11.5 Exercises

1. (a) What is an alternating series?
(b) Under what conditions does an alternating series converge?
(c) If these conditions are satisfied, what can you say about the remainder after n terms?

2–20 Test the series for convergence or divergence.

2. $\frac{2}{3} - \frac{2}{5} + \frac{2}{7} - \frac{2}{9} + \frac{2}{11} - \cdots$

3. $-\frac{2}{5} + \frac{4}{6} - \frac{6}{7} + \frac{8}{8} - \frac{10}{9} + \cdots$

4. $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{6}} - \cdots$

5. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$

6. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+4)}$

7. $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$

8. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+2}}$

9. $\sum_{n=1}^{\infty} (-1)^n e^{-n}$

10. $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{2n+3}$

11. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+4}$

12. $\sum_{n=1}^{\infty} (-1)^{n+1} n e^{-n}$

13. $\sum_{n=1}^{\infty} (-1)^{n-1} e^{2/n}$

14. $\sum_{n=1}^{\infty} (-1)^{n-1} \arctan n$

15. $\sum_{n=0}^{\infty} \frac{\sin\left(n+\frac{1}{2}\right)\pi}{1+\sqrt{n}}$

16. $\sum_{n=1}^{\infty} \frac{n \cos n\pi}{2^n}$

17. $\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{\pi}{n}\right)$

18. $\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{\pi}{n}\right)$

19. $\sum_{n=1}^{\infty} (-1)^n \frac{n^n}{n!}$

20. $\sum_{n=1}^{\infty} (-1)^n \left(\sqrt{n+1} - \sqrt{n}\right)$

22. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{8^n}$

23–26 Show that the series is convergent. How many terms of the series do we need to add in order to find the sum to the indicated accuracy?

23. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^6}$ $(|\text{error}| < 0.00005)$

24. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\,5^n}$ $(|\text{error}| < 0.0001)$

25. $\sum_{n=0}^{\infty} \frac{(-1)^n}{10^n n!}$ $(|\text{error}| < 0.000005)$

26. $\sum_{n=1}^{\infty} (-1)^{n-1} n e^{-n}$ $(|\text{error}| < 0.01)$

27–30 Approximate the sum of the series correct to four decimal places.

27. $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}$

28. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^6}$

29. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{10^n}$

30. $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n!}$

31. Is the 50th partial sum s_{50} of the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1}/n$ an overestimate or an underestimate of the total sum? Explain.

32–34 For what values of p is each series convergent?

32. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$