ANSWERS:

1)	*Solution.* Here $a_n = \frac{n}{n+2}$. Then the limit is $$\lim_{n\to\infty}\frac{n}{n+2} = \lim_{n\to\infty}\frac{n+2-2}{n+2} = \lim_{n\to\infty}\left(1-\frac{2}{n+2}\right) = \lim_{n\to\infty}1 - \lim_{n\to\infty}\frac{2}{n+2} = 1-0 =$$ Thus, the sequence converges to 1.
2)	*Solution.* We easily can see that nth term of the sequence is given by the formula $a_n = \frac{(-1)^{n-1}n}{2^{n-1}}$. Since $-n \le (-1)^{n-1}n \le n$, we can write: $$\frac{-n}{2^{n-1}} \le \frac{(-1)^{n-1}n}{2^{n-1}} \le \frac{n}{2^{n-1}}.$$ Using L'Hopital's rule, we obtain $$\lim_{x\to\infty}\left(\pm\frac{x}{2^{x-1}}\right) = \pm\lim_{x\to\infty}\frac{x}{2^{x-1}} = \pm\lim_{x\to\infty}\frac{1}{2^{x-1}\ln 2} = 0.$$ Hence, by the squeezing theorem, the limit of the initial sequence is $$\lim_{n\to\infty}\frac{(-1)^{n-1}n}{2^{n-1}} = 0.$$
3)	*Solution.* Divide by the highest power in the numerator and denominator: $$\lim_{n\to\infty}\frac{2n+3}{5n-7} = \lim_{n\to\infty}\frac{\frac{2n+3}{n}}{\frac{5n-7}{n}} = \lim_{n\to\infty}\frac{2+\frac{3}{n}}{5-\frac{7}{n}} = \frac{2}{5}.$$ Hence, the sequence converges to $\frac{2}{5}$.

4)	*Solution.* As L'Hopital's rule yields $$\lim_{x\to\infty}\left(\pm\frac{x}{2^{x-1}}\right)=\pm\lim_{x\to\infty}\frac{x}{2^{x-1}}=\pm\lim_{x\to\infty}\frac{1}{2^{x-1}\ln 2}=0.$$ Since the limit is finite, the given sequence converges.
5)	Determine whether the sequence $\{\sqrt{n+2}-\sqrt{n+1}\}$ converges or diverges. *Solution.* Multiply this expression by the quotient $\frac{\sqrt{n+2}+\sqrt{n+1}}{\sqrt{n+2}+\sqrt{n+1}}=1$. We obtain: $$\lim_{n\to\infty}(\sqrt{n+2}-\sqrt{n+1})=\lim_{n\to\infty}(\sqrt{n+2}-\sqrt{n+1})\frac{\sqrt{n+2}+\sqrt{n+1}}{\sqrt{n+2}+\sqrt{n+1}}$$ $$=\lim_{n\to\infty}\frac{(\sqrt{n+2})^2-(\sqrt{n+1})^2}{\sqrt{n+2}+\sqrt{n+1}}=\lim_{n\to\infty}\frac{n+2-(n+1)}{\sqrt{n+2}+\sqrt{n+1}}=\lim_{n\to\infty}\frac{1}{\sqrt{n+2}+\sqrt{n+1}}=0$$ This means that the sequence converges.
6)	*Solution.* The $(n+1)$th term of the sequence is given by the formula $$a_{n+1}=\frac{5(n+1)-7}{3(n+1)+4}=\frac{5n-2}{3n+7}.$$ Check the inequality $a_n\le a_{n+1}$: $$\frac{5n-7}{3n+4}\le\frac{5n-2}{3n+7},\ \Rightarrow\ \frac{5n-7}{3n+4}-\frac{5n-2}{3n+7}\le 0,\ \Rightarrow\ \frac{(5n-7)(3n+7)-(5n-2)(3n+4)}{(3n+4)(3n+7)}\le 0,$$ $$\Rightarrow\frac{15n^2-21n+35n-49-(15n^2-6n+20n-8)}{(3n+4)(3n+7)}\le 0,$$ $$\Rightarrow\frac{15n^2-21n+35n-49-15n^2+6n-20n+8}{(3n+4)(3n+7)}\le 0,\ \Rightarrow\ \frac{-41}{(3n+4)(3n+7)}\le 0.$$ The last inequality is obvious, since the numerator is negative and $3n+4\ge 0$ and $3n+7\ge 0$ for $n\ge 1$. The this sequence is increasing.

7) *Solution.*
Write out the first few terms of the sequence:

$$\left\{\frac{2^n+3}{2^n+1}\right\} = \left\{\frac{5}{3}, \frac{7}{5}, \frac{11}{9}, \frac{19}{17}, \frac{35}{33}, \ldots\right\}.$$

We see that this is a decreasing sequence. To confirm this, we prove the inequality $a_n \geq a_{n+1}$. W

$$a_n = \frac{2^n+3}{2^n+1}, \quad a_{n+1} = \frac{2^{n+1}+3}{2^{n+1}+1}.$$

Then the condition $a_n \geq a_{n+1}$ implies that

$$\frac{2^n+3}{2^n+1} \geq \frac{2^{n+1}+3}{2^{n+1}+1}.$$

Multiply both sides of the inequality by $\left(2^n+1\right)\left(2^{n+1}+1\right)$:

$$\left(2^n+3\right)\left(2^{n+1}+1\right) \geq \left(2^{n+1}+3\right)\left(2^n+1\right),$$
$$\Rightarrow 2^n 2^{n+1} + 3 \cdot 2^{n+1} + 2^n + 3 \geq 2^n 2^{n+1} + 3 \cdot 2^n + 2^{n+1} + 3, \quad \Rightarrow 2 \cdot 2^{n+1} \geq 2 \cdot 2^n,$$
$$\Rightarrow 2^{n+1} \geq 2^n, \quad \Rightarrow 2 \geq 1.$$

Since the last inequality is true, we can conclude that the sequence is decreasing.