Problems 127–128, use the figure to approximate the value of the six trigonometric functions at t to the nearest tenth. Then use a calculator to approximate each of the six trigonometric functions at t.



127. (a) $t = 1$ (b) $t = 5.1$

128. (a) $t = 2$ (b) $t = 4$

Discussion and Writing

129. Write a brief paragraph that explains how to quickly compute the trigonometric functions of 30°, 45°, and 60°.

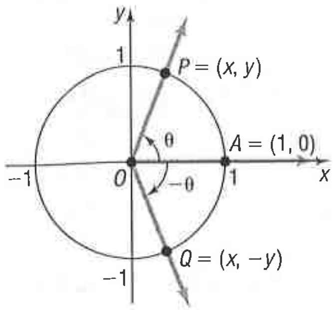
130. Write a brief paragraph that explains how to quickly compute the trigonometric functions of 0°, 90°, 180°, and 270°.

131. How would you explain the meaning of the sine function to a fellow student who has just completed college algebra?

'Are You Prepared?' Answers

1. $c^2 = a^2 + b^2$ **2.** 8 **3.** True **4.** equal; proportional **5.** $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **6.** $-\frac{1}{2}$

Figure 43



Proof Let $P = (x, y)$ be the point on the unit circle that corresponds to the angle θ. See Figure 43. Using symmetry, the point Q on the unit circle that corresponds to the angle $-\theta$ will have coordinates $(x, -y)$. Using the definition of the trigonometric functions, we have

$$\sin\theta = y \qquad \sin(-\theta) = -y \qquad \cos\theta = x \qquad \cos(-\theta) = x$$

so

$$\sin(-\theta) = -y = -\sin\theta \qquad \cos(-\theta) = x = \cos\theta$$

Now, using these results and some of the fundamental identities, we have

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta \qquad \cot(-\theta) = \frac{1}{\tan(-\theta)} = \frac{1}{-\tan\theta} = -\cot\theta$$

$$\sec(-\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos\theta} = \sec\theta \qquad \csc(-\theta) = \frac{1}{\sin(-\theta)} = \frac{1}{-\sin\theta} = -\csc\theta$$

■

EXAMPLE 7 **Finding Exact Values Using Even–Odd Properties**

Find the exact value of:

(a) $\sin(-45°)$ (b) $\cos(-\pi)$ (c) $\cot\left(-\frac{3\pi}{2}\right)$ (d) $\tan\left(-\frac{37\pi}{4}\right)$

Solution

(a) $\sin(-45°) = -\sin 45° = -\frac{\sqrt{2}}{2}$ (Odd function)

(b) $\cos(-\pi) = \cos\pi = -1$ (Even function)

(c) $\cot\left(-\frac{3\pi}{2}\right) = -\cot\frac{3\pi}{2} = 0$ (Odd function)

(d) $\tan\left(-\frac{37\pi}{4}\right) = -\tan\frac{37\pi}{4} = -\tan\left(\frac{\pi}{4} + 9\pi\right) = -\tan\frac{\pi}{4} = -1$ (Odd function; Period is π.)

■

Now Work PROBLEM 59

2.3 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. The domain of the function $f(x) = \frac{x+1}{2x+1}$ is ________. (pp. 22–35)
2. A function for which $f(x) = f(-x)$ for all x in the domain of f is called an ________ function. (pp. 41–43)
3. ***True or False*** The function $f(x) = \sqrt{x}$ is even. (pp. 41–43)
4. ***True or False*** The equation $x^2 + 2x = (x+1)^2 - 1$ is an identity. (p. A25)

Concepts and Vocabulary

5. The sine, cosine, cosecant, and secant functions have period ________; the tangent and cotangent functions have period ________.
6. The domain of the tangent function is ________.
7. The range of the sine function is ________.
8. ***True or False*** The only even trigonometric functions are the cosine and secant functions.
9. ***True or False*** All the trigonometric functions are periodic, with period 2π.
10. ***True or False*** The range of the secant function is the set of positive real numbers.

2.3

Skill Building

In Problems 11–26, use the fact that the trigonometric functions are periodic to find the exact value of each expression. Do not use calculator.

11. $\sin 405°$ **12.** $\cos 420°$ **13.** $\tan 405°$ **14.** $\sin 390°$ **15.** $\csc 450°$ **16.** $\sec 540°$

17. $\cot 390°$ **18.** $\sec 420°$ **19.** $\cos \frac{33\pi}{4}$ **20.** $\sin \frac{9\pi}{4}$ **21.** $\tan(21\pi)$ **22.** $\csc \frac{9\pi}{2}$

23. $\sec \frac{17\pi}{4}$ **24.** $\cot \frac{17\pi}{4}$ **25.** $\tan \frac{19\pi}{6}$ **26.** $\sec \frac{25\pi}{6}$

In Problems 27–34, name the quadrant in which the angle θ lies.

27. $\sin \theta > 0, \quad \cos \theta < 0$ **28.** $\sin \theta < 0, \quad \cos \theta > 0$ **29.** $\sin \theta < 0, \quad \tan \theta < 0$ **30.** $\cos \theta > 0, \quad \tan \theta > 0$

31. $\cos \theta > 0, \quad \tan \theta < 0$ **32.** $\cos \theta < 0, \quad \tan \theta > 0$ **33.** $\sec \theta < 0, \quad \sin \theta > 0$ **34.** $\csc \theta > 0, \quad \cos \theta < 0$

In Problems 35–42, sin θ and cos θ are given. Find the exact value of each of the four remaining trigonometric functions.

35. $\sin \theta = -\frac{3}{5}, \quad \cos \theta = \frac{4}{5}$ **36.** $\sin \theta = \frac{4}{5}, \quad \cos \theta = -\frac{3}{5}$ **37.** $\sin \theta = \frac{2\sqrt{5}}{5}, \quad \cos \theta = \frac{\sqrt{5}}{5}$

38. $\sin \theta = -\frac{\sqrt{5}}{5}, \quad \cos \theta = -\frac{2\sqrt{5}}{5}$ **39.** $\sin \theta = \frac{1}{2}, \quad \cos \theta = \frac{\sqrt{3}}{2}$ **40.** $\sin \theta = \frac{\sqrt{3}}{2}, \quad \cos \theta = \frac{1}{2}$

41. $\sin \theta = -\frac{1}{3}, \quad \cos \theta = \frac{2\sqrt{2}}{3}$ **42.** $\sin \theta = \frac{2\sqrt{2}}{3}, \quad \cos \theta = -\frac{1}{3}$

In Problems 43–58, find the exact value of each of the remaining trigonometric functions of θ.

43. $\sin \theta = \frac{12}{13}$, θ in quadrant II **44.** $\cos \theta = \frac{3}{5}$, θ in quadrant IV **45.** $\cos \theta = -\frac{4}{5}$, θ in quadrant III

46. $\sin \theta = -\frac{5}{13}$, θ in quadrant III **47.** $\sin \theta = \frac{5}{13}, \quad 90° < \theta < 180°$ **48.** $\cos \theta = \frac{4}{5}, \quad 270° < \theta < 360°$

49. $\cos \theta = -\frac{1}{3}, \quad \frac{\pi}{2} < \theta < \pi$ **50.** $\sin \theta = -\frac{2}{3}, \quad \pi < \theta < \frac{3\pi}{2}$ **51.** $\sin \theta = \frac{2}{3}, \quad \tan \theta < 0$

52. $\cos \theta = -\frac{1}{4}, \quad \tan \theta > 0$ **53.** $\sec \theta = 2, \quad \sin \theta < 0$ **54.** $\csc \theta = 3, \quad \cot \theta < 0$

55. $\tan \theta = \frac{3}{4}, \quad \sin \theta < 0$ **56.** $\cot \theta = \frac{4}{3}, \quad \cos \theta < 0$ **57.** $\tan \theta = -\frac{1}{3}, \quad \sin \theta > 0$

58. $\sec \theta = -2, \quad \tan \theta > 0$

In Problems 59–76, use the even–odd properties to find the exact value of each expression. Do not use a calculator.

59. $\sin(-60°)$ **60.** $\cos(-30°)$ **61.** $\tan(-30°)$ **62.** $\sin(-135°)$ **63.** $\sec(-60°)$ **64.** $\csc(-30°)$

65. $\sin(-90°)$ **66.** $\cos(-270°)$ **67.** $\tan\left(-\frac{\pi}{4}\right)$ **68.** $\sin(-\pi)$ **69.** $\cos\left(-\frac{\pi}{4}\right)$ **70.** $\sin\left(-\frac{\pi}{3}\right)$

71. $\tan(-\pi)$ **72.** $\sin\left(-\frac{3\pi}{2}\right)$ **73.** $\csc\left(-\frac{\pi}{4}\right)$ **74.** $\sec(-\pi)$ **75.** $\sec\left(-\frac{\pi}{6}\right)$ **76.** $\csc\left(-\frac{\pi}{3}\right)$

In Problems 77–88, use properties of the trigonometric functions to find the exact value of each expression. Do not use a calculator.

77. $\sin^2 40° + \cos^2 40°$ **78.** $\sec^2 18° - \tan^2 18°$ **79.** $\sin 80° \csc 80°$ **80.** $\tan 10° \cot 10°$

81. $\tan 40° - \frac{\sin 40°}{\cos 40°}$ **82.** $\cot 20° - \frac{\cos 20°}{\sin 20°}$ **83.** $\cos 400° \cdot \sec 40°$ **84.** $\tan 200° \cdot \cot 20°$

85. $\sin\left(-\frac{\pi}{12}\right) \csc \frac{25\pi}{12}$ **86.** $\sec\left(-\frac{\pi}{18}\right) \cdot \cos \frac{37\pi}{18}$ **87.** $\frac{\sin(-20°)}{\cos 380°} + \tan 200°$ **88.** $\frac{\sin 70°}{\cos(-430°)} + \tan(-70$

89. If $\sin \theta = 0.3$, find the value of:
$\sin \theta + \sin(\theta + 2\pi) + \sin(\theta + 4\pi)$

90. If $\cos \theta = 0.2$, find the value of:
$\cos \theta + \cos(\theta + 2\pi) + \cos(\theta + 4\pi)$

91. If $\tan \theta = 3$, find the value of:
$\tan \theta + \tan(\theta + \pi) + \tan(\theta + 2\pi)$

92. If $\cot \theta = -2$, find the value of:
$\cot \theta + \cot(\theta - \pi) + \cot(\theta - 2\pi)$

93. Find the exact value of:
$\sin 1° + \sin 2° + \sin 3° + \cdots + \sin 358° + \sin 359°$

94. Find the exact value of:
$\cos 1° + \cos 2° + \cos 3° + \cdots + \cos 358° + \cos 359°$

95. What is the domain of the sine function?
96. What is the domain of the cosine function?
97. For what numbers θ is $f(\theta) = \tan\theta$ not defined?
98. For what numbers θ is $f(\theta) = \cot\theta$ not defined?
99. For what numbers θ is $f(\theta) = \sec\theta$ not defined?
100. For what numbers θ is $f(\theta) = \csc\theta$ not defined?
101. What is the range of the sine function?
102. What is the range of the cosine function?
103. What is the range of the tangent function?
104. What is the range of the cotangent function?
105. What is the range of the secant function?
106. What is the range of the cosecant function?
107. Is the sine function even, odd, or neither? Is its graph symmetric? With respect to what?
108. Is the cosine function even, odd, or neither? Is its graph symmetric? With respect to what?
109. Is the tangent function even, odd, or neither? Is its graph symmetric? With respect to what?
110. Is the cotangent function even, odd, or neither? Is its graph symmetric? With respect to what?
111. Is the secant function even, odd, or neither? Is its graph symmetric? With respect to what?
112. Is the cosecant function even, odd, or neither? Is its graph symmetric? With respect to what?

Applications and Extensions

In Problems 113–118, use the periodic and even–odd properties.

113. If $f(\theta) = \sin\theta$ and $f(a) = \frac{1}{3}$, find the exact value of:
(a) $f(-a)$ (b) $f(a) + f(a + 2\pi) + f(a + 4\pi)$

114. If $f(\theta) = \cos\theta$ and $f(a) = \frac{1}{4}$, find the exact value of:
(a) $f(-a)$ (b) $f(a) + f(a + 2\pi) + f(a - 2\pi)$

115. If $f(\theta) = \tan\theta$ and $f(a) = 2$, find the exact value of:
(a) $f(-a)$ (b) $f(a) + f(a + \pi) + f(a + 2\pi)$

116. If $f(\theta) = \cot\theta$ and $f(a) = -3$, find the exact value of:
(a) $f(-a)$ (b) $f(a) + f(a + \pi) + f(a + 4\pi)$

117. If $f(\theta) = \sec\theta$ and $f(a) = -4$, find the exact value of:
(a) $f(-a)$ (b) $f(a) + f(a + 2\pi) + f(a + 4\pi)$

118. If $f(\theta) = \csc\theta$ and $f(a) = 2$, find the exact value of:
(a) $f(-a)$ (b) $f(a) + f(a + 2\pi) + f(a + 4\pi)$

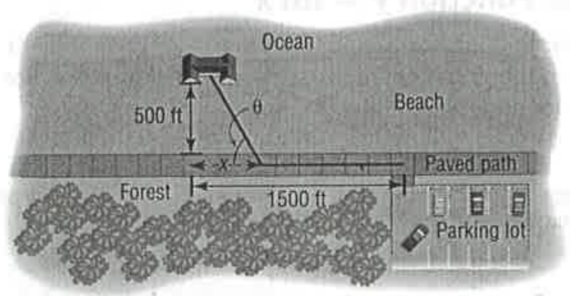
119. **Calculating the Time of a Trip** From a parking lot, you want to walk to a house on the beach. The house is located 1500 feet down a paved path that parallels the ocean, which is 500 feet away. See the illustration. Along the path you can walk 300 feet per minute, but in the sand on the beach you can only walk 100 feet per minute.

The time T to get from the parking lot to the beach house can be expressed as a function of the angle θ shown in the illustration and is

$$T(\theta) = 5 - \frac{5}{3\tan\theta} + \frac{5}{\sin\theta}, \quad 0 < \theta < \frac{\pi}{2}$$

Calculate the time T if you walk directly from the parking lot to the house.

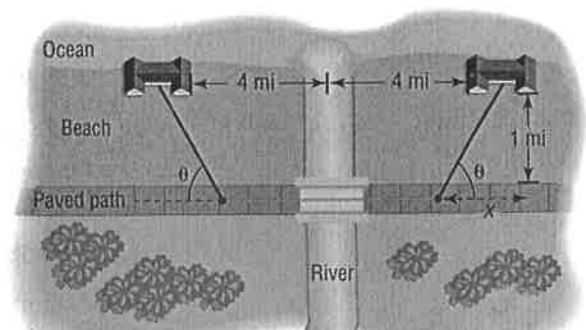
[**Hint:** $\tan\theta = \frac{500}{1500}$.]



120. **Calculating the Time of a Trip** Two oceanfront homes are located 8 miles apart on a straight stretch of beach, each a distance of 1 mile from a paved road that parallels the ocean. Sally can jog 8 miles per hour along the paved road, but only 3 miles per hour in the sand on the beach. Because a river flows directly between the two houses, it is necessary to jog in the sand to the road, continue on the road, and then jog directly back in the sand to get from one house to the other. See the illustration. The time T to get from one house to the other as a function of the angle θ shown in the illustration is

$$T(\theta) = 1 + \frac{2}{3\sin\theta} - \frac{1}{4\tan\theta} \qquad 0 < \theta < \frac{\pi}{2}$$

(a) Calculate the time T for $\tan\theta = \frac{1}{4}$.
(b) Describe the path taken.
(c) Explain why θ must be larger than 14°.

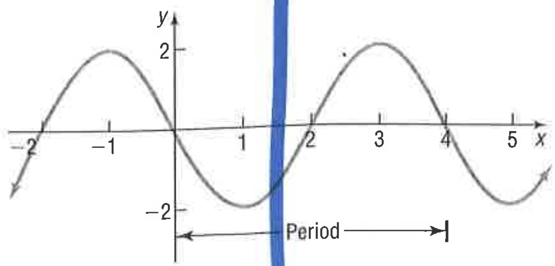


121. Show that the range of the tangent function is the set of all real numbers.
122. Show that the range of the cotangent function is the set of all real numbers.
123. Show that the period of $f(\theta) = \sin\theta$ is 2π.
[**Hint:** Assume that $0 < p < 2\pi$ exists so that $\sin(\theta + p) = \sin\theta$ for all θ. Let $\theta = 0$ to find p. Then let $\theta = \frac{\pi}{2}$ to obtain a contradiction.]
124. Show that the period of $f(\theta) = \cos\theta$ is 2π.
125. Show that the period of $f(\theta) = \sec\theta$ is 2π.
126. Show that the period of $f(\theta) = \csc\theta$ is 2π.

EXAMPLE 9 **Finding an Equation for a Sinusoidal Graph**

Find an equation for the graph shown in Figure 61.

Figure 61



Solution The graph is sinusoidal, with amplitude $|A| = 2$. The period is 4, so $\frac{2\pi}{\omega} = 4$ or $\omega = \frac{\pi}{2}$. Since the graph passes through the origin, it is easiest to view the equation as a sine function,* but notice that the graph is actually the reflection of a sine function about the x-axis (since the graph is decreasing near the origin). This requires that $A = -2$. The sine function whose graph is given in Figure 61 is

$$y = A\sin(\omega x) = -2\sin\left(\frac{\pi}{2}x\right)$$

✓Check: Graph $Y_1 = -2\sin\left(\frac{\pi}{2}x\right)$ and compare the result with Figure 61.

Now Work PROBLEMS 67 AND 71

2.4 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. Use transformations to graph $y = 3x^2$ (pp. 57–66)

2. Use transformations to graph $y = -x^2$. (pp. 57–66)

Concepts and Vocabulary

3. The maximum value of $y = \sin x, 0 \le x \le 2\pi$, is ______ and occurs at $x =$ ______.

4. The function $y = A\sin(\omega x)$, $A > 0$, has amplitude 3 and period 2; then $A =$ ______ and $\omega =$ ______.

5. The function $y = 3\cos(6x)$ has amplitude ______ and period ______.

6. ***True or False*** The graphs of $y = \sin x$ and $y = \cos x$ are identical except for a horizontal shift.

7. ***True or False*** For $y = 2\sin(\pi x)$, the amplitude is 2 and the period is $\frac{\pi}{2}$.

8. ***True or False*** The graph of the sine function has infinitely many x-intercepts.

Skill Building

In Problems 9–18, if necessary, refer to a graph to answer each question.

9. What is the y-intercept of $y = \sin x$?

10. What is the y-intercept of $y = \cos x$?

11. For what numbers x, $-\pi \le x \le \pi$, is the graph of $y = \sin x$ increasing?

12. For what numbers x, $-\pi \le x \le \pi$, is the graph of $y = \cos x$ decreasing?

* The equation could also be viewed as a cosine function with a horizontal shift, but viewing it as a sine function is easier.

2·4

13. What is the largest value of $y = \sin x$?

14. What is the smallest value of $y = \cos x$?

15. For what numbers $x, 0 \leq x \leq 2\pi$, does $\sin x = 0$?

16. For what numbers $x, 0 \leq x \leq 2\pi$, does $\cos x = 0$?

17. For what numbers $x, -2\pi \leq x \leq 2\pi$, does $\sin x = 1$? Where does $\sin x = -1$?

18. For what numbers $x, -2\pi \leq x \leq 2\pi$, does $\cos x = 1$? Where does $\cos x = -1$?

In Problems 19–28, determine the amplitude and period of each function without graphing.

19. $y = 2 \sin x$

20. $y = 3 \cos x$

21. $y = 4 \cos(2x)$

22. $y = -\sin\left(\frac{1}{2}x\right)$

23. $y = 6 \sin(\pi x)$

24. $y = -3 \cos(3x)$

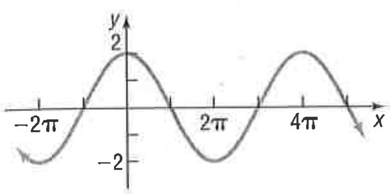
25. $y = -\frac{1}{2}\cos\left(\frac{3}{2}x\right)$

26. $y = \frac{4}{3}\sin\left(\frac{2}{3}x\right)$

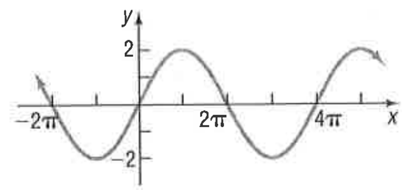
27. $y = \frac{5}{3}\sin\left(-\frac{2\pi}{3}x\right)$

28. $y = \frac{9}{5}\cos\left(-\frac{3\pi}{2}x\right)$

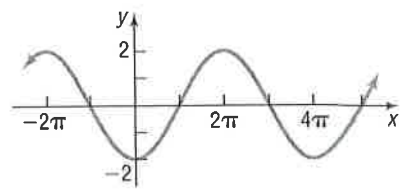
In Problems 29–38, match the given function to one of the graphs (A)–(J).



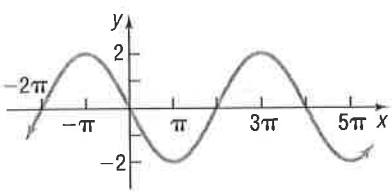
(A)



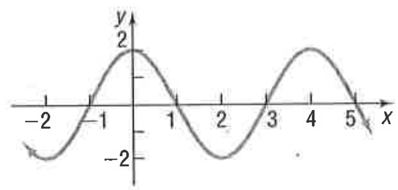
(B)



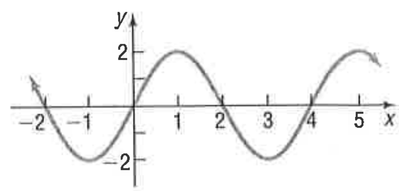
(C)



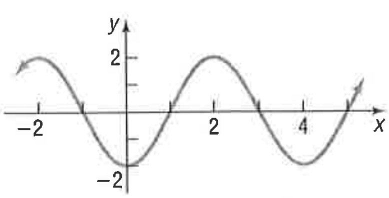
(D)



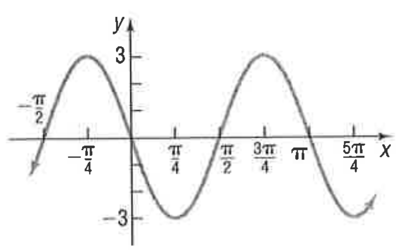
(E)



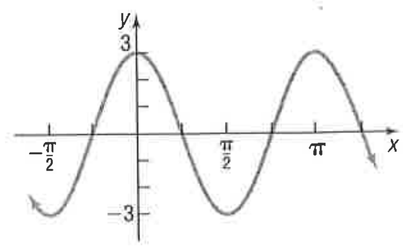
(F)



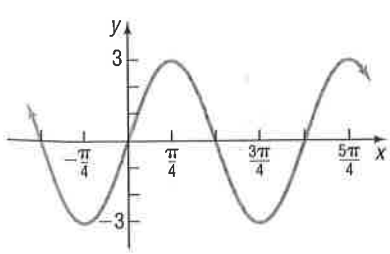
(G)



(H)



(I)



(J)

29. $y = 2 \sin\left(\frac{\pi}{2}x\right)$

30. $y = 2 \cos\left(\frac{\pi}{2}x\right)$

31. $y = 2 \cos\left(\frac{1}{2}x\right)$

32. $y = 3 \cos(2x)$

33. $y = -3 \sin(2x)$

34. $y = 2 \sin\left(\frac{1}{2}x\right)$

35. $y = -2 \cos\left(\frac{1}{2}x\right)$

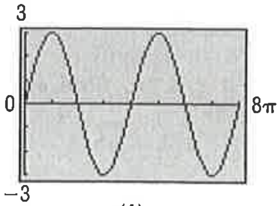
36. $y = -2 \cos\left(\frac{\pi}{2}x\right)$

37. $y = 3 \sin(2x)$

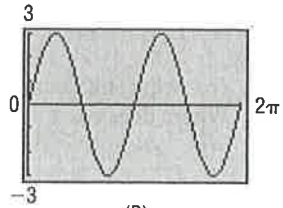
38. $y = -2 \sin\left(\frac{1}{2}x\right)$

2.4

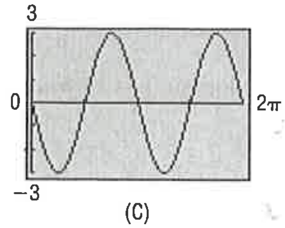
In Problems 39–42, match the given function to one of the graphs (A)–(D).



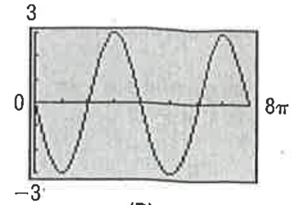
(A)



(B)



(C)



(D)

39. $y = 3\sin\left(\frac{1}{2}x\right)$ **40.** $y = -3\sin(2x)$ **41.** $y = 3\sin(2x)$ **42.** $y = -3\sin\left(\frac{1}{2}x\right)$

In Problems 43–66, graph each function. Be sure to label key points and show at least two cycles.

43. $y = 4\cos x$ **44.** $y = 3\sin x$ **45.** $y = -4\sin x$ **46.** $y = -3\cos x$

47. $y = \cos(4x)$ **48.** $y = \sin(3x)$ **49.** $y = \sin(-2x)$ **50.** $y = \cos(-2x)$

51. $y = 2\sin\left(\frac{1}{2}x\right)$ **52.** $y = 2\cos\left(\frac{1}{4}x\right)$ **53.** $y = -\frac{1}{2}\cos(2x)$ **54.** $y = -4\sin\left(\frac{1}{8}x\right)$

55. $y = 2\sin x + 3$ **56.** $y = 3\cos x + 2$ **57.** $y = 5\cos(\pi x) - 3$ **58.** $y = 4\sin\left(\frac{\pi}{2}x\right) - 2$

59. $y = -6\sin\left(\frac{\pi}{3}x\right) + 4$ **60.** $y = -3\cos\left(\frac{\pi}{4}x\right) + 2$ **61.** $y = 5 - 3\sin(2x)$ **62.** $y = 2 - 4\cos(3x)$

63. $y = \frac{5}{3}\sin\left(-\frac{2\pi}{3}x\right)$ **64.** $y = \frac{9}{5}\cos\left(-\frac{3\pi}{2}x\right)$ **65.** $y = -\frac{3}{2}\cos\left(\frac{\pi}{4}x\right) + \frac{1}{2}$ **66.** $y = -\frac{1}{2}\sin\left(\frac{\pi}{8}x\right) + \frac{3}{2}$

In Problems 67–70, write the equation of a sine function that has the given characteristics.

67. Amplitude: 3
Period: π

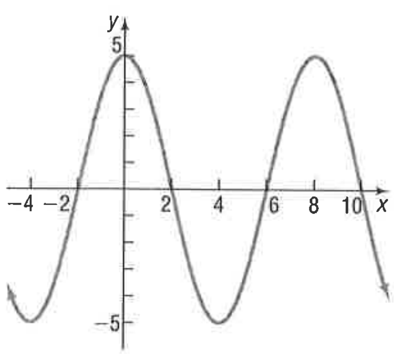
68. Amplitude: 2
Period: 4π

69. Amplitude: 3
Period: 2

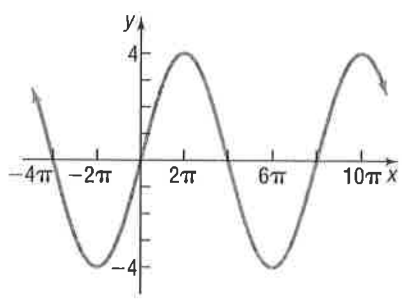
70. Amplitude: 4
Period: 1

In Problems 71–84, find an equation for each graph.

71.



72.



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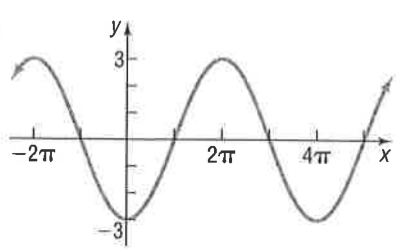

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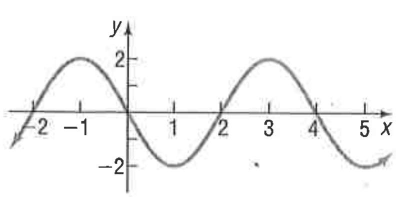

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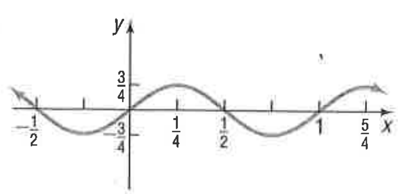
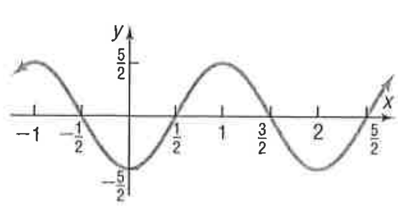
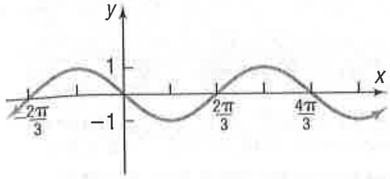


76.

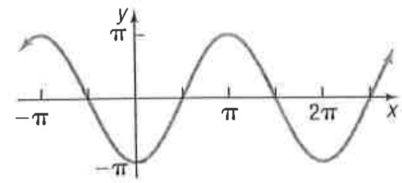


2.4

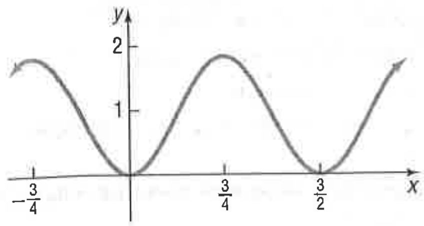
77.



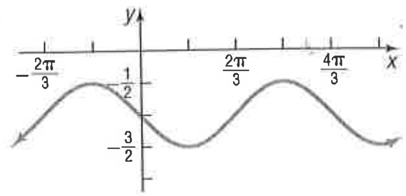
78.



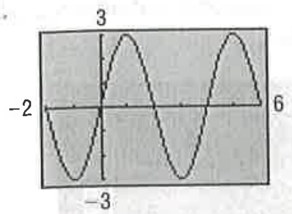
79.



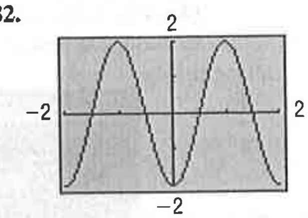
80.



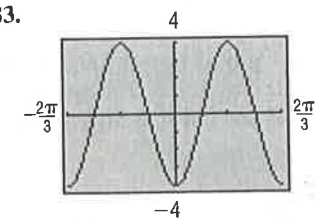
81.



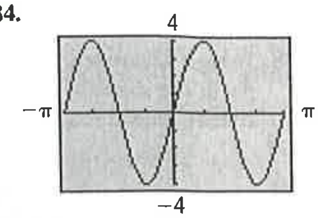
82.



83.



84.



Applications and Extensions

In Problems 85–88, find the average rate of change of f from 0 to $\frac{\pi}{2}$.

85. $f(x) = \sin x$

86. $f(x) = \cos x$

87. $f(x) = \sin\left(\frac{x}{2}\right)$

88. $f(x) = \cos(2x)$

In Problems 89–92, find $(f \circ g)(x)$ and $(g \circ f)(x)$ and graph each of these functions.

89. $f(x) = \sin x$
$g(x) = 4x$

90. $f(x) = \cos x$
$g(x) = \frac{1}{2}x$

91. $f(x) = -2x$
$g(x) = \cos x$

92. $f(x) = -3x$
$g(x) = \sin x$

93. **Alternating Current (ac) Circuits** The current I, in amperes, flowing through an ac (alternating current) circuit at time t in seconds, is

$$I(t) = 220\sin(60\pi t) \qquad t \geq 0$$

What is the period? What is the amplitude? Graph this function over two periods.

94. **Alternating Current (ac) Circuits** The current I, in amperes, flowing through an ac (alternating current) circuit at time t, in seconds, is

$$I(t) = 120\sin(30\pi t) \qquad t \geq 0$$

What is the period? What is the amplitude? Graph this function over two periods.

95. **Alternating Current (ac) Generators** The voltage V, in volts, produced by an ac generator at time t, in seconds, is

$$V(t) = 220\sin(120\pi t)$$

(a) What is the amplitude? What is the period?
(b) Graph V over two periods, beginning at $t = 0$.
(c) If a resistance of $R = 10$ ohms is present, what is the current I?
[**Hint:** Use Ohm's Law, $V = IR$.]
(d) What is the amplitude and period of the current I?
(e) Graph I over two periods, beginning at $t = 0$.

96. **Alternating Current (ac) Generators** The voltage V, in volts, produced by an ac generator at time t, in seconds, is

$$V(t) = 120\sin(120\pi t)$$

(a) What is the amplitude? What is the period?
(b) Graph V over two periods, beginning at $t = 0$.
(c) If a resistance of $R = 20$ ohms is present, what is the current I?
[**Hint:** Use Ohm's Law, $V = IR$.]
(d) What is the amplitude and period of the current I?
(e) Graph I over two periods, beginning at $t = 0$.

97. **Alternating Current (ac) Generators** The voltage V produced by an ac generator is sinusoidal. As a function of time, the voltage V is

$$V(t) = V_0\sin(2\pi f t)$$

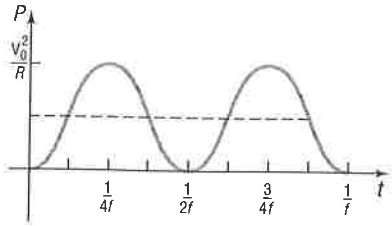
where f is the **frequency,** the number of complete oscillations (cycles) per second. [In the United States and Canada, f is 60 hertz (Hz).] The **power** P delivered to a resistance R at any time t is defined as

$$P(t) = \frac{[V(t)]^2}{R}$$

2.4

(a) Show that $P(t) = \frac{V_0^2}{R}\sin^2(2\pi f t)$.

(b) The graph of P is shown in the figure. Express P as a sinusoidal function.

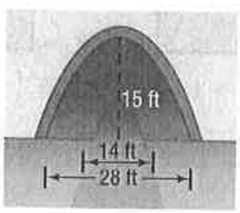


Power in an ac generator

(c) Deduce that

$$\sin^2(2\pi f t) = \frac{1}{2}[1 - \cos(4\pi f t)]$$

98. Bridge Clearance A one-lane highway runs through a tunnel in the shape of one-half a sine curve cycle. The opening is 28 feet wide at road level and is 15 feet tall at its highest point.



(a) Find an equation for the sine curve that fits the opening. Place the origin at the left end of the sine curve.

(b) If the road is 14 feet wide with 7-foot shoulders on each side, what is the height of the tunnel at the edge of the road?

Sources: *en.wikipedia.org/wiki/Interstate_Highway_standards* and *Ohio Revised Code*

99. Biorhythms In the theory of biorhythms, a sine function of the form

$$P(t) = 50\sin(\omega t) + 50$$

is used to measure the percent P of a person's potential at time t, where t is measured in days and $t = 0$ is the person's birthday. Three characteristics are commonly measured:

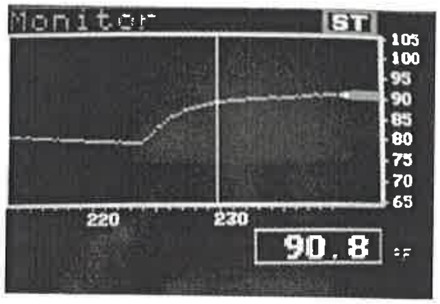
Physical potential: period of 23 days
Emotional potential: period of 28 days
Intellectual potential: period of 33 days

(a) Find ω for each characteristic.

(b) Using a graphing utility, graph all three functions on the same screen.

(c) Is there a time t when all three characteristics have 100% potential? When is it?

(d) Suppose that you are 20 years old today ($t = 7305$ days). Describe your physical, emotional, and intellectual potential for the next 30 days.



100. Graph $y = |\cos x|$, $-2\pi \le x \le 2\pi$.

101. Graph $y = |\sin x|$, $-2\pi \le x \le 2\pi$.

Discussion and Writing

102. Explain how you would scale the x-axis and y-axis before graphing $y = 3\cos(\pi x)$.

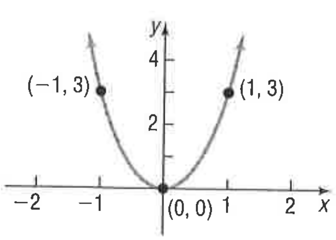
103. Explain the term *amplitude* as it relates to the graph of a sinusoidal function.

104. Explain how the amplitude and period of a sinusoidal graph are used to establish the scale on each coordinate axis.

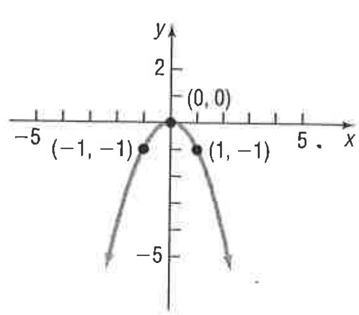
105. Find an application in your major field that leads to a sinusoidal graph. Write a paper about your findings.

'Are You Prepared?' Answers

1. Vertical stretch by a factor of 3

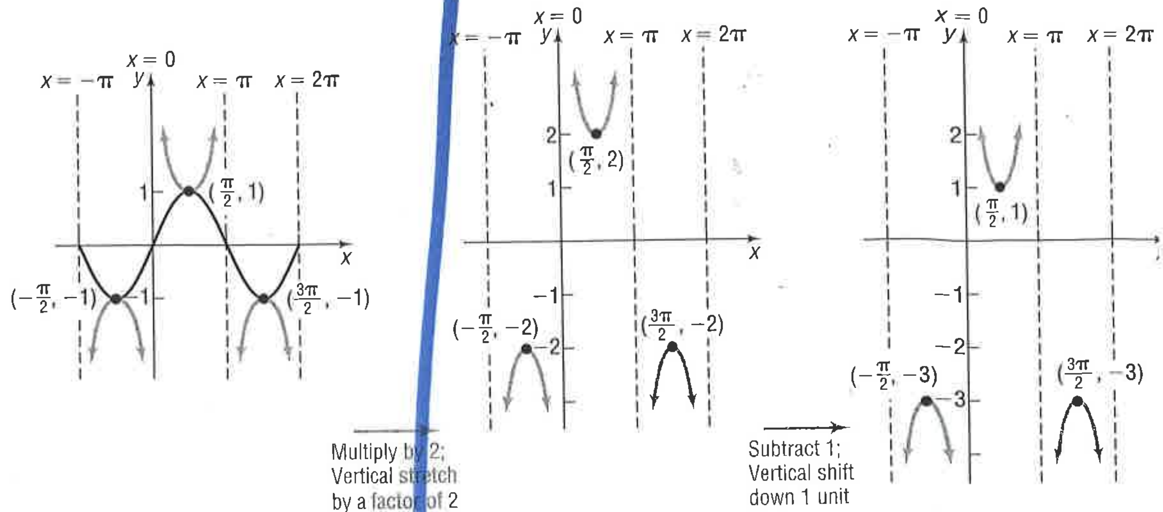


2. Reflection about the x-axis



Solution Using Transformations Figure 69 shows the required steps.

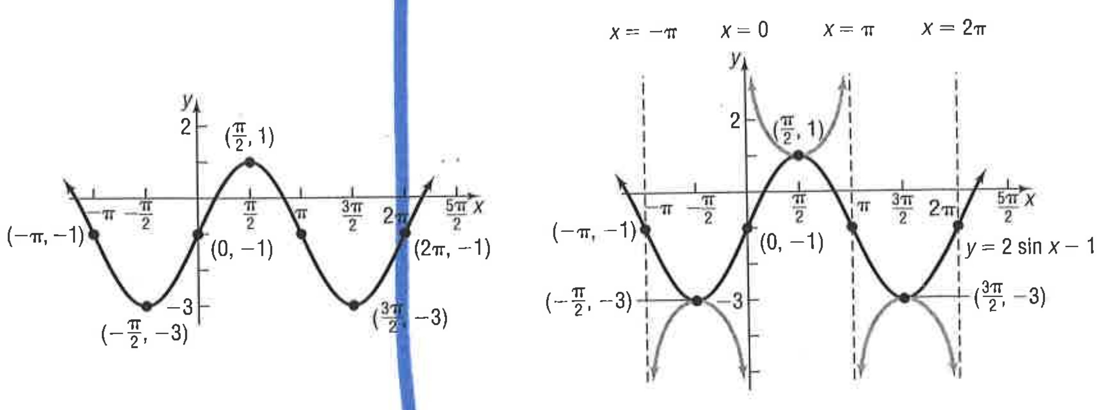
Figure 69



(a) $y = \csc x$ (b) $y = 2\csc x$ (c) $y = 2\csc x - 1$

Solution Using the Reciprocal Function We graph $y = 2\csc x - 1$ by first graphing the reciprocal function $y = 2\sin x -$ and then filling in the graph of $y = 2\csc x - 1$, using the idea of reciprocals. Se Figure 70.

Figure 70



(a) $y = 2\sin x - 1$ (b) $y = 2\csc x - 1$

Now Work PROBLEM 29

2.5 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. The graph of $y = \dfrac{3x - 6}{x - 4}$ has a vertical asymptote. What is it? (pp. 66–67)

2. ***True or False*** A function f has at most one vertical asympto (pp. 66–67)

Concepts and Vocabulary

3. The graph of $y = \tan x$ is symmetric with respect to the _____ and has vertical asymptotes at _____.

4. The graph of $y = \sec x$ is symmetric with respect to the _____ and has vertical asymptotes at _____.

5. It is easiest to graph $y = \sec x$ by first sketching the gra of _____.

6. ***True or False*** The graphs of $y = \tan x$, $y = \cot x$, $y = \sec$ and $y = \csc x$ each have infinitely many vertical asymptote

Skill Building

In Problems 7–16, if necessary, refer to the graphs to answer each question.

7. What is the y-intercept of $y = \tan x$?

8. What is the y-intercept of $y = \cot x$?

9. What is the y-intercept of $y = \sec x$?

10. What is the y-intercept of $y = \csc x$?

11. For what numbers x, $-2\pi \le x \le 2\pi$, does $\sec x = 1$? For what numbers x does $\sec x = -1$?

12. For what numbers x, $-2\pi \le x \le 2\pi$, does $\csc x = 1$? For what numbers x does $\csc x = -1$?

13. For what numbers x, $-2\pi \le x \le 2\pi$, does the graph of $y = \sec x$ have vertical asymptotes?

14. For what numbers x, $-2\pi \le x \le 2\pi$, does the graph of $y = \csc x$ have vertical asymptotes?

15. For what numbers x, $-2\pi \le x \le 2\pi$, does the graph of $y = \tan x$ have vertical asymptotes?

16. For what numbers x, $-2\pi \le x \le 2\pi$, does the graph of $y = \cot x$ have vertical asymptotes?

In Problems 17–40, graph each function. Be sure to label key points and show at least two cycles.

17. $y = 3\tan x$
18. $y = -2\tan x$
19. $y = 4\cot x$
20. $y = -3\cot x$
21. $y = \tan\left(\frac{\pi}{2}x\right)$
22. $y = \tan\left(\frac{1}{2}x\right)$
23. $y = \cot\left(\frac{1}{4}x\right)$
24. $y = \cot\left(\frac{\pi}{4}x\right)$
25. $y = 2\sec x$
26. $y = \frac{1}{2}\csc x$
27. $y = -3\csc x$
28. $y = -4\sec x$
29. $y = 4\sec\left(\frac{1}{2}x\right)$
30. $y = \frac{1}{2}\csc(2x)$
31. $y = -2\csc(\pi x)$
32. $y = -3\sec\left(\frac{\pi}{2}x\right)$
33. $y = \tan\left(\frac{1}{4}x\right) + 1$
34. $y = 2\cot x - 1$
35. $y = \sec\left(\frac{2\pi}{3}x\right) + 2$
36. $y = \csc\left(\frac{3\pi}{2}x\right)$
37. $y = \frac{1}{2}\tan\left(\frac{1}{4}x\right) - 2$
38. $y = 3\cot\left(\frac{1}{2}x\right) - 2$
39. $y = 2\csc\left(\frac{1}{3}x\right) - 1$
40. $y = 3\sec\left(\frac{1}{4}x\right) + 1$

Applications and Extensions

In Problems 41–44, find the average rate of change of f from 0 to $\frac{\pi}{6}$.

41. $f(x) = \tan x$
42. $f(x) = \sec x$
43. $f(x) = \tan(2x)$
44. $f(x) = \sec(2x)$

In Problems 45–48, find $(f \circ g)(x)$ and $(g \circ f)(x)$ and graph each of these functions.

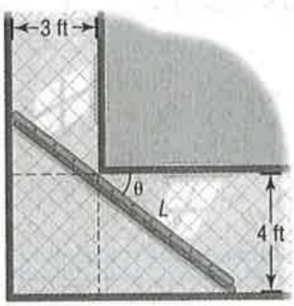
45. $f(x) = \tan x$
$g(x) = 4x$

46. $f(x) = 2\sec x$
$g(x) = \frac{1}{2}x$

47. $f(x) = -2x$
$g(x) = \cot x$

48. $f(x) = \frac{1}{2}x$
$g(x) = 2\csc x$

49. **Carrying a Ladder around a Corner** Two hallways, one of width 3 feet, the other of width 4 feet, meet at a right angle. See the illustration.



(a) Show that the length L of the line segment shown as a function of the angle θ is

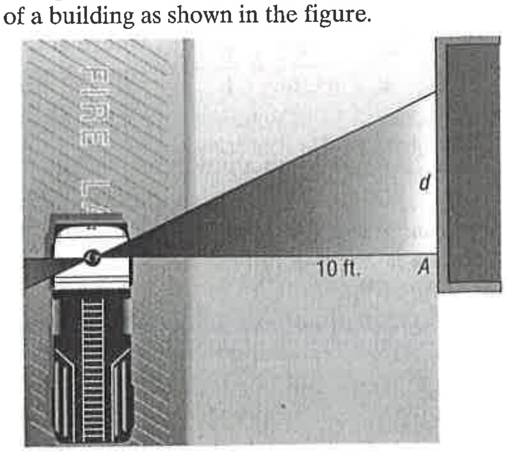
$$L(\theta) = 3\sec\theta + 4\csc\theta$$

(b) Graph $L = L(\theta)$, $0 < \theta < \frac{\pi}{2}$.

(c) For what value of θ is L the least?

(d) What is the length of the longest ladder that can be carried around the corner? Why is this also the least value of L?

50. **A Rotating Beacon** Suppose that a fire truck is parked in front of a building as shown in the figure.



The beacon light on top of the fire truck is located 10 feet from the wall and has a light on each side. If the beacon light rotates 1 revolution every 2 seconds, then a model for determining the distance d that the beacon of light is from point A on the wall after t seconds is given by

$$d(t) = |10\tan(\pi t)|$$

(a) Graph $d(t) = |10\tan(\pi t)|$ for $0 \le t \le 2$.

The output that the utility provides shows the equation

$$y = a\sin(bx + c) + d$$

The sinusoidal function of best fit is

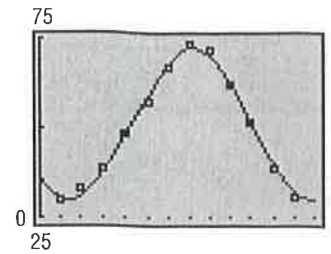
$$y = 21.15\sin(0.55x - 2.35) + 51.19$$

where x represents the month and y represents the average temperature.

Figure 85 shows the graph of the sinusoidal function of best fit on the scatter diagram.

Figure 84

Figure 85



Now Work PROBLEMS 29(d) AND (e)

2.6 Assess Your Understanding

Concepts and Vocabulary

1. For the graph of $y = A\sin(\omega x - \phi)$, the number $\frac{\phi}{\omega}$ is called the ______.

2. ***True or False*** Only two data points are required by a graphing utility to find the sine function of best fit.

Skill Building

In Problems 3–14, find the amplitude, period, and phase shift of each function. Graph each function. Be sure to label key points. Show at least two periods.

3. $y = 4\sin(2x - \pi)$
4. $y = 3\sin(3x - \pi)$
5. $y = 2\cos\left(3x + \frac{\pi}{2}\right)$
6. $y = 3\cos(2x + \pi)$
7. $y = -3\sin\left(2x + \frac{\pi}{2}\right)$
8. $y = -2\cos\left(2x - \frac{\pi}{2}\right)$
9. $y = 4\sin(\pi x + 2) - 5$
10. $y = 2\cos(2\pi x + 4) + 4$
11. $y = 3\cos(\pi x - 2) + 5$
12. $y = 2\cos(2\pi x - 4) - 1$
13. $y = -3\sin\left(-2x + \frac{\pi}{2}\right)$
14. $y = -3\cos\left(-2x + \frac{\pi}{2}\right)$

In Problems 15–18, write the equation of a sine function that has the given characteristics.

15. Amplitude: 2
Period: π
Phase shift: $\frac{1}{2}$

16. Amplitude: 3
Period: $\frac{\pi}{2}$
Phase shift: 2

17. Amplitude: 3
Period: 3π
Phase shift: $-\frac{1}{3}$

18. Amplitude: 2
Period: π
Phase shift: -2

Applications and Extensions

In Problems 19–26, apply the methods of this and the previous section to graph each function. Be sure to label key points and show at least two periods.

19. $y = 2\tan(4x - \pi)$
20. $y = \frac{1}{2}\cot(2x - \pi)$
21. $y = 3\csc\left(2x - \frac{\pi}{4}\right)$
22. $y = \frac{1}{2}\sec(3x - \pi)$
23. $y = -\cot\left(2x + \frac{\pi}{2}\right)$
24. $y = -\tan\left(3x + \frac{\pi}{2}\right)$
25. $y = -\sec(2\pi x + \pi)$
26. $y = -\csc\left(-\frac{1}{2}\pi x + \frac{\pi}{4}\right)$

2·6

27. Alternating Current (ac) Circuits The current I, in amperes, flowing through an ac (alternating current) circuit at time t, in seconds, is

$$I(t) = 120 \sin\left(30\pi t - \frac{\pi}{3}\right), \qquad t \geq 0$$

What is the period? What is the amplitude? What is the phase shift? Graph this function over two periods.

28. Alternating Current (ac) Circuits The current I, in amperes, flowing through an ac (alternating current) circuit at time t, in seconds, is

$$I(t) = 220 \sin\left(60\pi t - \frac{\pi}{6}\right), \qquad t \geq 0$$

What is the period? What is the amplitude? What is the phase shift? Graph this function over two periods.

29. Monthly Temperature The following data represent the average monthly temperatures for Juneau, Alaska.

Month, x	Average Monthly Temperature, °F
January, 1	24.2
February, 2	28.4
March, 3	32.7
April, 4	39.7
May, 5	47.0
June, 6	53.0
July, 7	56.0
August, 8	55.0
September, 9	49.4
October, 10	42.2
November, 11	32.0
December, 12	27.1

SOURCE: U.S. National Oceanic and Atmospheric Administration

(a) Draw a scatter diagram of the data for one period.
(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.
(c) Draw the sinusoidal function found in part (b) on the scatter diagram.
(d) Use a graphing utility to find the sinusoidal function of best fit.
(e) Draw the sinusoidal function of best fit on a scatter diagram of the data.

30. Monthly Temperature The following data represent the average monthly temperatures for Washington, D.C.

(a) Draw a scatter diagram of the data for one period.
(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.
(c) Draw the sinusoidal function found in part (b) on the scatter diagram.
(d) Use a graphing utility to find the sinusoidal function of best fit.
(e) Graph the sinusoidal function of best fit on a scatter diagram of the data.

Month, x	Average Monthly Temperature, °F
January, 1	34.6
February, 2	37.5
March, 3	47.2
April, 4	56.5
May, 5	66.4
June, 6	75.6
July, 7	80.0
August, 8	78.5
September, 9	71.3
October, 10	59.7
November, 11	49.8
December, 12	39.4

SOURCE: U.S. National Oceanic and Atmospheric Administration

31. Monthly Temperature The following data represent the average monthly temperatures for Indianapolis, Indiana.

Month, x	Average Monthly Temperature, °F
January, 1	25.5
February, 2	29.6
March, 3	41.4
April, 4	52.4
May, 5	62.8
June, 6	71.9
July, 7	75.4
August, 8	73.2
September, 9	66.6
October, 10	54.7
November, 11	43.0
December, 12	30.9

SOURCE: U.S. National Oceanic and Atmospheric Administration

(a) Draw a scatter diagram of the data for one period.
(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.
(c) Draw the sinusoidal function found in part (b) on the scatter diagram.
(d) Use a graphing utility to find the sinusoidal function of best fit.
(e) Graph the sinusoidal function of best fit on a scatter diagram of the data.

32. Monthly Temperature The data on the following page represent the average monthly temperatures for Baltimore, Maryland.

(a) Draw a scatter diagram of the data for one period.

2.6

Month, x	Average Monthly Temperature, °F
January, 1	31.8
February, 2	34.8
March, 3	44.1
April, 4	53.4
May, 5	63.4
June, 6	72.5
July, 7	77.0
August, 8	75.6
September, 9	68.5
October, 10	56.6
November, 11	46.8
December, 12	36.7

SOURCE: U.S. National Oceanic and Atmospheric Administration

(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.

(c) Draw the sinusoidal function found in part (b) on the scatter diagram.

(d) Use a graphing utility to find the sinusoidal function of best fit.

(e) Graph the sinusoidal function of best fit on a scatter diagram of the data.

33. Tides Suppose that the length of time between consecutive high tides is approximately 12.5 hours. According to the National Oceanic and Atmospheric Administration, on Saturday, August 7, 2004, in Savannah, Georgia, high tide occurred at 3:38 AM (3.6333 hours) and low tide occurred at 10:08 AM (10.1333 hours). Water heights are measured as the amounts above or below the mean lower low water. The height of the water at high tide was 8.2 feet, and the height of the water at low tide was -0.6 foot.

(a) Approximately when will the next high tide occur?

(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.

(c) Draw a graph of the function found in part (b).

(d) Use the function found in part (b) to predict the height of the water at the next high tide.

34. Tides Suppose that the length of time between consecutive high tides is approximately 12.5 hours. According to the National Oceanic and Atmospheric Administration, on Saturday, August 7, 2004, in Juneau, Alaska, high tide occurred at 8:11 AM (8.1833 hours) and low tide occurred at 2:14 PM (14.2333 hours). Water heights are measured as the amounts above or below the mean lower low water. The height of the water at high tide was 13.2 feet, and the height of the water at low tide was 2.2 feet.

(a) Approximately when will the next high tide occur?

(b) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.

(c) Draw a graph of the function found in part (b).

(d) Use the function found in part (b) to predict the height of the water at the next high tide.

35. Hours of Daylight According to the *Old Farmer's Almanac,* in Miami, Florida, the number of hours of sunlight on the summer solstice of 2005 is 13.75 and the number of hours of sunlight on the winter solstice is 10.53.

(a) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.

(b) Use the function found in part (a) to predict the number of hours of sunlight on April 1, the 91st day of the year.

(c) Draw a graph of the function found in part (a).

(d) Look up the number of hours of sunlight for April 1 in the *Old Farmer's Almanac,* and compare the actual hours of daylight to the results found in part (c).

36. Hours of Daylight According to the *Old Farmer's Almanac,* in Detroit, Michigan, the number of hours of sunlight on the summer solstice of 2005 is 15.30 and the number of hours of sunlight on the winter solstice is 9.08.

(a) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.

(b) Use the function found in part (a) to predict the number of hours of sunlight on April 1, the 91st day of the year.

(c) Draw a graph of the function found in part (a).

(d) Look up the number of hours of sunlight for April 1 in the *Old Farmer's Almanac,* and compare the actual hours of daylight to the results found in part (c).

37. Hours of Daylight According to the *Old Farmer's Almanac,* in Anchorage, Alaska, the number of hours of sunlight on the summer solstice of 2005 is 19.42 and the number of hours of sunlight on the winter solstice is 5.47.

(a) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.

(b) Use the function found in part (a) to predict the number of hours of sunlight on April 1, the 91st day of the year.

(c) Draw a graph of the function found in part (a).

(d) Look up the number of hours of sunlight for April 1 in the *Old Farmer's Almanac,* and compare the actual hours of daylight to the results found in part (c).

38. Hours of Daylight According to the *Old Farmer's Almanac,* in Honolulu, Hawaii, the number of hours of sunlight on the summer solstice of 2005 is 13.43 and the number of hours of sunlight on the winter solstice is 10.85.

(a) Find a sinusoidal function of the form $y = A\sin(\omega x - \phi) + B$ that fits the data.

(b) Use the function found in part (a) to predict the number of hours of sunlight on April 1, the 91st day of the year.

(c) Draw a graph of the function found in part (a).

(d) Look up the number of hours of sunlight for April 1 in the *Old Farmer's Almanac,* and compare the actual hours of daylight to the results found in part (c).

Discussion and Writing

39. Explain how the amplitude and period of a sinusoidal graph are used to establish the scale on each coordinate axis.

40. Find an application in your major field that leads to a sinusoidal graph. Write a paper about your findings.