Chapter 1

1.1 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. On the real number line the origin is assigned the number _____. (pp. A1–A10)
2. If -3 and 5 are the coordinates of two points on the real number line, the distance between these points is _____. (pp. A1–A10)
3. If 3 and 4 are the legs of a right triangle, the hypotenuse is _____. (pp. A14–A19)
4. Use the converse of the Pythagorean Theorem to show that a triangle whose sides are of lengths 11, 60, and 61 is a right triangle. (pp. A14–A19)

Concepts and Vocabulary

5. If (x, y) are the coordinates of a point P in the xy-plane, then x is called the _____ of P and y is the _____ of P.
6. The coordinate axes divide the xy-plane into four sections called _____.
7. If three distinct points P, Q, and R all lie on a line and if $d(P, Q) = d(Q, R)$, then Q is called the _____ of the line segment from P to R.
8. ***True or False*** The distance between two points is sometimes a negative number.
9. ***True or False*** The point $(-1, 4)$ lies in quadrant IV of the Cartesian plane.
10. ***True or False*** The midpoint of a line segment is found by averaging the x-coordinates and averaging the y-coordinates of the endpoints.

Skill Building

In Problems 11 and 12, plot each point in the xy-plane. Tell in which quadrant or on what coordinate axis each point lies.

11. (a) $A = (-3, 2)$ (b) $B = (6, 0)$ (c) $C = (-2, -2)$ (d) $D = (6, 5)$ (e) $E = (0, -3)$ (f) $F = (6, -3)$
12. (a) $A = (1, 4)$ (b) $B = (-3, -4)$ (c) $C = (-3, 4)$ (d) $D = (4, 1)$ (e) $E = (0, 1)$ (f) $F = (-3, 0)$

13. Plot the points $(2, 0)$, $(2, -3)$, $(2, 4)$, $(2, 1)$, and $(2, -1)$. Describe the set of all points of the form $(2, y)$, where y is a real number.

14. Plot the points $(0, 3)$, $(1, 3)$, $(-2, 3)$, $(5, 3)$, and $(-4, 3)$. Describe the set of all points of the form $(x, 3)$, where x is a real number.

In Problems 15–28, find the distance $d(P_1, P_2)$ between the points P_1 and P_2.

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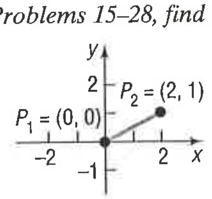

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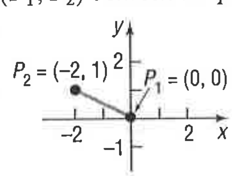

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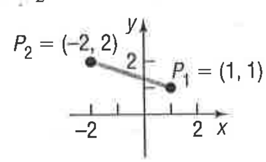

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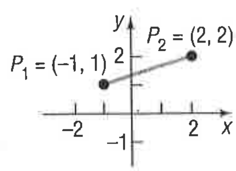


19. $P_1 = (3, -4)$; $P_2 = (5, 4)$
20. $P_1 = (-1, 0)$; $P_2 = (2, 4)$
21. $P_1 = (-3, 2)$; $P_2 = (6, 0)$
22. $P_1 = (2, -3)$; $P_2 = (4, 2)$
23. $P_1 = (4, -3)$; $P_2 = (6, 4)$
24. $P_1 = (-4, -3)$; $P_2 = (6, 2)$
25. $P_1 = (-0.2, 0.3)$; $P_2 = (2.3, 1.1)$
26. $P_1 = (1.2, 2.3)$; $P_2 = (-0.3, 1.1)$
27. $P_1 = (a, b)$; $P_2 = (0, 0)$
28. $P_1 = (a, a)$; $P_2 = (0, 0)$

In Problems 29–34, plot each point and form the triangle ABC. Verify that the triangle is a right triangle. Find its area.

29. $A = (-2, 5)$; $B = (1, 3)$; $C = (-1, 0)$
30. $A = (-2, 5)$; $B = (12, 3)$; $C = (10, -11)$
31. $A = (-5, 3)$; $B = (6, 0)$; $C = (5, 5)$
32. $A = (-6, 3)$; $B = (3, -5)$; $C = (-1, 5)$
33. $A = (4, -3)$; $B = (0, -3)$; $C = (4, 2)$
34. $A = (4, -3)$; $B = (4, 1)$; $C = (2, 1)$

In Problems 35–44, find the midpoint of the line segment joining the points P_1 and P_2.

35. $P_1 = (3, -4)$; $P_2 = (5, 4)$
36. $P_1 = (-2, 0)$; $P_2 = (2, 4)$
37. $P_1 = (-3, 2)$; $P_2 = (6, 0)$
38. $P_1 = (2, -3)$; $P_2 = (4, 2)$

1.1

39. $P_1 = (4, -3)$; $P_2 = (6, 1)$

40. $P_1 = (-4, -3)$; $P_2 = (2, 2)$

41. $P_1 = (-0.2, 0.3)$; $P_2 = (2.3, 1.1)$

42. $P_1 = (1.2, 2.3)$; $P_2 = (-0.3, 1.1)$

43. $P_1 = (a, b)$; $P_2 = (0, 0)$

44. $P_1 = (a, a)$; $P_2 = (0, 0)$

Applications and Extensions

45. Find all points having an x-coordinate of 2 whose distance from the point $(-2, -1)$ is 5.

46. Find all points having a y-coordinate of -3 whose distance from the point $(1, 2)$ is 13.

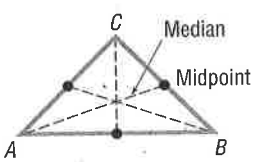
47. Find all points on the x-axis that are 5 units from the point $(4, -3)$.

48. Find all points on the y-axis that are 5 units from the point $(4, 4)$.

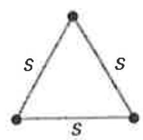
49. The midpoint of the line segment from P_1 to P_2 is $(-1, 4)$. If $P_1 = (-3, 6)$, what is P_2?

50. The midpoint of the line segment from P_1 to P_2 is $(5, -4)$. If $P_2 = (7, -2)$, what is P_1?

51. **Geometry** The **medians** of a triangle are the line segments from each vertex to the midpoint of the opposite side (see the figure). Find the lengths of the medians of the triangle with vertices at $A = (0, 0)$, $B = (6, 0)$, and $C = (4, 4)$.



52. **Geometry** An **equilateral triangle** is one in which all three sides are of equal length. If two vertices of an equilateral triangle are $(0, 4)$ and $(0, 0)$, find the third vertex. How many of these triangles are possible?



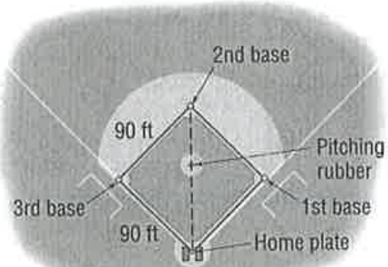
53. **Geometry** Find the midpoint of each diagonal of a square with side of length s. Draw the conclusion that the diagonals of a square intersect at their midpoints.
[**Hint:** Use $(0, 0)$, $(0, s)$, $(s, 0)$, and (s, s) as the vertices of the square.]

54. **Geometry** Verify that the points $(0, 0)$, $(a, 0)$, and $\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$ are the vertices of an equilateral triangle. Then show that the midpoints of the three sides are the vertices of a second equilateral triangle (refer to Problem 52).

In Problems 55–58, find the length of each side of the triangle determined by the three points P_1, P_2, and P_3. State whether the triangle is an isosceles triangle, a right triangle, neither of these, or both. (An ***isosceles triangle*** *is one in which at least two of the sides are of equal length.)*

55. $P_1 = (2, 1)$; $P_2 = (-4, 1)$; $P_3 = (-4, -3)$
56. $P_1 = (-1, 4)$; $P_2 = (6, 2)$; $P_3 = (4, -5)$
57. $P_1 = (-2, -1)$; $P_2 = (0, 7)$; $P_3 = (3, 2)$
58. $P_1 = (7, 2)$; $P_2 = (-4, 0)$; $P_3 = (4, 6)$

59. **Baseball** A major league baseball "diamond" is actually a square, 90 feet on a side (see the figure). What is the distance directly from home plate to second base (the diagonal of the square)?



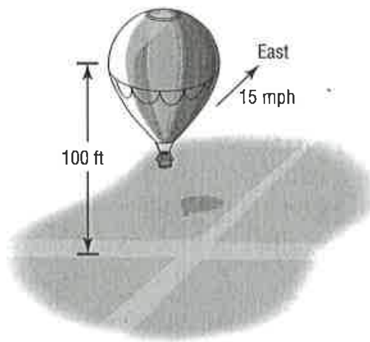
60. **Little League Baseball** The layout of a Little League playing field is a square, 60 feet on a side. How far is it directly from home plate to second base (the diagonal of the square)?
Source: Little League Baseball, Official Regulations and Playing Rules, 2006.

61. **Baseball** Refer to Problem 59. Overlay a rectangular coordinate system on a major league baseball diamond so that the origin is at home plate, the positive x-axis lies in the direction from home plate to first base, and the positive y-axis lies in the direction from home plate to third base.
(a) What are the coordinates of first base, second base, and third base? Use feet as the unit of measurement.
(b) If the right fielder is located at $(310, 15)$, how far is it from the right fielder to second base?
(c) If the center fielder is located at $(300, 300)$, how far is it from the center fielder to third base?

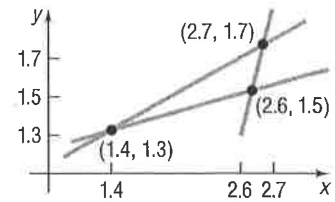
62. **Little League Baseball** Refer to Problem 60. Overlay a rectangular coordinate system on a Little League baseball diamond so that the origin is at home plate, the positive x-axis lies in the direction from home plate to first base, and the positive y-axis lies in the direction from home plate to third base.
(a) What are the coordinates of first base, second base, and third base? Use feet as the unit of measurement.
(b) If the right fielder is located at $(180, 20)$, how far is it from the right fielder to second base?
(c) If the center fielder is located at $(220, 220)$, how far is it from the center fielder to third base?

63. Distance between Moving Objects A Dodge Neon and a Mack truck leave an intersection at the same time. The Neon heads east at an average speed of 30 miles per hour, while the truck heads south at an average speed of 40 miles per hour. Find an expression for their distance apart d (in miles) at the end of t hours.

64. Distance of a Moving Object from a Fixed Point A hot-air balloon, headed due east at an average speed of 15 miles per hour and at a constant altitude of 100 feet, passes over an intersection (see the figure). Find an expression for the distance d (measured in feet) from the balloon to the intersection t seconds later.



65. Drafting Error When a draftsman draws three lines that are to intersect at one point, the lines may not intersect as intended and subsequently will form an **error triangle.** If this error triangle is long and thin, one estimate for the location of the desired point is the midpoint of the shortest side. The figure shows one such error triangle.



(a) Find an estimate for the desired intersection point.

(b) Find the length of the median for the midpoint found in part (a). See Problem 51.

Source: *www.uwgb.edu/dutchs/STRUCTGE/sl00.htm*

66. Net Sales The figure illustrates how net sales of Wal-Mart Stores, Inc., have grown from 2002 through 2006. Use the midpoint formula to estimate the net sales of Wal-Mart Stores, Inc., in 2004. How does your result compare to the reported value of \$256 billion?

Source: *Wal-Mart Stores, Inc., 2006 Annual Report*

'Are You Prepared?' Answers

1. 0 **2.** 8 **3.** 5 **4.** $11^2 + 60^2 = 121 + 3600 = 3721 = 61^2$

1.2 Graphs of Equations in Two Variables; Circles

PREPARING FOR THIS SECTION *Before getting started, review the following:*

- Completing the Square (Appendix A, Section A.4, pp. A29–A30)
- Square Root Method (Appendix A, Section A.4, p. A29)

Now Work the 'Are You Prepared' problems on page 19.

OBJECTIVES
1. Graph Equations by Plotting Points (p. 9)
2. Find Intercepts from a Graph (p. 10)
3. Find Intercepts from an Equation (p. 11)
4. Test an Equation for Symmetry with Respect to the x-Axis, the y-Axis, and the Origin (p. 12)
5. Know How to Graph Key Equations (p. 13)
6. Write the Standard Form of the Equation of a Circle (p. 15)
7. Graph a Circle (p. 16)
8. Work with the General Form of the Equation of a Circle (p. 17)

1.2 Assess Your Understanding

Are You Prepared? *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. To complete the square of $x^2 + 10x$, you would (*add/subtract*) the number ____. (pp. A29–A30)

2. Use the Square Root Method to solve the equation $(x - 2)^2 = 9$. (p. A29)

Concepts and Vocabulary

3. The points, if any, at which a graph crosses or touches the coordinate axes are called ________.

4. If for every point (x, y) on the graph of an equation the point $(-x, y)$ is also on the graph, then the graph is symmetric with respect to the ________.

5. If the graph of an equation is symmetric with respect to the origin and $(3, -4)$ is a point on the graph, then ________ is also a point on the graph.

6. ***True or False*** To find the y-intercepts of the graph of an equation, let $x = 0$ and solve for y.

7. ***True or False*** If a graph is symmetric with respect to the x-axis, then it cannot be symmetric with respect to the y-axis.

8. For a circle, the ____ is the distance from the center to any point on the circle.

9. ***True or False*** The radius of the circle $x^2 + y^2 = 9$ is 3.

10. ***True or False*** The center of the circle

$$(x + 3)^2 + (y - 2)^2 = 13$$

is $(3, -2)$.

Skill Building

In Problems 11–16, determine which of the given points are on the graph of the equation.

11. Equation: $y = x^4 - \sqrt{x}$
Points: $(0, 0); (1, 1); (-1, 0)$

12. Equation: $y = x^3 - 2\sqrt{x}$
Points: $(0, 0); (1, 1); (1, -1)$

13. Equation: $y^2 = x^2 + 9$
Points: $(0, 3); (3, 0); (-3, 0)$

14. Equation: $y^3 = x + 1$
Points: $(1, 2); (0, 1); (-1, 0)$

15. Equation: $x^2 + y^2 = 4$
Points: $(0, 2); (-2, 2); (\sqrt{2}, \sqrt{2})$

16. Equation: $x^2 + 4y^2 = 4$
Points: $(0, 1); (2, 0); \left(2, \frac{1}{2}\right)$

In Problems 17–28, find the intercepts and graph each equation by plotting points. Be sure to label the intercepts.

17. $y = x + 2$
18. $y = x - 6$
19. $y = 2x + 8$
20. $y = 3x - 9$
21. $y = x^2 - 1$
22. $y = x^2 - 9$
23. $y = -x^2 + 4$
24. $y = -x^2 + 1$
25. $2x + 3y = 6$
26. $5x + 2y = 10$
27. $9x^2 + 4y = 36$
28. $4x^2 + y = 4$

In Problems 29–38, plot each point. Then plot the point that is symmetric to it with respect to (a) the x-axis; (b) the y-axis; (c) the origin.

29. $(3, 4)$
30. $(5, 3)$
31. $(-2, 1)$
32. $(4, -2)$
33. $(5, -2)$
34. $(-1, -1)$
35. $(-3, -4)$
36. $(4, 0)$
37. $(0, -3)$
38. $(-3, 0)$

In Problems 39–50, the graph of an equation is given. (a) Find the intercepts. (b) Indicate whether the graph is symmetric with respect to the x-axis, the y-axis, or the origin.

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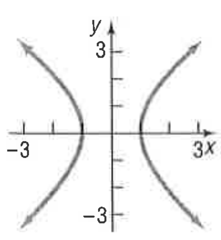

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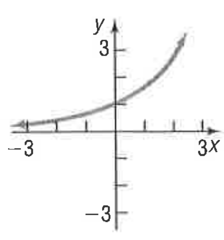

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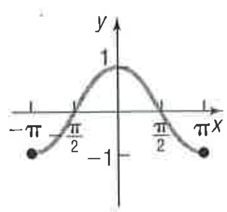

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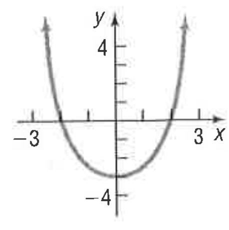

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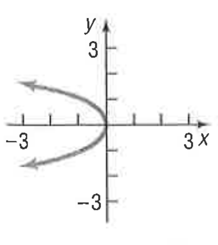

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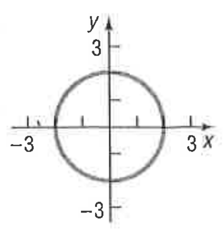

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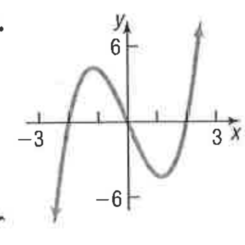

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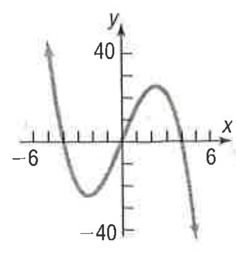

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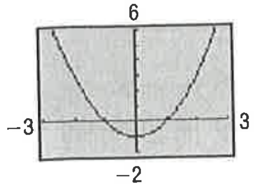
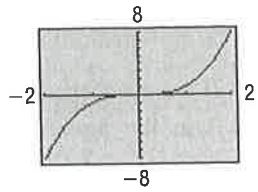
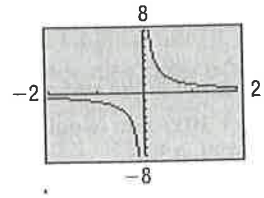
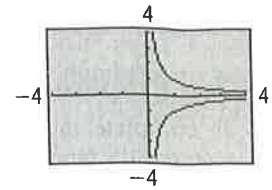
47. 

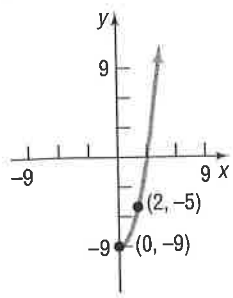
48. 

49. 

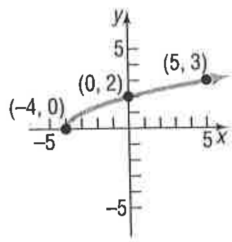
50. 

In Problems 51–54, draw a complete graph so that it has the type of symmetry indicated.

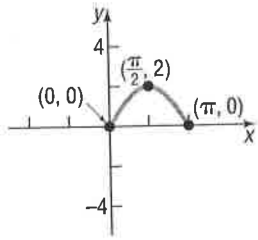
51. y-axis



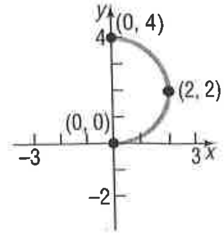
52. x-axis



53. Origin



54. y-axis



In Problems 55–70, list the intercepts and test for symmetry.

55. $y^2 = x + 4$

56. $y^2 = x + 9$

57. $y = \sqrt[3]{x}$

58. $y = \sqrt[5]{x}$

59. $x^2 + y - 9 = 0$

60. $x^2 - y - 4 = 0$

61. $9x^2 + 4y^2 = 36$

62. $4x^2 + y^2 = 4$

63. $y = x^3 - 27$

64. $y = x^4 - 1$

65. $y = x^2 - 3x - 4$

66. $y = x^2 + 4$

67. $y = \dfrac{3x}{x^2 + 9}$

68. $y = \dfrac{x^2 - 4}{2x}$

69. $y = \dfrac{-x^3}{x^2 - 9}$

70. $y = \dfrac{x^4 + 1}{2x^5}$

In Problems 71–74, draw a quick sketch of each equation.

71. $y = x^3$

72. $x = y^2$

73. $y = \sqrt{x}$

74. $y = \dfrac{1}{x}$

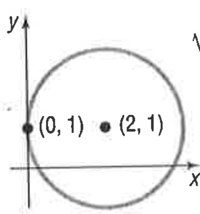
75. If $(3, b)$ is a point on the graph of $y = 4x + 1$, what is b?

76. If $(-2, b)$ is a point on the graph of $2x + 3y = 2$, what is b?

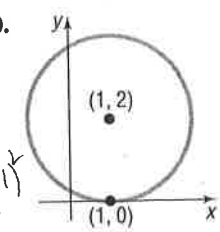
77. If $(a, 4)$ is a point on the graph of $y = x^2 + 3x$, what is a?

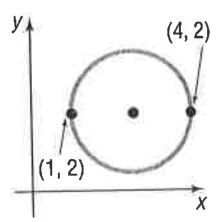
78. If $(a, -5)$ is a point on the graph of $y = x^2 + 6x$, what is a?

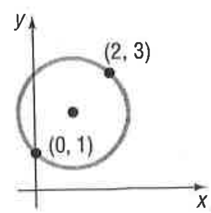
In Problems 79–82, find the center and radius of each circle. Write the standard form of the equation.

79. 

$r=2$
$h=2,$
$k=1$
$(x-2)^2+(y-1)^2=2^2$

80. 

81. 

82. 

In Problems 83–92, write the standard form of the equation and the general form of the equation of each circle of radius r and center (h, k). Graph each circle.

83. $r = 2$; $(h, k) = (0, 0)$

84. $r = 3$; $(h, k) = (0, 0)$

85. $r = 2$; $(h, k) = (0, 2)$

86. $r = 3$; $(h, k) = (1, 0)$

87. $r = 5$; $(h, k) = (4, -3)$

88. $r = 4$; $(h, k) = (2, -3)$

89. $r = 4$; $(h, k) = (-2, 1)$

90. $r = 7$; $(h, k) = (-5, -2)$

91. $r = \dfrac{1}{2}$; $(h, k) = \left(\dfrac{1}{2}, 0\right)$

92. $r = \dfrac{1}{2}$; $(h, k) = \left(0, -\dfrac{1}{2}\right)$

In Problems 93–106, (a) find the center (h, k) and radius r of each circle; (b) graph each circle; (c) find the intercepts, if any.

93. $x^2 + y^2 = 4$ **94.** $x^2 + (y - 1)^2 = 1$ **95.** $2(x - 3)^2 + 2y^2 = 8$

96. $3(x + 1)^2 + 3(y - 1)^2 = 6$ **97.** $x^2 + y^2 - 2x - 4y - 4 = 0$ **98.** $x^2 + y^2 + 4x + 2y - 20 = 0$

99. $x^2 + y^2 + 4x - 4y - 1 = 0$ **100.** $x^2 + y^2 - 6x + 2y + 9 = 0$ **101.** $x^2 + y^2 - x + 2y + 1 = 0$

102. $x^2 + y^2 + x + y - \frac{1}{2} = 0$ **103.** $2x^2 + 2y^2 - 12x + 8y - 24 = 0$ **104.** $2x^2 + 2y^2 + 8x + 7 = 0$

105. $2x^2 + 8x + 2y^2 = 0$ **106.** $3x^2 + 3y^2 - 12y = 0$

In Problems 107–114, find the standard form of the equation of each circle.

107. Center at the origin and containing the point $(-2, 3)$

108. Center $(1, 0)$ and containing the point $(-3, 2)$

109. Center $(2, 3)$ and tangent to the x-axis

110. Center $(-3, 1)$ and tangent to the y-axis

111. With endpoints of a diameter at $(1, 4)$ and $(-3, 2)$

112. With endpoints of a diameter at $(4, 3)$ and $(0, 1)$

113. Center $(-1, 3)$ and tangent to the line $y = 2$

114. Center $(4, -2)$ and tangent to the line $x = 1$

Applications and Extensions

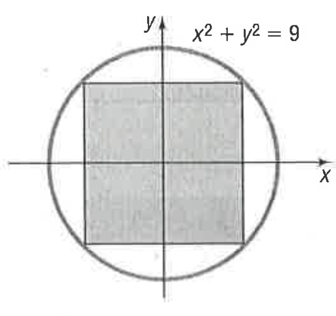
115. Given that the point $(1, 2)$ is on the graph of an equation that is symmetric with respect to the origin, what other point is on the graph?

116. If the graph of an equation is symmetric with respect to the y-axis and 6 is an x-intercept of this graph, name another x-intercept.

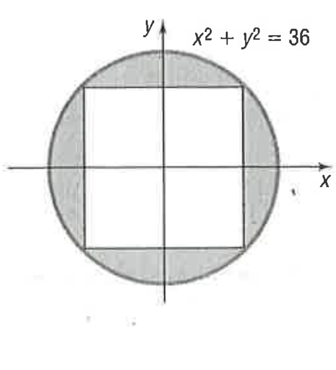
117. If the graph of an equation is symmetric with respect to the origin and -4 is an x-intercept of this graph, name another x-intercept.

118. If the graph of an equation is symmetric with respect to the x-axis and 2 is a y-intercept, name another y-intercept.

119. Find the area of the square in the figure.



120. Find the area of the shaded region in the figure assuming the quadrilateral inside the circle is a square.



121. Ferris Wheel The original Ferris wheel was built in 1893 by Pittsburg, Pennsylvania, bridge builder George W. Ferris. The Ferris wheel was originally built for the 1893 World's Fair in Chicago, but was also later reconstructed for the 1904 World's Fair in St. Louis. It had a maximum height of 264 feet and a wheel diameter of 250 feet. Find an equation for the wheel if the center of the wheel is on the y-axis.

Source: *inventors.about.com*

122. Ferris Wheel In 2006, the star of Nanchang (in the Jiangxi province) opened as the world's largest Ferris wheel. It has a maximum height of 160 meters and a diameter of 153 meters, with one full rotation taking approximately 30 minutes. Find an equation for the wheel if the center of the wheel is on the y-axis.

Source: *AsiaOne Travel*

Note: Two other even larger Ferris wheels are reportedly to be completed in Asia by 2008 in time for the 2008 summer Olympics

Unspecified domain If a function f is defined by an equation and no domain is specified, then the domain will be taken to be the largest set of real numbers for which the equation defines a real number.

Function notation $y = f(x)$
f is a symbol for the function.
x is the independent variable or argument.
y is the dependent variable.
$f(x)$ is the value of the function at x, or the image of x.

Graph of a function The collection of points (x, y) that satisfies the equation $y = f(x)$.

Vertical Line Test A collection of points is the graph of a function provided that every vertical line intersects the graph in at most one point.

1.3 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

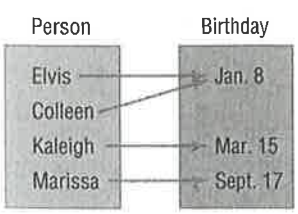
1. The inequality $-1 < x < 3$ can be written in interval notation as _____. (pp. A44–A45)
2. If $x = -2$, the value of the expression $3x^2 - 5x + \frac{1}{x}$ is _____. (pp. A6–A7)
3. The domain of the variable in the expression $\frac{x-3}{x+4}$ is _____. (pp. A6–A7)
4. Solve the inequality: $3 - 2x > 5$. Graph the solution set. (pp. A47–A50)

Concepts and Vocabulary

5. If f is a function defined by the equation $y = f(x)$, then x is called the _____ variable and y is the _____ variable.
6. The set of all images of the elements in the domain of a function is called the _____.
7. ***True or False*** Every relation is a function.
8. ***True or False*** The independent variable is sometimes referred to as the argument of the function.
9. ***True or False*** If no domain is specified for a function f, then the domain of f is taken to be the set of real numbers.
10. ***True or False*** The domain of the function $f(x) = \frac{x^2 - 4}{x}$ is $\{x \mid x \neq \pm 2\}$.
11. A set of points in the xy-plane is the graph of a function if and only if every _____ line intersects the graph in at most one point.
12. If the point $(5, -3)$ is a point on the graph of f, then $f(\_\_\_) = \_\_\_$.
13. Find a so that the point $(-1, 2)$ is on the graph of $f(x) = ax^2 + 4$.
14. ***True or False*** A function can have more than one y-intercept.
15. ***True or False*** The graph of a function $y = f(x)$ always crosses the y-axis.
16. ***True or False*** The y-intercept of the graph of the function $y = f(x)$, whose domain is all real numbers, is $f(0)$.

Skill Building

In Problems 17–28, determine whether each relation represents a function. For each function, state the domain and range.

17.


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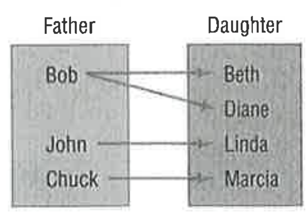

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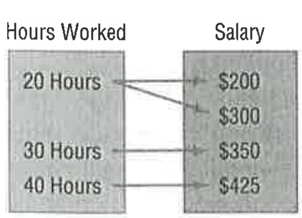

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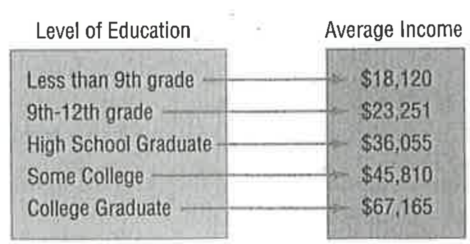


1.3

21. $\{(2, 6), (-3, 6), (4, 9), (2, 10)\}$ **22.** $\{(-2, 5), (-1, 3), (3, 7), (4, 12)\}$ **23.** $\{(1, 3), (2, 3), (3, 3), (4, 3)\}$

24. $\{(0, -2), (1, 3), (2, 3), (3, 7)\}$ **25.** $\{(-2, 4), (-2, 6), (0, 3), (3, 7)\}$ **26.** $\{(-4, 4), (-3, 3), (-2, 2), (-1, 1), (-4, 0)\}$

27. $\{(-2, 4), (-1, 1), (0, 0), (1, 1)\}$ **28.** $\{(-2, 16), (-1, 4), (0, 3), (1, 4)\}$

In Problems 29–40, determine whether the equation defines y as a function of x.

29. $y = x^2$ **30.** $y = x^3$ **31.** $y = \frac{1}{x}$ **32.** $y = |x|$

33. $y^2 = 4 - x^2$ **34.** $y = \pm\sqrt{1 - 2x}$ **35.** $x = y^2$ **36.** $x + y^2 = 1$

37. $y = 2x^2 - 3x + 4$ **38.** $y = \frac{3x - 1}{x + 2}$ **39.** $2x^2 + 3y^2 = 1$ **40.** $x^2 - 4y^2 = 1$

In Problems 41–48, find the following for each function:

(a) $f(0)$ *(b)* $f(1)$ *(c)* $f(-1)$ *(d)* $f(-x)$ *(e)* $-f(x)$ *(f)* $f(x + 1)$ *(g)* $f(2x)$ *(h)* $f(x + h)$

41. $f(x) = 3x^2 + 2x - 4$ **42.** $f(x) = -2x^2 + x - 1$ **43.** $f(x) = \frac{x}{x^2 + 1}$ **44.** $f(x) = \frac{x^2 - 1}{x + 4}$

45. $f(x) = |x| + 4$ **46.** $f(x) = \sqrt{x^2 + x}$ **47.** $f(x) = \frac{2x + 1}{3x - 5}$ **48.** $f(x) = 1 - \frac{1}{(x + 2)^2}$

In Problems 49–60, find the domain of each function.

49. $f(x) = -5x + 4$ **50.** $f(x) = x^2 + 2$ **51.** $f(x) = \frac{x}{x^2 + 1}$ **52.** $f(x) = \frac{x^2}{x^2 + 1}$

53. $g(x) = \frac{x}{x^2 - 16}$ **54.** $h(x) = \frac{2x}{x^2 - 4}$ **55.** $F(x) = \frac{x - 2}{x^3 + x}$ **56.** $G(x) = \frac{x + 4}{x^3 - 4x}$

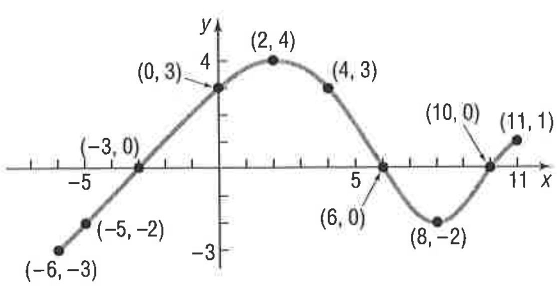
57. $h(x) = \sqrt{3x - 12}$ **58.** $G(x) = \sqrt{1 - x}$ **59.** $f(x) = \frac{4}{\sqrt{x - 9}}$ **60.** $f(x) = \frac{x}{\sqrt{x - 4}}$

In Problems 61–68, find the difference quotient of f; that is, find $\frac{f(x + h) - f(x)}{h}$, *$h \neq 0$, for each function. Be sure to simplify.*

61. $f(x) = 4x + 3$ **62.** $f(x) = -3x + 1$ **63.** $f(x) = x^2 - x + 4$ **64.** $f(x) = x^2 + 5x - 1$

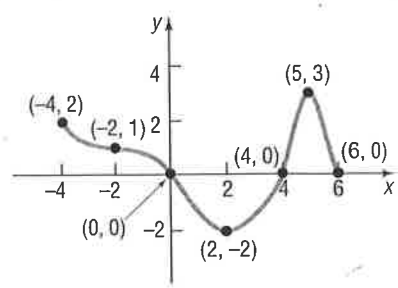
65. $f(x) = 3x^2 - 2x + 6$ **66.** $f(x) = 4x^2 + 5x - 7$ **67.** $f(x) = x^3 - 2$ **68.** $f(x) = \frac{1}{x + 3}$

69. Use the given graph of the function f to answer parts (a)–(n).



(a) Find $f(0)$ and $f(-6)$.
(b) Find $f(6)$ and $f(11)$.
(c) Is $f(3)$ positive or negative?
(d) Is $f(-4)$ positive or negative?
(e) For what values of x is $f(x) = 0$?
(f) For what values of x is $f(x) > 0$?
(g) What is the domain of f?
(h) What is the range of f?
(i) What are the x-intercepts?
(j) What is the y-intercept?
(k) How often does the line $y = \frac{1}{2}$ intersect the graph?
(l) How often does the line $x = 5$ intersect the graph?
(m) For what values of x does $f(x) = 3$?
(n) For what values of x does $f(x) = -2$?

70. Use the given graph of the function f to answer parts (a)–(n).

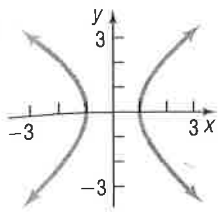


(a) Find $f(0)$ and $f(6)$.
(b) Find $f(2)$ and $f(-2)$.
(c) Is $f(3)$ positive or negative?
(d) Is $f(-1)$ positive or negative?
(e) For what values of x is $f(x) = 0$?
(f) For what values of x is $f(x) < 0$?
(g) What is the domain of f?
(h) What is the range of f?
(i) What are the x-intercepts?
(j) What is the y-intercept?
(k) How often does the line $y = -1$ intersect the graph?
(l) How often does the line $x = 1$ intersect the graph?
(m) For what value of x does $f(x) = 3$?
(n) For what value of x does $f(x) = -2$?

1.3

In Problems 71–82, determine whether the graph is that of a function by using the vertical-line test. If it is, use the graph to find:

(a) The domain and range
(b) The intercepts, if any
(c) Any symmetry with respect to the x-axis, the y-axis, or the origin

71.


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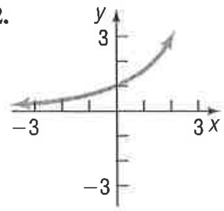

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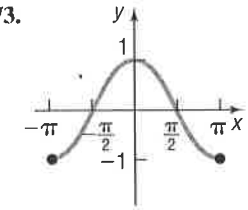

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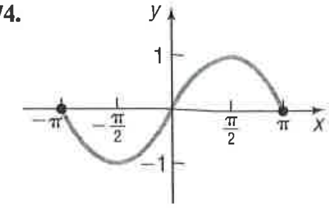

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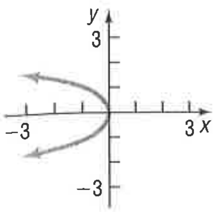

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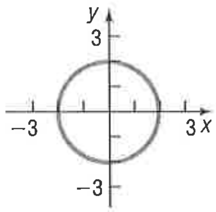

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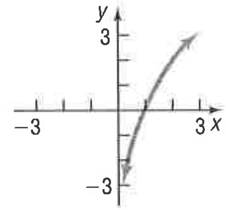

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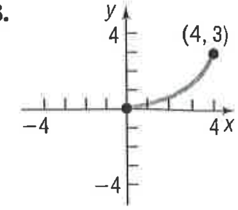

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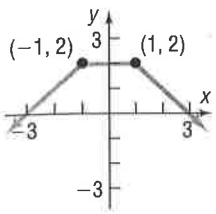

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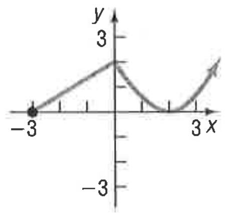

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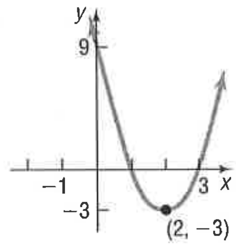

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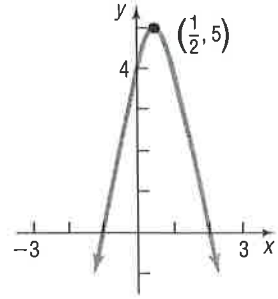


In Problems 83–88, answer the questions about the given function.

83. $f(x) = 2x^2 - x - 1$
(a) Is the point $(-1, 2)$ on the graph of f?
(b) If $x = -2$, what is $f(x)$? What point is on the graph of f?
(c) If $f(x) = -1$, what is x? What point(s) are on the graph of f?
(d) What is the domain of f?
(e) List the x-intercepts, if any, of the graph of f.
(f) List the y-intercept, if there is one, of the graph of f.

84. $f(x) = -3x^2 + 5x$
(a) Is the point $(-1, 2)$ on the graph of f?
(b) If $x = -2$, what is $f(x)$? What point is on the graph of f?
(c) If $f(x) = -2$, what is x? What point(s) are on the graph of f?
(d) What is the domain of f?
(e) List the x-intercepts, if any, of the graph of f.
(f) List the y-intercept, if there is one, of the graph of f.

85. $f(x) = \dfrac{x+2}{x-6}$
(a) Is the point $(3, 14)$ on the graph of f?
(b) If $x = 4$, what is $f(x)$? What point is on the graph of f?
(c) If $f(x) = 2$, what is x? What point(s) are on the graph of f?
(d) What is the domain of f?
(e) List the x-intercepts, if any, of the graph of f.
(f) List the y-intercept, if there is one, of the graph of f.

86. $f(x) = \dfrac{x^2+2}{x+4}$
(a) Is the point $\left(1, \frac{3}{5}\right)$ on the graph of f?
(b) If $x = 0$, what is $f(x)$? What point is on the graph of f?
(c) If $f(x) = \frac{1}{2}$, what is x? What point(s) are on the graph of f?
(d) What is the domain of f?
(e) List the x-intercepts, if any, of the graph of f.
(f) List the y-intercept, if there is one, of the graph of f.

87. $f(x) = \dfrac{2x^2}{x^4+1}$
(a) Is the point $(-1, 1)$ on the graph of f?
(b) If $x = 2$, what is $f(x)$? What point is on the graph of f?
(c) If $f(x) = 1$, what is x? What point(s) are on the graph of f?
(d) What is the domain of f?
(e) List the x-intercepts, if any, of the graph of f.
(f) List the y-intercept, if there is one, of the graph of f.

1.3

88. $f(x) = \dfrac{2x}{x-2}$

(a) Is the point $\left(\frac{1}{2}, -\frac{2}{3}\right)$ on the graph of f?

(b) If $x = 4$, what is $f(x)$? What point is on the graph of f?

(c) If $f(x) = 1$, what is x? What point(s) are on the graph of f?

(d) What is the domain of f?

(e) List the x-intercepts, if any, of the graph of f.

(f) List the y-intercept, if there is one, of the graph of f.

Applications and Extensions

89. If $f(x) = 2x^3 + Ax^2 + 4x - 5$ and $f(2) = 5$, what is the value of A?

90. If $f(x) = 3x^2 - Bx + 4$ and $f(-1) = 12$, what is the value of B?

91. If $f(x) = \dfrac{3x+8}{2x-A}$ and $f(0) = 2$, what is the value of A?

92. If $f(x) = \dfrac{2x-B}{3x+4}$ and $f(2) = \dfrac{1}{2}$, what is the value of B?

93. If $f(x) = \dfrac{2x-A}{x-3}$ and $f(4) = 0$, what is the value of A? Where is f not defined?

94. If $f(x) = \dfrac{x-B}{x-A}$, $f(2) = 0$ and $f(1)$ is undefined, what are the values of A and B?

95. Geometry Express the area A of a rectangle as a function of the length x if the length of the rectangle is twice its width.

96. Geometry Express the area A of an isosceles right triangle as a function of the length x of one of the two equal sides.

97. Constructing Functions Express the gross salary G of a person who earns \$10 per hour as a function of the number x of hours worked.

98. Constructing Functions Tiffany, a commissioned salesperson, earns \$100 base pay plus \$10 per item sold. Express her gross salary G as a function of the number x of items sold.

99. Population as a Function of Age The function

$$P(a) = 0.015a^2 - 4.962a + 290.580$$

represents the population P (in millions) of Americans in 2005 that are a years of age or older.

Source: *U.S. Census Bureau*

(a) Identify the dependent and independent variable.

(b) Evaluate $P(20)$. Provide a verbal explanation of the meaning of $P(20)$.

(c) Evaluate $P(0)$. Provide a verbal explanation of the meaning of $P(0)$.

100. Number of Rooms The function

$$N(r) = -1.44r^2 + 14.52r - 14.96$$

represents the number N of housing units (in millions) in 2005 that have r rooms, where r is an integer and $2 \le r \le 9$.

Source: *U.S. Census Bureau*

(a) Identify the dependent and independent variable.

(b) Evaluate $N(3)$. Provide a verbal explanation of the meaning of $N(3)$.

101. Effect of Gravity on Earth If a rock falls from a height of 20 meters on Earth, the height H (in meters) after x seconds is approximately

$$H(x) = 20 - 4.9x^2$$

(a) What is the height of the rock when $x = 1$ second? $x = 1.1$ seconds? $x = 1.2$ seconds? $x = 1.3$ seconds?

(b) When is the height of the rock 15 meters? When is it 10 meters? When is it 5 meters?

(c) When does the rock strike the ground?

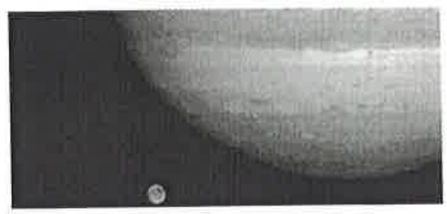
102. Effect of Gravity on Jupiter If a rock falls from a height of 20 meters on the planet Jupiter, its height H (in meters) after x seconds is approximately

$$H(x) = 20 - 13x^2$$

(a) What is the height of the rock when $x = 1$ second? $x = 1.1$ seconds? $x = 1.2$ seconds?

(b) When is the height of the rock 15 meters? When is it 10 meters? When is it 5 meters?

(c) When does the rock strike the ground?



103. Cost of Trans-Atlantic Travel A Boeing 747 crosses the Atlantic Ocean (3000 miles) with an airspeed of 500 miles per hour. The cost C (in dollars) per passenger is given by

$$C(x) = 100 + \frac{x}{10} + \frac{36{,}000}{x}$$

where x is the ground speed (airspeed $\pm$ wind).

(a) What is the cost per passenger for quiescent (no wind) conditions?

(b) What is the cost per passenger with a head wind of 50 miles per hour?

(c) What is the cost per passenger with a tail wind of 100 miles per hour?

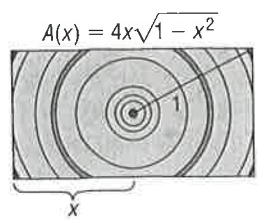
(d) What is the cost per passenger with a head wind of 100 miles per hour?

104. Cross-sectional Area The cross-sectional area of a beam cut from a log with radius 1 foot is given by the function $A(x) = 4x\sqrt{1 - x^2}$, where x represents the length, in feet, of half the base of the beam. See the figure. Determine the cross-sectional area of the beam if the length of half the base of the beam is as follows:

(a) One-third of a foot

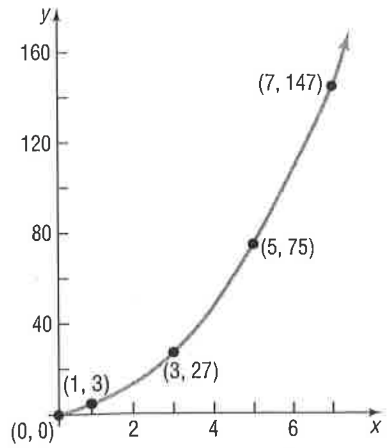
(b) One-half of a foot

(c) Two-thirds of a foot



1.4

Figure 48



(c) The average rate of change of $f(x) = 3x^2$ from 1 to 7 is

$$\frac{\Delta y}{\Delta x} = \frac{f(7) - f(1)}{7 - 1} = \frac{147 - 3}{7 - 1} = \frac{144}{6} = 24$$

See Figure 48 for a graph of $f(x) = 3x^2$. The function f is increasing for $x > 0$. The fact that the average rates of change are getting larger indicates that the graph is getting steeper; that is, it is increasing at an increasing rate.

Now Work PROBLEM 51

1.4 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

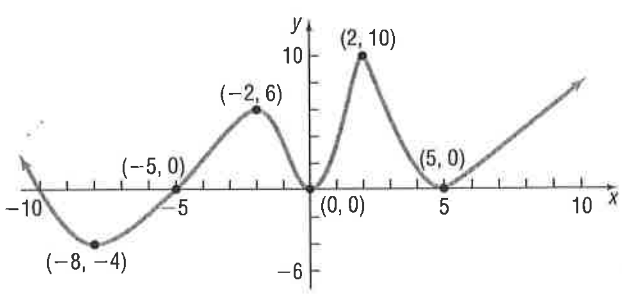
1. The interval (2, 5) can be written as the inequality _____. (pp. A44–A45)
2. The slope of the line containing the points $(-2, 3)$ and $(3, 8)$ is _____. (pp. A62–A65)
3. Write the point–slope form of the line with slope 5 containing the point $(3, -2)$. (p. A66)

Concepts and Vocabulary

4. A function f is _____ on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.
5. A(n) _____ function f is one for which $f(-x) = f(x)$ for every x in the domain of f; (an) _____ function f is one for which $f(-x) = -f(x)$ for every x in the domain of f.
6. ***True or False*** A function f is decreasing on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) > f(x_2)$.
7. ***True or False*** A function f has a local maximum at c if there is an open interval I containing c so that, for all x not equal to c in I, $f(x) < f(c)$.
8. ***True or False*** Even functions have graphs that are symmetric with respect to the origin.

Skill Building

In Problems 9–18, use the graph of the function f given.

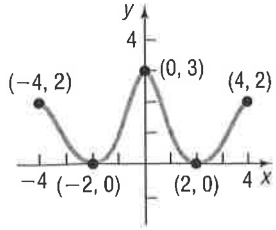


9. Is f increasing on the interval $(-8, -2)$?
10. Is f decreasing on the interval $(-8, -4)$?
11. Is f increasing on the interval $(2, 10)$?
12. Is f decreasing on the interval $(2, 5)$?
13. List the interval(s) on which f is increasing.
14. List the interval(s) on which f is decreasing.
15. Is there a local maximum at 2? If yes, what is it?
16. Is there a local maximum at 5? If yes, what is it?
17. List the numbers at which f has a local maximum. What are these local maxima?
18. List the numbers at which f has a local minimum. What are these local minima?

1.4

In Problems 19–26, the graph of a function is given. Use the graph to find:

(a) The intercepts, if any
(b) The domain and range
(c) The intervals on which it is increasing, decreasing, or constant
(d) Whether it is even, odd, or neither

19.


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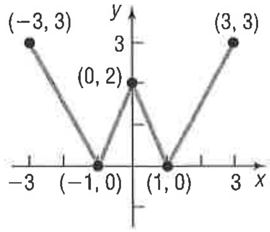

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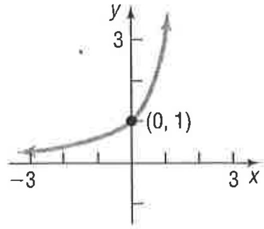

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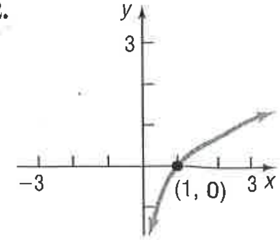

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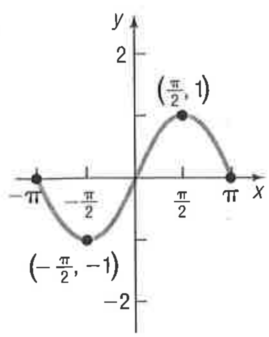

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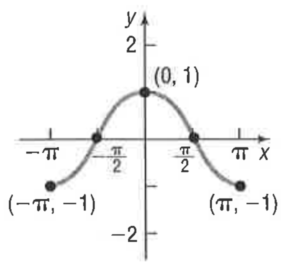

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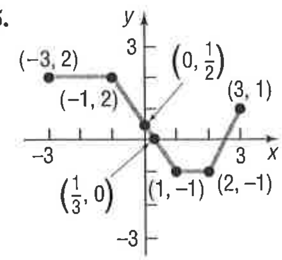

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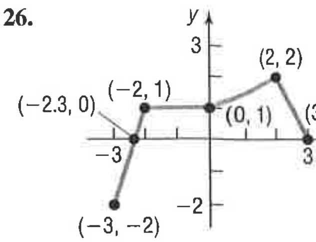
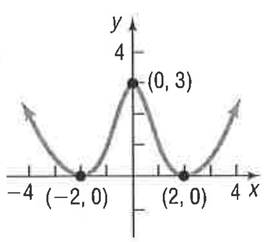


In Problems 27–30, the graph of a function f is given. Use the graph to find:

(a) The numbers, if any, at which f has a local maximum. What are these local maxima?
(b) The numbers, if any, at which f has a local minimum. What are these local minima?

27.


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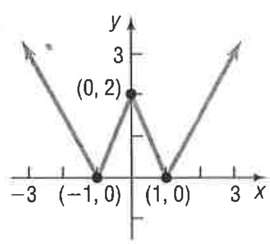

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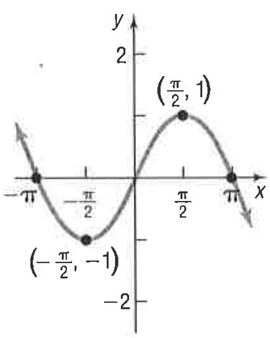

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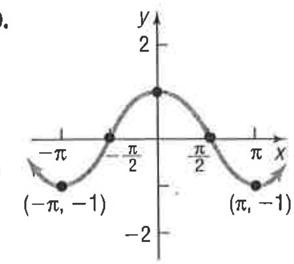


In Problems 31–42, determine algebraically whether each function is even, odd, or neither.

31. $f(x) = 4x^3$ **32.** $f(x) = 2x^4 - x^2$ **33.** $g(x) = -3x^2 - 5$ **34.** $h(x) = 3x^3 + 5$

35. $F(x) = \sqrt[3]{x}$ **36.** $G(x) = \sqrt{x}$ **37.** $f(x) = x + |x|$ **38.** $f(x) = \sqrt[3]{2x^2 + 1}$

39. $g(x) = \frac{1}{x^2}$ **40.** $h(x) = \frac{x}{x^2 - 1}$ **41.** $h(x) = \frac{-x^3}{3x^2 - 9}$ **42.** $F(x) = \frac{2x}{|x|}$

In Problems 43–50, use a graphing utility to graph each function over the indicated interval and approximate any local maxima and local minima. Determine where the function is increasing and where it is decreasing. Round answers to two decimal places.

43. $f(x) = x^3 - 3x + 2 \quad (-2, 2)$ **44.** $f(x) = x^3 - 3x^2 + 5 \quad (-1, 3)$

45. $f(x) = x^5 - x^3 \quad (-2, 2)$ **46.** $f(x) = x^4 - x^2 \quad (-2, 2)$

47. $f(x) = -0.2x^3 - 0.6x^2 + 4x - 6 \quad (-6, 4)$ **48.** $f(x) = -0.4x^3 + 0.6x^2 + 3x - 2 \quad (-4, 5)$

49. $f(x) = 0.25x^4 + 0.3x^3 - 0.9x^2 + 3 \quad (-3, 2)$ **50.** $f(x) = -0.4x^4 - 0.5x^3 + 0.8x^2 - 2 \quad (-3, 2)$

51. Find the average rate of change of $f(x) = -2x^2 + 4$
   (a) From 0 to 2
   (b) From 1 to 3
   (c) From 1 to 4

52. Find the average rate of change of $f(x) = -x^3 + 1$
   (a) From 0 to 2
   (b) From 1 to 3
   (c) From -1 to 1

53. Find the average rate of change of $g(x) = x^3 - 2x + 1$
   (a) From -3 to -2
   (b) From -1 to 1
   (c) From 1 to 3

54. Find the average rate of change of $h(x) = x^2 - 2x + 3$
   (a) From -1 to 1
   (b) From 0 to 2
   (c) From 2 to 5

55. $f(x) = 5x - 2$
   (a) Find the average rate of change from 1 to 3.
   (b) Find an equation of the secant line containing $(1, f(1))$ and $(3, f(3))$.

56. $f(x) = -4x + 1$
   (a) Find the average rate of change from 2 to 5.
   (b) Find an equation of the secant line containing $(2, f(2))$ and $(5, f(5))$.

57. $g(x) = x^2 - 2$
   (a) Find the average rate of change from -2 to 1.
   (b) Find an equation of the secant line containing $(-2, g(-2))$ and $(1, g(1))$.

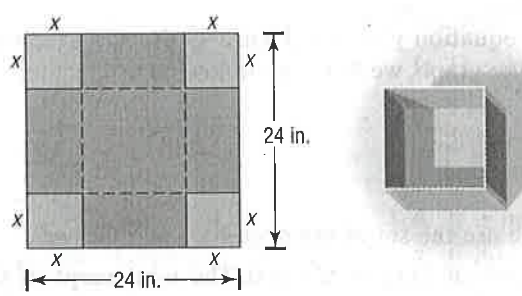
58. $g(x) = x^2 + 1$
   (a) Find the average rate of change from -1 to 2.
   (b) Find an equation of the secant line containing $(-1, g(-1))$ and $(2, g(2))$.

59. $h(x) = x^2 - 2x$
   (a) Find the average rate of change from 2 to 4.
   (b) Find an equation of the secant line containing $(2, h(2))$ and $(4, h(4))$.

60. $h(x) = -2x^2 + x$
   (a) Find the average rate of change from 0 to 3.
   (b) Find an equation of the secant line containing $(0, h(0))$ and $(3, h(3))$.

Applications and Extensions

61. **Constructing an Open Box** An open box with a square base is to be made from a square piece of cardboard 24 inches on a side by cutting out a square from each corner and turning up the sides. See the figure.



   (a) Express the volume V of the box as a function of the length x of the side of the square cut from each corner.
   (b) What is the volume if a 3-inch square is cut out?
   (c) What is the volume if a 10-inch square is cut out?
   (d) Graph $V = V(x)$. For what value of x is V largest?

62. **Constructing an Open Box** An open box with a square base is required to have a volume of 10 cubic feet.
   (a) Express the amount A of material used to make such a box as a function of the length x of a side of the square base.
   (b) How much material is required for a base 1 foot by 1 foot?
   (c) How much material is required for a base 2 feet by 2 feet?
   (d) Use a graphing utility to graph $A = A(x)$. For what value of x is A smallest?

63. **Maximum Height of a Ball** The height s of a ball (in feet) thrown with an initial velocity of 80 feet per second from an initial height of 6 feet is given as a function of the time t (in seconds) by

$$s(t) = -16t^2 + 80t + 6$$

   (a) Use a graphing utility to graph $s = s(t)$.
   (b) Determine the time at which height is maximum.
   (c) What is the maximum height?

64. **Maximum Height of a Ball** On July 1, 2004, the Cassini probe became the first spacecraft to orbit the planet Saturn. Although Saturn is about 764 times the size of Earth, it has a very similar gravitational force. The height s of an object thrown upward from Saturn's surface with an initial velocity of 100 feet per second is given as a function of time t (in seconds) by $s(t) = -17.28t^2 + 100t$.
   (a) Use a graphing utility to graph $s = s(t)$.
   (b) Determine the time at which height is a maximum.
   (c) What is the maximum height?
   (d) The same object thrown from the surface of Earth would have a height given by $s(t) = -16t^2 + 100t$. Determine the maximum height of the object on Earth and compare this to your result from part (c).

65. **Minimum Average Cost** The average cost per hour in dollars of producing x riding lawn mowers is given by

$$\overline{C}(x) = 0.3x^2 + 21x - 251 + \frac{2500}{x}$$

   (a) Use a graphing utility to graph $\overline{C} = \overline{C}(x)$
   (b) Determine the number of riding lawn mowers to produce in order to minimize average cost.
   (c) What is the minimum average cost?

66. **Medicine Concentration** The concentration C of a medication in the bloodstream t hours after being administered is given by

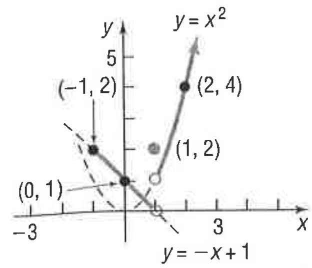
$$C(t) = -0.002x^4 + 0.039t^3 - 0.285t^2 + 0.766t + 0.085$$

   (a) After how many hours will the concentration be highest?
   (b) A woman nursing a child must wait until the concentration is below 0.5 before she can feed him. After taking the medication, how long must she wait before feeding her child?

When $x = 2$, the equation for f is $f(x) = x^2$. So

$$f(2) = 2^2 = 4$$

Figure 61



(b) To find the domain of f, we look at its definition. We conclude that the domain of f is $\{x|x \geq -1\}$, or the interval $[-1, \infty)$.

(c) To graph f, we graph "each piece." First we graph the line $y = -x + 1$ and keep only the part for which $-1 \leq x < 1$. Then we plot the point $(1, 2)$ because, when $x = 1$, $f(x) = 2$. Finally, we graph the parabola $y = x^2$ and keep only the part for which $x > 1$. See Figure 61.

(d) From the graph, we conclude that the range of f is $\{y|y > 0\}$, or the interval $(0, \infty)$.

Now Work PROBLEM 25

1.5 Assess Your Understanding

Concepts and Vocabulary

1. When functions are defined by more than one equation, they are called _____ functions.

2. ***True or False*** The cube function is odd and is increasing on the interval $(-\infty, \infty)$.

3. ***True or False*** The cube root function is odd and is decreasing on the interval $(-\infty, \infty)$.

4. ***True or False*** The domain and the range of the reciprocal function are the set of all real numbers.

Skill Building

In Problems 5–12, match each graph to its function.

A. Constant function *B. Identity function* *C. Square function*
D. Cube function *E. Square root function* *F. Reciprocal function*
G. Absolute value function *H. Cube root function*

5. 

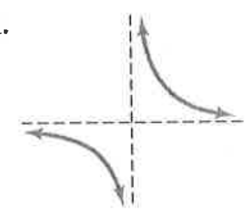
6. 

7. 

8. 

9. 

10. 

11. 

12. 

In Problems 13–20, sketch the graph of each function. Be sure to label three points on the graph.

13. $f(x) = x$

14. $f(x) = x^2$

15. $f(x) = x^3$

16. $f(x) = \sqrt{x}$

17. $f(x) = \dfrac{1}{x}$

18. $f(x) = |x|$

19. $f(x) = \sqrt[3]{x}$

20. $f(x) = 3$

21. If $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

find: (a) $f(-2)$ (b) $f(0)$ (c) $f(2)$

22. If $f(x) = \begin{cases} -3x & \text{if } x < -1 \\ 0 & \text{if } x = -1 \\ 2x^2 + 1 & \text{if } x > -1 \end{cases}$

find: (a) $f(-2)$ (b) $f(-1)$ (c) $f(0)$

23. If $f(x) = \begin{cases} 2x - 4 & \text{if } -1 \leq x \leq 2 \\ x^3 - 2 & \text{if } 2 < x \leq 3 \end{cases}$

find: (a) $f(0)$ (b) $f(1)$ (c) $f(2)$ (d) $f(3)$

24. If $f(x) = \begin{cases} x^3 & \text{if } -2 \leq x < 1 \\ 3x + 2 & \text{if } 1 \leq x \leq 4 \end{cases}$

find: (a) $f(-1)$ (b) $f(0)$ (c) $f(1)$ (d) $f(3)$

1.5

In Problems 25–34:

(a) Find the domain of each function.
(b) Locate any intercepts.
(c) Graph each function.
(d) Based on the graph, find the range.

25. $f(x) = \begin{cases} 2x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

26. $f(x) = \begin{cases} 3x & \text{if } x \neq 0 \\ 4 & \text{if } x = 0 \end{cases}$

27. $f(x) = \begin{cases} -2x + 3 & x < 1 \\ 3x - 2 & x \geq 1 \end{cases}$

28. $f(x) = \begin{cases} x + 3 & \text{if } x < -2 \\ -2x - 3 & \text{if } x \geq -2 \end{cases}$

29. $f(x) = \begin{cases} x + 3 & \text{if } -2 \leq x < 1 \\ 5 & \text{if } x = 1 \\ -x + 2 & \text{if } x > 1 \end{cases}$

30. $f(x) = \begin{cases} 2x + 5 & \text{if } -3 \leq x < 0 \\ -3 & \text{if } x = 0 \\ -5x & \text{if } x > 0 \end{cases}$

31. $f(x) = \begin{cases} 1 + x & \text{if } x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

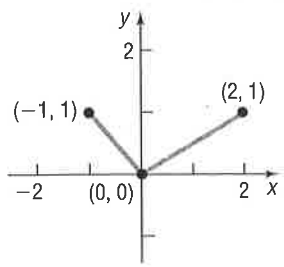
32. $f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 0 \\ \sqrt[3]{x} & \text{if } x \geq 0 \end{cases}$

33. $f(x) = \begin{cases} |x| & \text{if } -2 \leq x < 0 \\ x^3 & \text{if } x > 0 \end{cases}$

34. $f(x) = \begin{cases} 2 - x & \text{if } -3 \leq x < 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$

In Problems 35–38, the graph of a piecewise-defined function is given. Write a definition for each function.

35.



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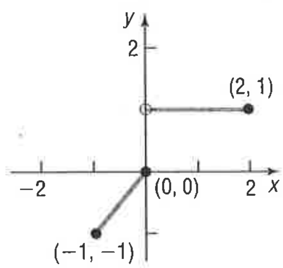

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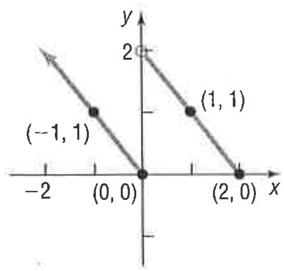

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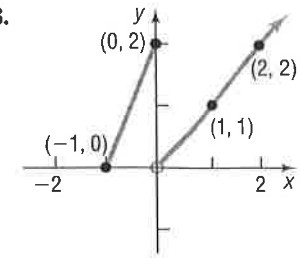


Applications and Extensions

39. Cell Phone Service Sprint PCS offers a monthly cellular phone plan for \$35. It includes 300 anytime minutes and charges \$0.40 per minute for additional minutes. The following function is used to compute the monthly cost for a subscriber:

$$C(x) = \begin{cases} 35 & \text{if } 0 < x \leq 300 \\ 0.40x - 85 & \text{if } x > 300 \end{cases}$$

where x is the number of anytime minutes used. Compute the monthly cost of the cellular phone for use of the following anytime minutes:

(a) 200 (b) 365 (c) 301

***Source:** Sprint PCS*

40. Parking at O'Hare International Airport The short-term (no more than 24 hours) parking fee F (in dollars) for parking x hours at O'Hare International Airport's main parking garage can be modeled by the function

$$F(x) = \begin{cases} 3 & \text{if } 0 < x \leq 3 \\ 5\,\text{int}(x + 1) + 1 & \text{if } 3 < x < 9 \\ 50 & \text{if } 9 \leq x \leq 24 \end{cases}$$

Determine the fee for parking in the short-term parking garage for

(a) 2 hours
(b) 7 hours
(c) 15 hours
(d) 8 hours and 24 minutes

***Source:** O'Hare International Airport*

41. Cost of Natural Gas In May 2006, Peoples Energy had the following rate schedule for natural gas usage in single-family residences:

Monthly service charge	\$9.45
Per therm service charge	
1st 50 therms	\$0.36375/therm
Over 50 therms	\$0.11445/therm
Gas charge	\$0.7958/therm

(a) What is the charge for using 50 therms in a month?
(b) What is the charge for using 500 therms in a month?
(c) Construct a function that relates the monthly charge C for x therms of gas.
(d) Graph this function.

***Source:** Peoples Energy, Chicago, Illinois, 2006*

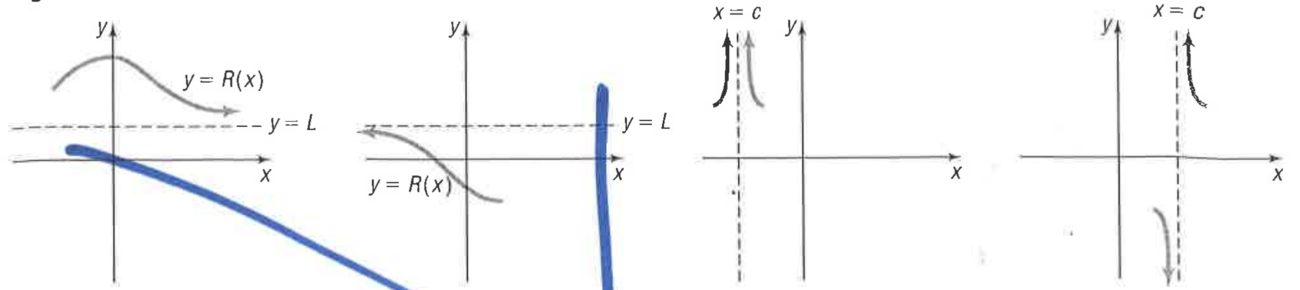
42. Cost of Natural Gas In May 2006, Nicor Gas had the following rate schedule for natural gas usage in single-family residences:

Monthly customer charge	\$8.85
Distribution charge	
1st 20 therms	\$0.1557/therm
Next 30 therms	\$0.0663/therm
Over 50 therms	\$0.0519/therm
Gas supply charge	\$0.66/therm

(a) What is the charge for using 40 therms in a month?
(b) What is the charge for using 202 therms in a month?
(c) Construct a function that gives the monthly charge C for x therms of gas.
(d) Graph this function.

***Source:** Nicor Gas, Aurora, Illinois, 2006*

Figure 78



(a) End behavior: As $x \to \infty$, the values of $R(x)$ approach L [symbolized by $\lim_{x \to \infty} R(x) = L$]. That is, the points on the graph of R are getting closer to the line $y = L$; $y = L$ is a horizontal asymptote.

(b) End behavior: As $x \to -\infty$, the values of $R(x)$ approach L [symbolized by $\lim_{x \to -\infty} R(x) = L$]. That is, the points on the graph of R are getting closer to the line $y = L$; $y = L$ is a horizontal asymptote.

(c) As x approaches c, the values of $R(x) \to \infty$ [for $x < c$, this is symbolized by $\lim_{x \to c^-} R(x) = \infty$; for $x > c$, this is symbolized by $\lim_{x \to c^+} R(x) = \infty$]. That is, the points on the graph of R are getting closer to the line $x = c$; $x = c$ is a vertical asymptote.

(d) As x approaches c, the values of $|R(x)| \to \infty$ [for $x < c$, this is symbolized by $\lim_{x \to c^-} R(x) = -\infty$; for $x > c$, this is symbolized by $\lim_{x \to c^+} R(x) = \infty$]. That is, the points on the graph of R are getting closer to the line $x = c$; $x = c$ is a vertical asymptote.

A horizontal asymptote, when it occurs, describes the end behavior of the graph as $x \to \infty$ or as $x \to -\infty$. **The graph of a function may intersect a horizontal asymptote.**

A vertical asymptote, when it occurs, describes the behavior of the graph when x is close to some number c. **The graph of a function will never intersect a vertical asymptote.**

1.6 Assess Your Understanding

Concepts and Vocabulary

1. Suppose that the graph of a function f is known. Then the graph of $y = f(x - 2)$ may be obtained by a(n) _____ shift of the graph of f to the _____ a distance of 2 units.

2. Suppose that the graph of a function f is known. Then the graph of $y = f(-x)$ may be obtained by a reflection about the _____-axis of the graph of the function $y = f(x)$.

3. Suppose that the x-intercepts of the graph of $y = f(x)$ are $-2, 1$, and 5. The x-intercepts of $y = f(x + 3)$ are _____, _____, and _____.

4. ***True or False*** The graph of $y = -f(x)$ is the reflection about the x-axis of the graph of $y = f(x)$.

5. ***True or False*** To obtain the graph of $y = f(x + 2) - 3$, shift the graph of $y = f(x)$ horizontally to the right 2 units and vertically down 3 units.

6. ***True or False*** Suppose that the x-intercepts of the graph of $y = f(x)$ are -3 and 2. Then the x-intercepts of the graph of $y = 2f(x)$ are -3 and 2.

Skill Building

In Problems 7–18, match each graph to one of the following functions:

A. $y = x^2 + 2$ | *B.* $y = -x^2 + 2$ | *C.* $y = |x| + 2$ | *D.* $y = -|x| + 2$

E. $y = (x - 2)^2$ | *F.* $y = -(x + 2)^2$ | *G.* $y = |x - 2|$ | *H.* $y = -|x + 2|$

I. $y = 2x^2$ | *J.* $y = -2x^2$ | *K.* $y = 2|x|$ | *L.* $y = -2|x|$

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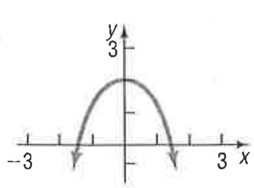

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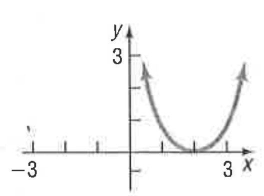

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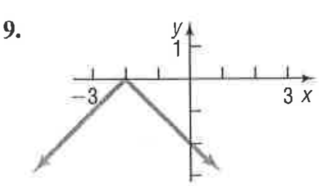

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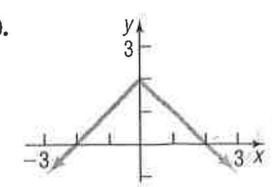

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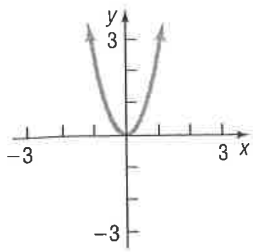

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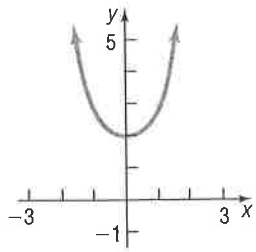

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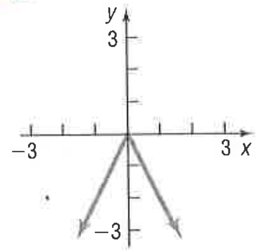

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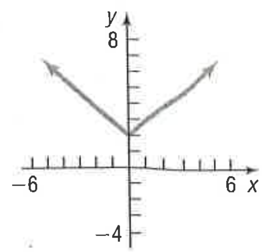

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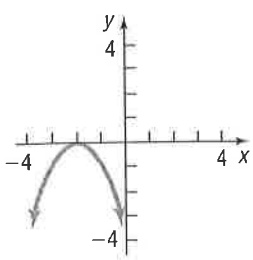

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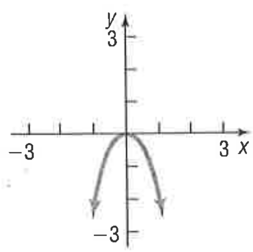

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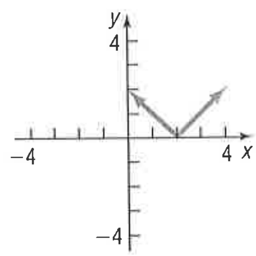

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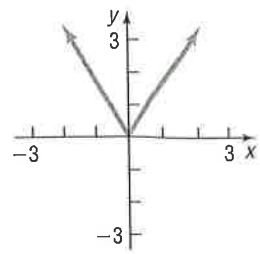


In Problems 19–26, write the function whose graph is the graph of $y = x^3$, but is:

19. Shifted to the right 4 units

20. Shifted to the left 4 units

21. Shifted up 4 units

22. Shifted down 4 units

23. Reflected about the y-axis

24. Reflected about the x-axis

25. Vertically stretched by a factor of 4

26. Horizontally stretched by a factor of 4

In Problems 27–30, find the function that is finally graphed after the following transformations are applied to the graph of $y = \sqrt{x}$.

27. (1) Shift up 2 units
(2) Reflect about the x-axis
(3) Reflect about the y-axis

28. (1) Reflect about the x-axis
(2) Shift right 3 units
(3) Shift down 2 units

29. (1) Reflect about the x-axis
(2) Shift up 2 units
(3) Shift left 3 units

30. (1) Shift up 2 units
(2) Reflect about the y-axis
(3) Shift left 3 units

31. If $(3, 0)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = -f(x)$?
(a) $(0, 3)$ (b) $(0, -3)$
(c) $(3, 0)$ (d) $(-3, 0)$

32. If $(3, 0)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = f(-x)$?
(a) $(0, 3)$ (b) $(0, -3)$
(c) $(3, 0)$ (d) $(-3, 0)$

33. If $(0, 3)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = 2f(x)$?
(a) $(0, 3)$ (b) $(0, 2)$
(c) $(0, 6)$ (d) $(6, 0)$

34. If $(3, 0)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = \frac{1}{2}f(x)$?
(a) $(3, 0)$ (b) $\left(\frac{3}{2}, 0\right)$
(c) $\left(0, \frac{3}{2}\right)$ (d) $\left(\frac{1}{2}, 0\right)$

In Problems 35–64, graph each function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function (for example, $y = x^2$) and show all stages.

35. $f(x) = x^2 - 1$

36. $f(x) = x^2 + 4$

37. $g(x) = x^3 + 1$

38. $g(x) = x^3 - 1$

39. $h(x) = \sqrt{x - 2}$

40. $h(x) = \sqrt{x + 1}$

41. $f(x) = (x - 1)^3 + 2$

42. $f(x) = (x + 2)^3 - 3$

43. $g(x) = 4\sqrt{x}$

44. $g(x) = \frac{1}{2}\sqrt{x}$

45. $h(x) = \frac{1}{2x}$

46. $h(x) = 3\sqrt[3]{x}$

47. $f(x) = -\sqrt[3]{x}$

48. $f(x) = -\sqrt{x}$

49. $g(x) = \sqrt[3]{-x}$

50. $g(x) = -\frac{1}{x}$

51. $h(x) = -x^3 + 2$

52. $h(x) = \frac{1}{-x} + 2$

53. $f(x) = 2(x + 1)^2 - 3$

54. $f(x) = 3(x - 2)^2 + 1$

55. $g(x) = \sqrt{x - 2} + 1$

56. $g(x) = |x + 1| - 3$

57. $h(x) = \sqrt{-x} - 2$

58. $h(x) = \dfrac{4}{x} + 2$

59. $f(x) = -(x + 1)^3 - 1$

60. $f(x) = -4\sqrt{x - 1}$

61. $g(x) = 2|1 - x|$

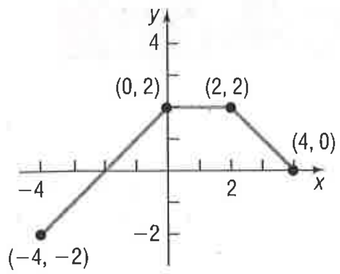
62. $g(x) = 4\sqrt{2 - x}$

In Problems 63–66, the graph of a function f is illustrated. Use the graph of f as the first step toward graphing each of the following functions:

(a) $F(x) = f(x) + 3$ *(b)* $G(x) = f(x + 2)$ *(c)* $P(x) = -f(x)$ *(d)* $H(x) = f(x + 1) - 2$

(e) $Q(x) = \frac{1}{2}f(x)$ *(f)* $g(x) = f(-x)$ *(g)* $h(x) = f(2x)$

63.



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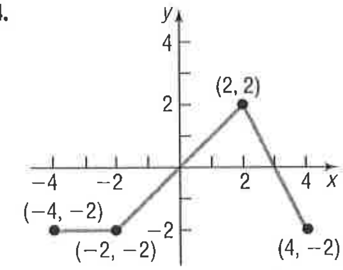

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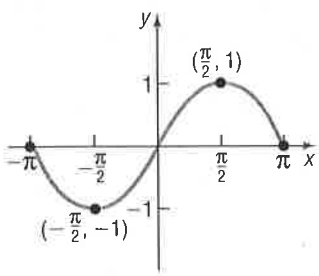

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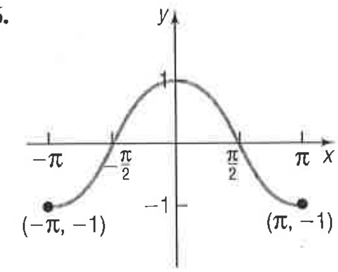


Applications and Extensions

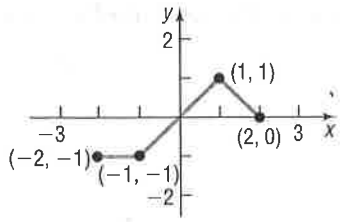
67. Suppose that the x-intercepts of the graph of $y = f(x)$ are -5 and 3.
(a) What are the x-intercepts of the graph of $y = f(x + 2)$?
(b) What are the x-intercepts of the graph of $y = f(x - 2)$?
(c) What are the x-intercepts of the graph of $y = 4f(x)$?
(d) What are the x-intercepts of the graph of $y = f(-x)$?

68. Suppose that the x-intercepts of the graph of $y = f(x)$ are -8 and 1.
(a) What are the x-intercepts of the graph of $y = f(x + 4)$?
(b) What are the x-intercepts of the graph of $y = f(x - 3)$?
(c) What are the x-intercepts of the graph of $y = 2f(x)$?
(d) What are the x-intercepts of the graph of $y = f(-x)$?

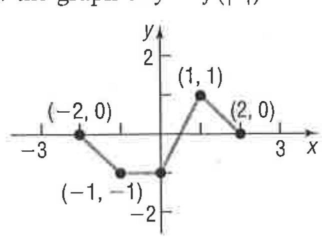
69. Suppose that the function $y = f(x)$ is increasing on the interval $(-1, 5)$.
(a) Over what interval is the graph of $y = f(x + 2)$ increasing?
(b) Over what interval is the graph of $y = f(x - 5)$ increasing?
(c) What can be said about the graph of $y = -f(x)$?
(d) What can be said about the graph of $y = f(-x)$?

70. Suppose that the function $y = f(x)$ is decreasing on the interval $(-2, 7)$.
(a) Over what interval is the graph of $y = f(x + 2)$ decreasing?
(b) Over what interval is the graph of $y = f(x - 5)$ decreasing?
(c) What can be said about the graph of $y = -f(x)$?
(d) What can be said about the graph of $y = f(-x)$?

71. The graph of a function f is illustrated in the figure.
(a) Draw the graph of $y = |f(x)|$.
(b) Draw the graph of $y = f(|x|)$.

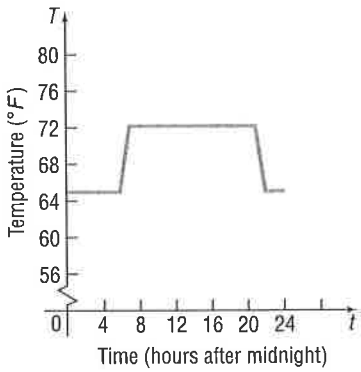


72. The graph of a function f is illustrated in the figure.
(a) Draw the graph of $y = |f(x)|$.
(b) Draw the graph of $y = f(|x|)$.



1.6

73. Thermostat Control Energy conservation experts estimate that homeowners can save 5 to 10 percent on winter heating bills by programming their thermostats 5 to 10 degrees lower while sleeping. In the given graph, the temperature T (in degrees Fahrenheit) of a home is given as a function of time t (in hours after midnight) over a 24-hour period.



(a) At what temperature is the thermostat set during daytime hours? At what temperature is the thermostat set overnight?

(b) The homeowner reprograms the thermostat to $y = T(t) - 2$. Explain how this affects the temperature in the house. Graph this new function.

(c) The homeowner reprograms the thermostat to $y = T(t + 1)$. Explain how this affects the temperature in the house. Graph this new function.

Source: Roger Albright, *547 Ways to be Fuel Smart,* 2000.

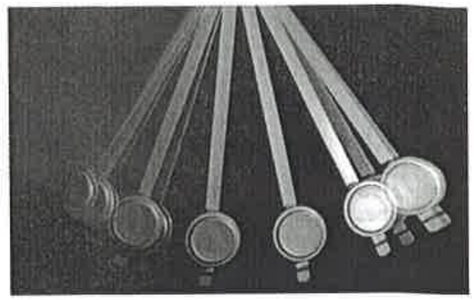
74. Temperature Measurements The relationship between the Celsius (°C) and Fahrenheit (°F) scales for measuring temperature is given by the equation

$$F = \frac{9}{5}C + 32$$

The relationship between the Celsius (°C) and Kelvin (K) scales is $K = C + 273$. Graph the equation $F = \frac{9}{5}C + 32$ using degrees Fahrenheit on the y-axis and degrees Celsius on the x-axis. Use the techniques introduced in this section to obtain the graph showing the relationship between Kelvin and Fahrenheit temperatures.

75. Period of a Pendulum The period T (in seconds) of a simple pendulum is a function of its length l (in feet) defined by the equation

$$T = 2\pi\sqrt{\frac{l}{g}}$$



where $g \approx 32.2$ feet per second per second is the acceleration of gravity.

(a) Use a graphing utility to graph the function $T = T(l)$.

(b) Now graph the functions $T = T(l + 1)$, $T = T(l + 2)$, and $T = T(l + 3)$.

(c) Discuss how adding to the length l changes the period T.

(d) Now graph the functions $T = T(2l)$, $T = T(3l)$, and $T = T(4l)$.

(e) Discuss how multiplying the length l by factors of 2, 3, and 4 changes the period T.

76. Cigar Company Profits The daily profits of a cigar company from selling x cigars are given by

$$p(x) = -0.05x^2 + 100x - 2000$$

The government wishes to impose a tax on cigars (sometimes called a *sin tax*) that gives the company the option of either paying a flat tax of \$10,000 per day or a tax of 10% on profits. As chief financial officer (CFO) of the company, you need to decide which tax is the better option for the company.

(a) On the same screen, graph $Y_1 = p(x) - 10{,}000$ and $Y_2 = (1 - 0.10)p(x)$.

(b) Based on the graph, which option would you select? Why?

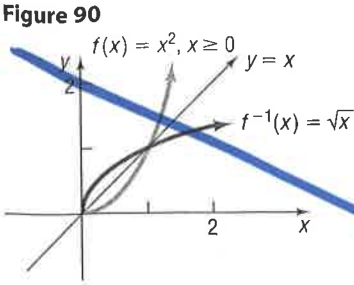
(c) Using the terminology learned in this section, describe each graph in terms of the graph of $p(x)$.

(d) Suppose that the government offered the options of a flat tax of \$4800 or a tax of 10% on profits. Which would you select? Why?

Discussion and Writing

77. Suppose that the graph of a function f is known. Explain how the graph of $y = 4f(x)$ differs from the graph of $y = f(4x)$.

Figure 90



Step 2: We solve for y to get the explicit form of the inverse. Since $y \geq 0$, only one solution for y is obtained: $y = \sqrt{x}$. So $f^{-1}(x) = \sqrt{x}$.

Step 3: **Check:** $f^{-1}(f(x)) = f^{-1}(x^2) = \sqrt{x^2} = |x| = x$ since $x \geq 0$

$f(f^{-1}(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$

Figure 90 illustrates the graphs of $f(x) = x^2, x \geq 0$, and $f^{-1}(x) = \sqrt{x}$.

SUMMARY

1. If a function f is one-to-one, then it has an inverse function f^{-1}.
2. Domain of f = Range of f^{-1}; Range of f = Domain of f^{-1}.
3. To verify that f^{-1} is the inverse of f, show that $f^{-1}(f(x)) = x$ for every x in the domain of f and $f(f^{-1}(x)) = x$ for every x in the domain of f^{-1}.
4. The graphs of f and f^{-1} are symmetric with respect to the line $y = x$.
5. To find the range of a one-to-one function f, find the domain of its inverse function f^{-1}.

1.7 Assess Your Understanding

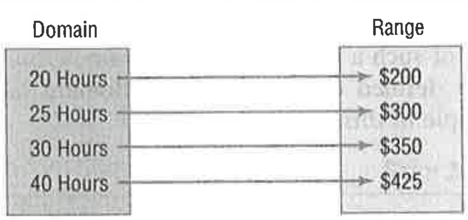
Concepts and Vocabulary

1. If every horizontal line intersects the graph of a function f at no more than one point, f is a(n) _____ function.
2. If f^{-1} denotes the inverse of a function f, then the graphs of f and f^{-1} are symmetric with respect to the line _____.
3. If the domain of a one-to-one function f is $[4, \infty)$, the range of its inverse, f^{-1}, is _____.
4. ***True or False*** If f and g are inverse functions, the domain of f is the same as the domain of g.

Skill Building

In Problems 5–12, determine whether the function is one-to-one.

5.



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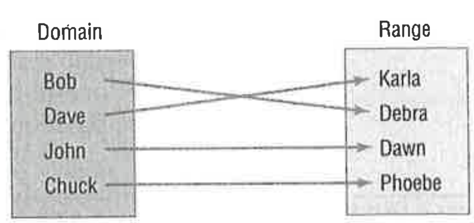

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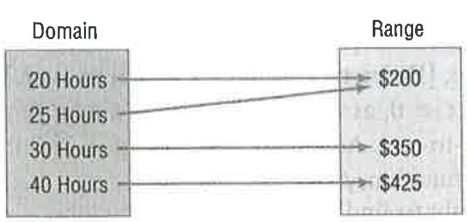

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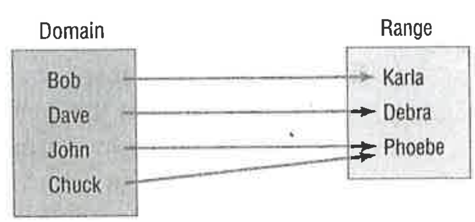


9. $\{(2, 6), (-3, 6), (4, 9), (1, 10)\}$

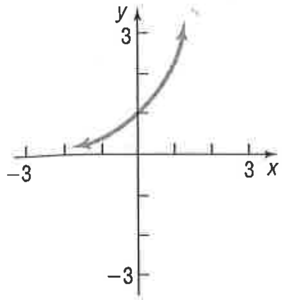
10. $\{(-2, 5), (-1, 3), (3, 7), (4, 12)\}$

11. $\{(0, 0), (1, 1), (2, 16), (3, 81)\}$

12. $\{(1, 2), (2, 8), (3, 18), (4, 32)\}$

In Problems 13–18, the graph of a function f is given. Use the horizontal-line test to determine whether f is one-to-one.

13.



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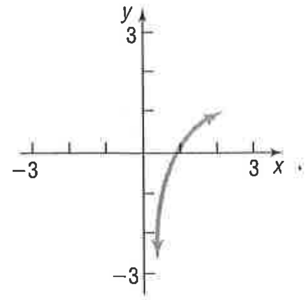

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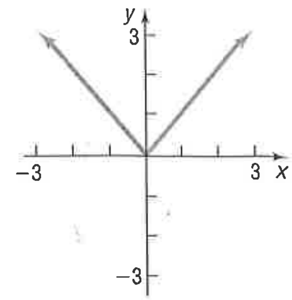

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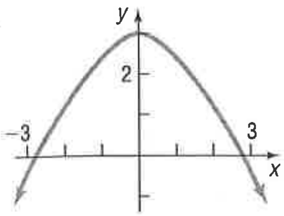

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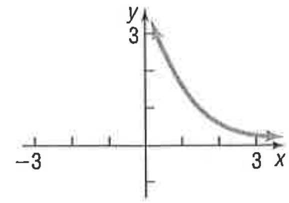

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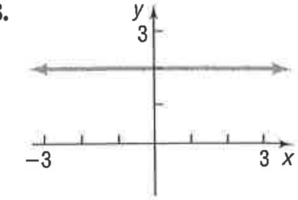
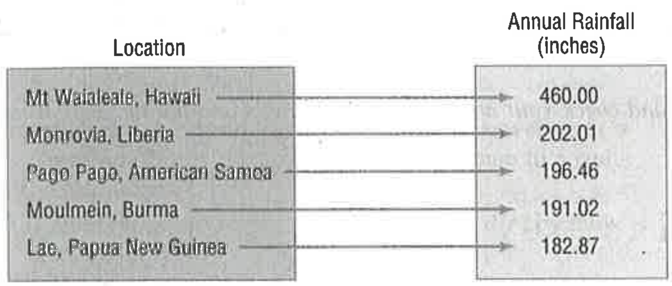


In Problems 19–26, find the inverse of each one-to-one function. State the domain and the range of each inverse function.

19.



Source: Information Please Almanac

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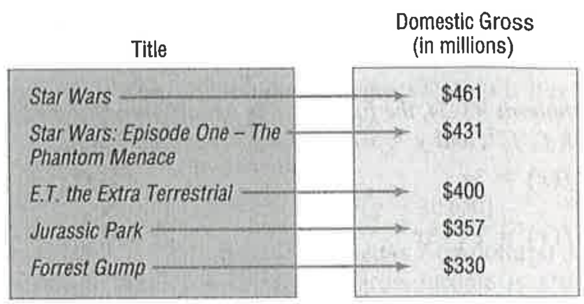

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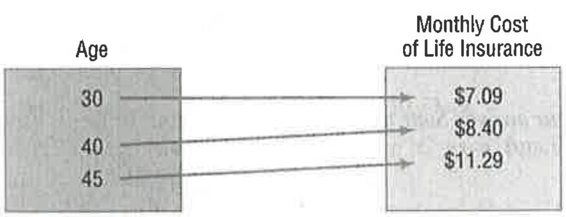
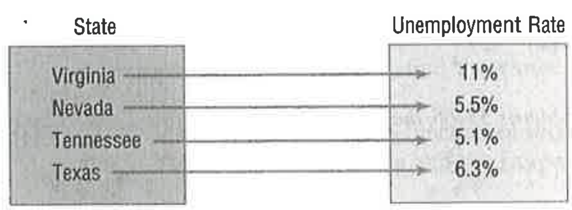


Source: eterm.com

22.



Source: United States Statistical Abstract

23. $\{(-3, 5), (-2, 9), (-1, 2), (0, 11), (1, -5)\}$

24. $\{(-2, 2), (-1, 6), (0, 8), (1, -3), (2, 9)\}$

25. $\{(-2, 1), (-3, 2), (-10, 0), (1, 9), (2, 4)\}$

26. $\{(-2, -8), (-1, -1), (0, 0), (1, 1), (2, 8)\}$

In Problems 27–36, verify that the functions f and g are inverses of each other by showing that $f(g(x)) = x$ and $g(f(x)) = x$. Give any values of x that need to be excluded.

27. $f(x) = 3x + 4; \quad g(x) = \frac{1}{3}(x - 4)$

28. $f(x) = 3 - 2x; \quad g(x) = -\frac{1}{2}(x - 3)$

29. $f(x) = 4x - 8; \quad g(x) = \frac{x}{4} + 2$

30. $f(x) = 2x + 6; \quad g(x) = \frac{1}{2}x - 3$

31. $f(x) = x^3 - 8; \quad g(x) = \sqrt[3]{x + 8}$

32. $f(x) = (x - 2)^2, x \geq 2; \quad g(x) = \sqrt{x} + 2$

33. $f(x) = \frac{1}{x}; \quad g(x) = \frac{1}{x}$

34. $f(x) = x; \quad g(x) = x$

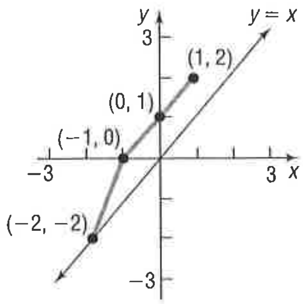
35. $f(x) = \frac{2x + 3}{x + 4}; \quad g(x) = \frac{4x - 3}{2 - x}$

36. $f(x) = \frac{x - 5}{2x + 3}; \quad g(x) = \frac{3x + 5}{1 - 2x}$

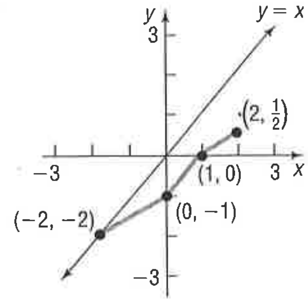
1.7

In Problems 37–42, the graph of a one-to-one function f is given. Draw the graph of the inverse function f^{-1}. For convenience (and as a hint), the graph of $y = x$ is also given.

37.



38.



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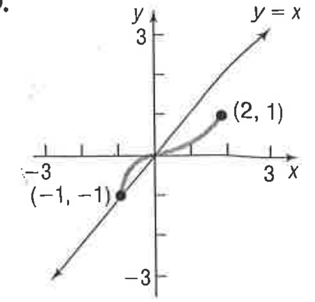

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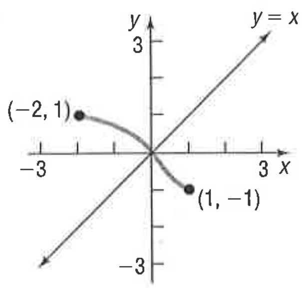

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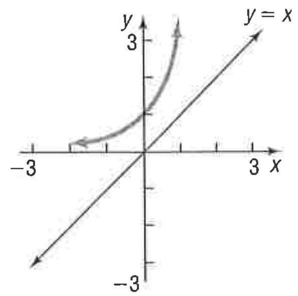

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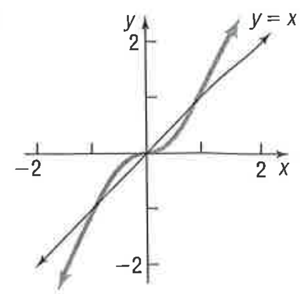

In Problems 43–54, the function f is one-to-one. Find its inverse and check your answer. State the domain and the range of f and f^{-1}. Graph f, f^{-1}, and $y = x$ on the same coordinate axes.

43. $f(x) = 3x$

44. $f(x) = -4x$

45. $f(x) = 4x + 2$

46. $f(x) = 1 - 3x$

47. $f(x) = x^3 - 1$

48. $f(x) = x^3 + 1$

49. $f(x) = x^2 + 4 \quad x \geq 0$

50. $f(x) = x^2 + 9 \quad x \geq 0$

51. $f(x) = \dfrac{4}{x}$

52. $f(x) = -\dfrac{3}{x}$

53. $f(x) = \dfrac{1}{x - 2}$

54. $f(x) = \dfrac{4}{x + 2}$

In Problems 55–66, the function f is one-to-one. Find its inverse and check your answer. State the domain of f and find its range using f^{-1}.

55. $f(x) = \dfrac{2}{3 + x}$

56. $f(x) = \dfrac{4}{2 - x}$

57. $f(x) = \dfrac{3x}{x + 2}$

58. $f(x) = -\dfrac{2x}{x - 1}$

59. $f(x) = \dfrac{2x}{3x - 1}$

60. $f(x) = -\dfrac{3x + 1}{x}$

61. $f(x) = \dfrac{3x + 4}{2x - 3}$

62. $f(x) = \dfrac{2x - 3}{x + 4}$

63. $f(x) = \dfrac{2x + 3}{x + 2}$

64. $f(x) = \dfrac{-3x - 4}{x - 2}$

65. $f(x) = \dfrac{x^2 - 4}{2x^2} \quad x > 0$

66. $f(x) = \dfrac{x^2 + 3}{3x^2} \quad x > 0$

Applications and Extensions

67. Use the graph of $y = f(x)$ given in Problem 37 to evaluate the following:
(a) $f(-1)$ (b) $f(1)$ (c) $f^{-1}(1)$ (d) $f^{-1}(2)$

68. Use the graph of $y = f(x)$ given in Problem 38 to evaluate the following:
(a) $f(2)$ (b) $f(1)$ (c) $f^{-1}(0)$ (d) $f^{-1}(-1)$

69. If $f(7) = 13$ and f is one-to-one, what is $f^{-1}(13)$?

70. If $g(-5) = 3$ and g is one-to-one, what is $g^{-1}(3)$?

71. The domain of a one-to-one function f is $[5, \infty)$, and its range is $[-2, \infty)$. State the domain and the range of f^{-1}.

72. The domain of a one-to-one function f is $[0, \infty)$, and its range is $[5, \infty)$. State the domain and the range of f^{-1}.

73. The domain of a one-to-one function g is the set of all real numbers, and its range is $[0, \infty)$. State the domain and the range of g^{-1}.

74. The domain of a one-to-one function g is $[0, 15]$, and its range is $(0, 8)$. State the domain and the range of g^{-1}.

75. A function $y = f(x)$ is increasing on the interval $(0, 5)$. What conclusions can you draw about the graph of $y = f^{-1}(x)$?

76. A function $y = f(x)$ is decreasing on the interval $(0, 5)$. What conclusions can you draw about the graph of $y = f^{-1}(x)$?

77. Find the inverse of the linear function

$$f(x) = mx + b \quad m \neq 0$$

78. Find the inverse of the function

$$f(x) = \sqrt{r^2 - x^2} \quad 0 \leq x \leq r$$

79. A function f has an inverse function. If the graph of f lies in quadrant I, in which quadrant does the graph of f^{-1} lie?

80. A function f has an inverse function. If the graph of f lies in quadrant II, in which quadrant does the graph of f^{-1} lie?

81. The function $f(x) = |x|$ is not one-to-one. Find a suitable restriction on the domain of f so that the new function that results is one-to-one. Then find the inverse of f.

82. The function $f(x) = x^4$ is not one-to-one. Find a suitable restriction on the domain of f so that the new function that results is one-to-one. Then find the inverse of f.

In applications, the symbols used for the independent and dependent variables are often based on common usage. So, rather than using $y = f(x)$ to represent a function, an applied problem might use $C = C(q)$ to represent the cost C of manufacturing q units of a good since, in economics, q is used for output. Because of this, the inverse notation f^{-1} used in a pure mathematics problem is not used when finding inverses of applied problems. Rather, the inverse of a function such as $C = C(q)$ will be $q = q(C)$. So $C = C(q)$ is a function that represents the cost C as a function of the output q, while $q = q(C)$ is a function that represents the output q as a function of the cost C. Problems 83–86 illustrate this idea.

83. Vehicle Stopping Distance Taking into account reaction time, the distance d (in feet) that a car requires to come to a complete stop while traveling r miles per hour is given by the function

$$d(r) = 6.97r - 90.39$$

(a) Express the speed r at which the car is traveling as a function of the distance d required to come to a complete stop.
(b) Verify that $r = r(d)$ is the inverse of $d = d(r)$ by showing that $r(d(r)) = r$ and $d(r(d)) = d$.
(c) Predict the speed that a car was traveling if the distance required to stop was 300 feet.

84. Height and Head Circumference The head circumference C of a child is related to the height H of the child (both in inches) through the function

$$H(C) = 2.15C - 10.53$$

(a) Express the head circumference C as a function of height H.
(b) Verify that $C = C(H)$ is the inverse of $H = H(C)$ by showing that $H(C(H)) = H$ and $C(H(C)) = C$.
(c) Predict the head circumference of a child who is 26 inches tall.

85. Ideal Body Weight The ideal body weight W for men (in kilograms) as a function of height h (in inches) is given by the function

$$W(h) = 50 + 2.3(h - 60)$$

(a) What is the ideal weight of a 6-foot male?
(b) Express the height h as a function of weight W.
(c) Verify that $h = h(W)$ is the inverse of $W = W(h)$ by showing that $h(W(h)) = h$ and $W(h(W)) = W$.
(d) What is the height of a male who is at his ideal weight of 80 kilograms?

Note: The ideal body weight W for women (in kilograms) as a function of height h (in inches) is given by $W(h) = 45.5 + 2.3(h - 60)$.

86. Temperature Conversion The function $F(C) = \frac{9}{5}C + 32$ converts a temperature from C degrees Celsius to F degrees Fahrenheit.

(a) Express the temperature in degrees Celsius C as a function of the temperature in degrees Fahrenheit F.
(b) Verify that $C = C(F)$ is the inverse of $F = F(C)$ by showing that $C(F(C)) = C$ and $F(C(F)) = F$.
(c) What is the temperature in degrees Celsius if it is 70 degrees Fahrenheit?

87. Income Taxes The function

$$T(g) = 4220 + 0.25(g - 30{,}650)$$

represents the 2006 federal income tax T (in dollars) due for a "single" filer whose adjusted gross income is g dollars, where $30{,}650 \leq g \leq 74{,}200$.

(a) What is the domain of the function T?
(b) Given that the tax due T is an increasing linear function of adjusted gross income g, find the range of the function T.
(c) Find adjusted gross income g as a function of federal income tax T. What are the domain and the range of this function?

88. Income Taxes The function

$$T(g) = 1510 + 0.15(g - 15{,}100)$$

represents the 2006 federal income tax T (in dollars) due for a "married filing jointly" filer whose adjusted gross income is g dollars, where $15{,}100 \leq g \leq 61{,}300$.

(a) What is the domain of the function T?
(b) Given that the tax due T is an increasing linear function of adjusted gross income g, find the range of the function T.
(c) Find adjusted gross income g as a function of federal income tax T. What are the domain and the range of this function?

89. Given

$$f(x) = \frac{ax + b}{cx + d}$$

find $f^{-1}(x)$. If $c \neq 0$, under what conditions on a, b, c, and d is $f = f^{-1}$?