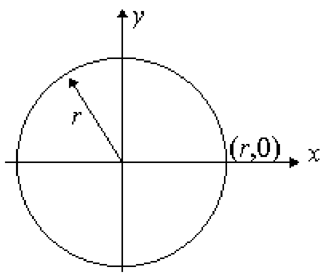
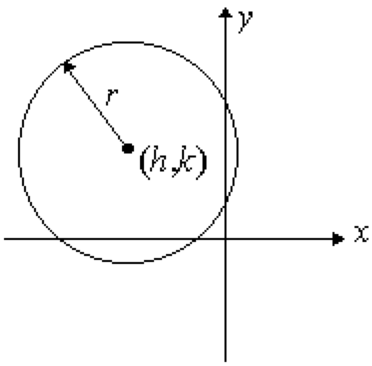
Circle



The circle with centre $(0, 0)$ and radius r has the equation:

$$x^2 + y^2 = r^2$$



The circle with centre (h, k) and radius r has the equation:

$$(x - h)^2 + (y - k)^2 = r^2$$

General Form of the Circle

An equation which can be written in the following form (with constants D, E, F) represents a **circle**:

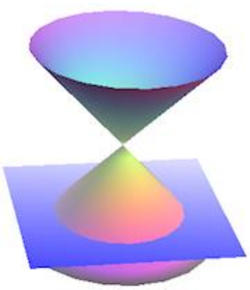
$$x^2 + y^2 + Dx + Ey + F = 0$$

Formal Definition

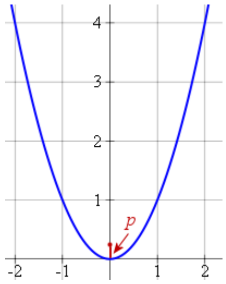
A circle is the locus of points that are equidistant from a fixed point (the center).

Conic Section

If we slice one of the cones with a plane at right angles to the axis of the cone, the shape formed is a circle.



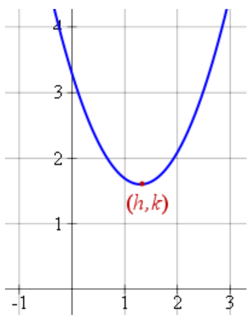
Parabola



Parabola with Vertical Axis

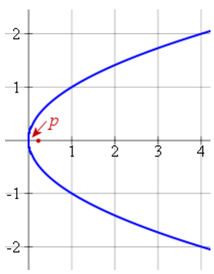
A parabola with focal distance p has equation:

$$x^2 = 4py$$



If the axis of a parabola is **vertical**, and the vertex is at (h, k), we have

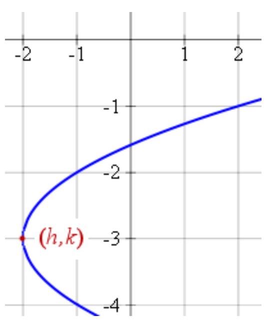
$$(x - h)^2 = 4p(y - k)$$



Parabola with Horizontal Axis

In this case, we have the relation:

$$y^2 = 4px$$



If the axis of a parabola is horizontal, and the vertex is at (h, k), the equation becomes

$$(y - k)^2 = 4p(x - h)$$

Formal Definition

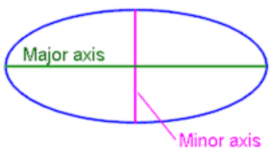
A parabola is the locus of points that are equidistant from a point (the focus) and a line (the directrix).

Conic Section

If we slice a cone parallel to the slant edge of the cone, the resulting shape is a parabola.

Ellipse

Horizontal Major Axis



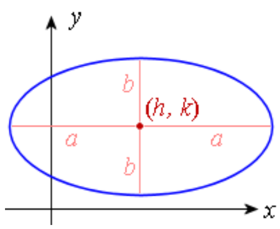
The equation for an ellipse with a horizontal major axis and center (0,0) is given by:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The **foci** (plural of 'focus') of the ellipse (with horizontal major axis) are at $(-c,0)$ and $(c,0)$ where c is given by:

$$c = \sqrt{a^2 - b^2}$$

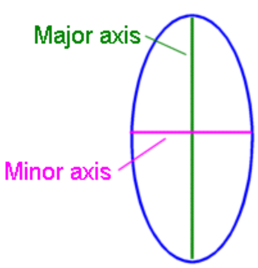
The **vertices** of an ellipse are at $(-a,0)$ and $(a,0)$.



A parabola with horizontal major axis and with center at (h, k) is given by:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Vertical Major Axis



If the major axis is **vertical**, then the formula becomes:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

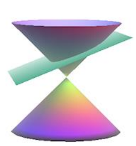
We always choose our a and b such that $a > b$.

Formal Definition

An ellipse is the locus of points whereby the sum of the distances from 2 fixed points (the foci) is constant..

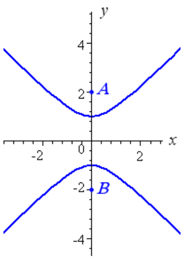
Conic Section

When we slice one of the cones at an angle to the sides of the cone, we get an **ellipse**, as seen in the view from the top (at right).



Hyperbola

North-south Opening

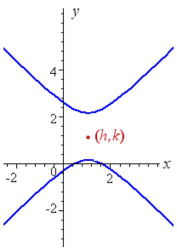


For a **north-south opening hyperbola:**

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

The slopes of the asymptotes are given by:

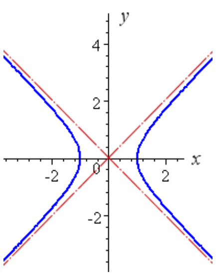
$$\pm \frac{a}{b}$$



For a "north-south" opening hyperbola with centre (h, k), we have:

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

East-west Opening

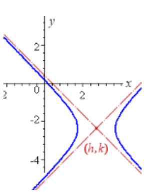


For an **east-west opening hyperbola:**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The slopes of the asymptotes are given by:

$$\pm \frac{b}{a}$$



For an "east-west" opening hyperbola with centre (h, k), we have:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Formal Definition

A hyperbola is the locus of points where the difference in the distance to two fixed foci is constant.

General Form of a Hyperbola

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

(such that $B^2 > 4AC$)

Conic Section

When we slice our double cone such that the plane passes througn both cones, we get a **hyperbola**.