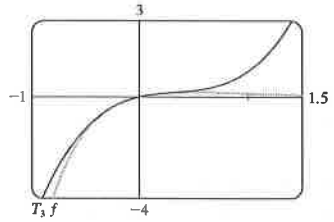
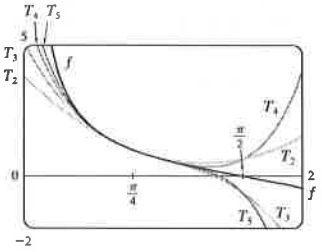
9. $x - 2x^2 + 2x^3$



11. $T_5(x) = 1 - 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 - \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3 + \frac{10}{3}\left(x - \frac{\pi}{4}\right)^4 - \frac{64}{15}\left(x - \frac{\pi}{4}\right)^5$



13. (a) $2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$ (b) 1.5625×10^{-5}

15. (a) $1 + \frac{2}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{4}{81}(x-1)^3$ (b) 0.000097

17. (a) $1 + \frac{1}{2}x^2$ (b) 0.0014

19. (a) $1 + x^2$ (b) 0.00006 **21.** (a) $x^2 - \frac{1}{6}x^4$ (b) 0.042

23. 0.17365 **25.** Four **27.** $-1.037 < x < 1.037$

29. $-0.86 < x < 0.86$ **31.** 21 m, no

37. (c) They differ by about 8×10^{-9} km.

CHAPTER 11 REVIEW ■ PAGE 802

True-False Quiz

1. False **3.** True **5.** False **7.** False **9.** False
11. True **13.** True **15.** False **17.** True
19. True **21.** True

Exercises

1. $\frac{1}{2}$ **3.** D **5.** 0 **7.** e^{12} **9.** 2 **11.** C **13.** C
15. D **17.** C **19.** C **21.** C **23.** CC **25.** AC
27. $\frac{1}{11}$ **29.** $\pi/4$ **31.** e^{-e} **35.** 0.9721
37. 0.18976224, error $< 6.4 \times 10^{-7}$
41. 4, $[-6, 2)$ **43.** 0.5, [2.5, 3.5)

45. $\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n\left[\frac{1}{(2n)!}\left(x - \frac{\pi}{6}\right)^{2n} + \frac{\sqrt{3}}{(2n+1)!}\left(x - \frac{\pi}{6}\right)^{2n+1}\right]$

47. $\sum_{n=0}^{\infty}(-1)^n x^{n+2}$, $R = 1$ **49.** $\ln 4 - \sum_{n=1}^{\infty}\frac{x^n}{n4^n}$, $R = 4$

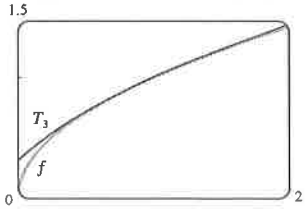
51. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{8n+4}}{(2n+1)!}$, $R = \infty$

53. $\frac{1}{2} + \sum_{n=1}^{\infty}\frac{1 \cdot 5 \cdot 9 \cdot \cdots \cdot (4n-3)}{n!\,2^{6n+1}}x^n$, $R = 16$

55. $C + \ln|x| + \sum_{n=1}^{\infty}\frac{x^n}{n \cdot n!}$

57. (a) $1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$

(b)



(c) 0.000006

59. $-\frac{1}{6}$

PROBLEMS PLUS ■ PAGE 805

1. $15!/5! = 10{,}897{,}286{,}400$
3. (b) 0 if $x = 0$, $(1/x) - \cot x$ if $x \neq k\pi$, k an integer
5. (a) $s_n = 3 \cdot 4^n$, $l_n = 1/3^n$, $p_n = 4^n/3^{n-1}$ (c) $\frac{2}{5}\sqrt{3}$
9. $(-1, 1)$, $\dfrac{x^3 + 4x^2 + x}{(1-x)^4}$

11. $\ln\frac{1}{2}$ **13.** (a) $\frac{250}{101}\pi(e^{-(n-1)\pi/5} - e^{-n\pi/5})$ (b) $\frac{250}{101}\pi$

19. $\dfrac{\pi}{2\sqrt{3}} - 1$

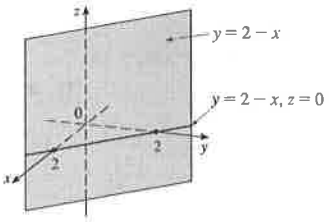
21. $-\left(\dfrac{\pi}{2} - \pi k\right)^2$ where k is a positive integer

CHAPTER 12

EXERCISES 12.1 ■ PAGE 814

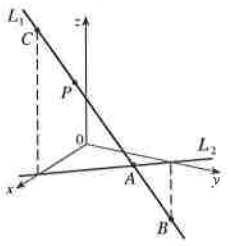
1. $(4, 0, -3)$ **3.** C; A

5. A vertical plane that intersects the xy-plane in the line $y = 2 - x$, $z = 0$



7. (a) $|PQ| = 6$, $|QR| = 2\sqrt{10}$, $|RP| = 6$; isosceles triangle
9. (a) No (b) Yes
11. $(x+3)^2 + (y-2)^2 + (z-5)^2 = 16$; $(y-2)^2 + (z-5)^2 = 7$, $x = 0$ (a circle)
13. $(x-3)^2 + (y-8)^2 + (z-1)^2 = 30$
15. $(1, 2, -4)$, 6 **17.** $(2, 0, -6)$, $9/\sqrt{2}$
19. (b) $\frac{5}{2}, \frac{1}{2}\sqrt{94}, \frac{1}{2}\sqrt{85}$
21. (a) $(x-2)^2 + (y+3)^2 + (z-6)^2 = 36$
(b) $(x-2)^2 + (y+3)^2 + (z-6)^2 = 4$
(c) $(x-2)^2 + (y+3)^2 + (z-6)^2 = 9$
23. A plane parallel to the yz-plane and 5 units in front of it

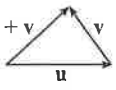
25. A half-space consisting of all points to the left of the plane $y = 8$
27. All points on or between the horizontal planes $z = 0$ and $z = 6$
29. All points on a circle with radius 2 with center on the z-axis that is contained in the plane $z = -1$
31. All points on or inside a sphere with radius $\sqrt{3}$ and center O
33. All points on or inside a circular cylinder of radius 3 with axis the y-axis
35. $0 < x < 5$ **37.** $r^2 < x^2 + y^2 + z^2 < R^2$
39. (a) (2, 1, 4) (b)



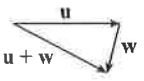
41. $14x - 6y - 10z = 9$, a plane perpendicular to AB
43. $2\sqrt{3} - 3$

EXERCISES 12.2 ■ PAGE 822

1. (a) Scalar (b) Vector (c) Vector (d) Scalar
3. $\overrightarrow{AB} = \overrightarrow{DC}, \overrightarrow{DA} = \overrightarrow{CB}, \overrightarrow{DE} = \overrightarrow{EB}, \overrightarrow{EA} = \overrightarrow{CE}$
5. (a)

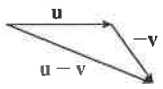


(b)

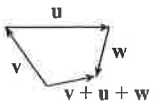


(c)

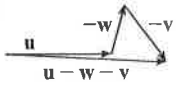
(d)



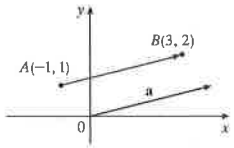
(e)



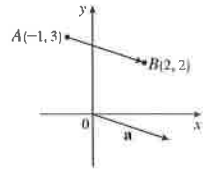
(f)



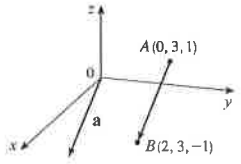
7. $\mathbf{c} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}, \mathbf{d} = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$
9. $\mathbf{a} = \langle 4, 1 \rangle$



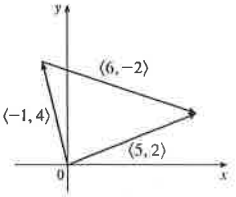
11. $\mathbf{a} = \langle 3, -1 \rangle$



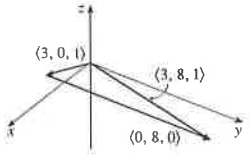
13. $\mathbf{a} = \langle 2, 0, -2 \rangle$



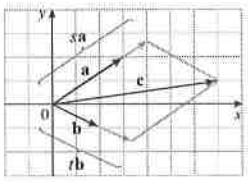
15. $\langle 5, 2 \rangle$



17. $\langle 3, 8, 1 \rangle$



19. $\langle 2, -18 \rangle, \langle 1, -42 \rangle$, 13, 10
21. $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}, -4\mathbf{i} + \mathbf{j} + 9\mathbf{k}, \sqrt{14}, \sqrt{82}$
23. $-\dfrac{3}{\sqrt{58}}\mathbf{i} + \dfrac{7}{\sqrt{58}}\mathbf{j}$ **25.** $\frac{8}{9}\mathbf{i} - \frac{1}{9}\mathbf{j} + \frac{4}{9}\mathbf{k}$ **27.** 60°
29. $\langle 2, 2\sqrt{3} \rangle$ **31.** ≈45.96 ft/s, ≈38.57 ft/s
33. $100\sqrt{7} \approx 264.6$ N, ≈139.1°
35. $\sqrt{493} \approx 22.2$ mi/h, N8°W
37. $\mathbf{T}_1 = -196\,\mathbf{i} + 3.92\,\mathbf{j}, \mathbf{T}_2 = 196\,\mathbf{i} + 3.92\,\mathbf{j}$
39. (a) At an angle of 43.4° from the bank, toward upstream
(b) 20.2 min
41. $\pm(\mathbf{i} + 4\,\mathbf{j})/\sqrt{17}$ **43.** 0
45. (a), (b)



(d) $s = \frac{9}{7}, t = \frac{11}{7}$
47. A sphere with radius 1, centered at (x_0, y_0, z_0)

EXERCISES 12.3 ■ PAGE 830

1. (b), (c), (d) are meaningful **3.** 14 **5.** 19 **7.** 1
9. −15 **11.** $\mathbf{u} \cdot \mathbf{v} = \frac{1}{2}, \mathbf{u} \cdot \mathbf{w} = -\frac{1}{2}$
15. $\cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right) \approx 63°$ **17.** $\cos^{-1}\left(\dfrac{5}{\sqrt{1015}}\right) \approx 81°$
19. $\cos^{-1}\left(\dfrac{7}{\sqrt{130}}\right) \approx 52°$ **21.** 48°, 75°, 57°
23. (a) Neither (b) Orthogonal
(c) Orthogonal (d) Parallel
25. Yes **27.** $(\mathbf{i} - \mathbf{j} - \mathbf{k})/\sqrt{3}$ $\left[\text{or } (-\mathbf{i} + \mathbf{j} + \mathbf{k})/\sqrt{3}\right]$
29. 45° **31.** 0° at (0, 0), 8.1° at (1, 1)
33. $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$; 48°, 71°, 48°
35. $1/\sqrt{14}, -2/\sqrt{14}, -3/\sqrt{14}$; 74°, 122°, 143°
37. $1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}$; 55°, 55°, 55° **39.** 4, $\langle -\frac{20}{13}, \frac{48}{13} \rangle$
41. $\frac{9}{7}, \langle \frac{27}{49}, \frac{54}{49}, -\frac{18}{49} \rangle$ **43.** $1/\sqrt{21}, \frac{2}{21}\mathbf{i} - \frac{1}{21}\mathbf{j} + \frac{4}{21}\mathbf{k}$
47. $\langle 0, 0, -2\sqrt{10} \rangle$ or any vector of the form $\langle s, t, 3s - 2\sqrt{10} \rangle, s, t \in \mathbb{R}$
49. 144 J **51.** 2400 cos(40°) ≈ 1839 ft-lb
53. $\frac{13}{5}$ **55.** $\cos^{-1}(1/\sqrt{3}) \approx 55°$

EXERCISES 12.4 ■ PAGE 838

1. $16\,\mathbf{i} + 48\,\mathbf{k}$ **3.** $15\,\mathbf{i} - 3\,\mathbf{j} + 3\,\mathbf{k}$ **5.** $\frac{1}{2}\mathbf{i} - \mathbf{j} + \frac{3}{2}\mathbf{k}$
7. $(1 - t)\,\mathbf{i} + (t^3 - t^2)\,\mathbf{k}$ **9.** 0 **11.** $\mathbf{i} + \mathbf{j} + \mathbf{k}$
13. (a) Scalar (b) Meaningless (c) Vector
(d) Meaningless (e) Meaningless (f) Scalar

15. $96\sqrt{3}$; into the page **17.** $\langle -7, 10, 8\rangle$, $\langle 7, -10, -8\rangle$

19. $\left\langle -\frac{1}{3\sqrt{3}}, -\frac{1}{3\sqrt{3}}, \frac{5}{3\sqrt{3}} \right\rangle, \left\langle \frac{1}{3\sqrt{3}}, \frac{1}{3\sqrt{3}}, -\frac{5}{3\sqrt{3}} \right\rangle$

27. 16 **29.** (a) $\langle 0, 18, -9\rangle$ (b) $\frac{9}{2}\sqrt{5}$

31. (a) $\langle 13, -14, 5\rangle$ (b) $\frac{1}{2}\sqrt{390}$

33. 9 **35.** 16 **39.** $10.8 \sin 80° \approx 10.6$ N · m

41. ≈417 N **43.** 60°

45. (b) $\sqrt{97/3}$ **53.** (a) No (b) No (c) Yes

EXERCISES 12.5 ■ PAGE 848

1. (a) True (b) False (c) True (d) False (e) False (f) True (g) False (h) True (i) True (j) False (k) True

3. $\mathbf{r} = (2\,\mathbf{i} + 2.4\,\mathbf{j} + 3.5\,\mathbf{k}) + t(3\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k})$; $x = 2 + 3t, y = 2.4 + 2t, z = 3.5 - t$

5. $\mathbf{r} = (\mathbf{i} + 6\mathbf{k}) + t(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$; $x = 1 + t, y = 3t, z = 6 + t$

7. $x = 2 + 2t, y = 1 + \frac{1}{2}t, z = -3 - 4t$; $(x - 2)/2 = 2y - 2 = (z + 3)/(-4)$

9. $x = -8 + 11t, y = 1 - 3t, z = 4$; $\dfrac{x + 8}{11} = \dfrac{y - 1}{-3}, z = 4$

11. $x = 1 + t, y = -1 + 2t, z = 1 + t$; $x - 1 = (y + 1)/2 = z - 1$

13. Yes

15. (a) $(x - 1)/(-1) = (y + 5)/2 = (z - 6)/(-3)$
(b) $(-1, -1, 0), \left(-\frac{3}{2}, 0, -\frac{3}{2}\right), (0, -3, 3)$

17. $\mathbf{r}(t) = (2\mathbf{i} - \mathbf{j} + 4\mathbf{k}) + t(2\mathbf{i} + 7\mathbf{j} - 3\mathbf{k}), 0 \leq t \leq 1$

19. Skew **21.** $(4, -1, -5)$ **23.** $x - 2y + 5z = 0$

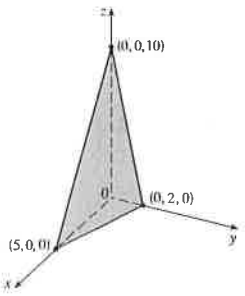
25. $x + 4y + z = 4$ **27.** $5x - y - z = 7$

29. $6x + 6y + 6z = 11$ **31.** $x + y + z = 2$

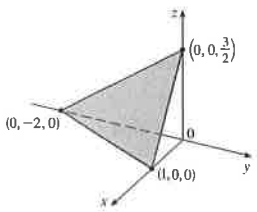
33. $-13x + 17y + 7z = -42$ **35.** $33x + 10y + 4z = 190$

37. $x - 2y + 4z = -1$ **39.** $3x - 8y - z = -38$

41.



43.



45. (2, 3, 5) **47.** (2, 3, 1) **49.** 1, 0, −1

51. Perpendicular **53.** Neither, $\cos^{-1}\left(\frac{1}{3}\right) \approx 70.5°$

55. Parallel

57. (a) $x = 1, y = -t, z = t$ (b) $\cos^{-1}\left(\dfrac{5}{3\sqrt{3}}\right) \approx 15.8°$

59. $x = 1, y - 2 = -z$ **61.** $x + 2y + z = 5$

63. $(x/a) + (y/b) + (z/c) = 1$

65. $x = 3t, y = 1 - t, z = 2 - 2t$

67. P_2 and P_3 are parallel, P_1 and P_4 are identical

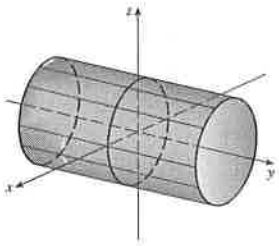
69. $\sqrt{61/14}$ **71.** $\frac{18}{7}$ **73.** $5/(2\sqrt{14})$ **77.** $1/\sqrt{6}$

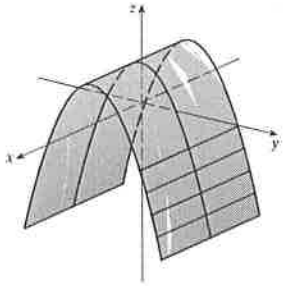
79. $13/\sqrt{69}$

EXERCISES 12.6 ■ PAGE 856

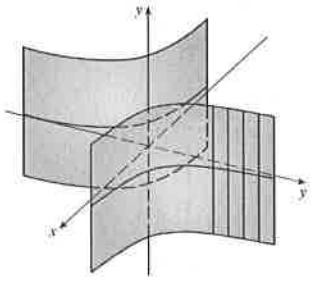
1. (a) Parabola
(b) Parabolic cylinder with rulings parallel to the z-axis
(c) Parabolic cylinder with rulings parallel to the x-axis

3. Circular cylinder **5.** Parabolic cylinder



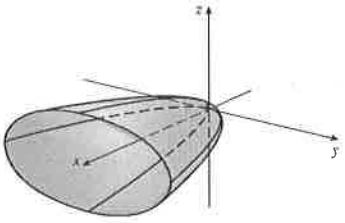


7. Hyperbolic cylinder

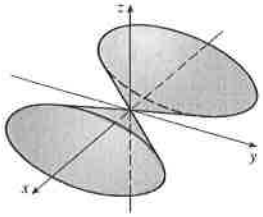


9. (a) $x = k, y^2 - z^2 = 1 - k^2$, hyperbola ($k \neq \pm 1$);
$y = k, x^2 - z^2 = 1 - k^2$, hyperbola ($k \neq \pm 1$);
$z = k, x^2 + y^2 = 1 + k^2$, circle
(b) The hyperboloid is rotated so that it has axis the y-axis
(c) The hyperboloid is shifted one unit in the negative y-direction

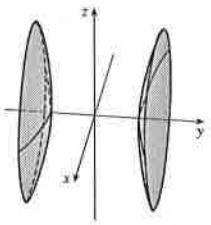
11. Elliptic paraboloid with axis the x-axis



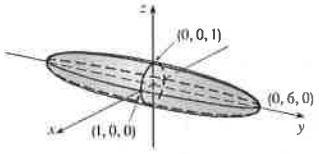
13. Elliptic cone with axis the x-axis



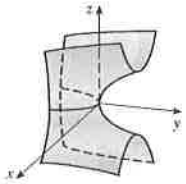
15. Hyperboloid of two sheets



17. Ellipsoid



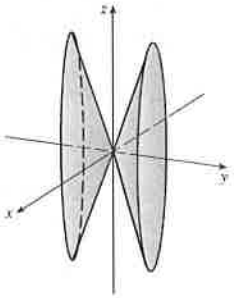
19. Hyperbolic paraboloid



21. VII **23.** II **25.** VI **27.** VIII

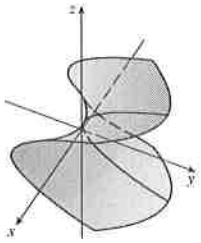
29. $y^2 = x^2 + \dfrac{z^2}{9}$

Elliptic cone with axis the y-axis



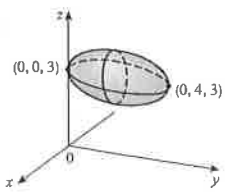
31. $y = z^2 - \dfrac{x^2}{2}$

Hyperbolic paraboloid

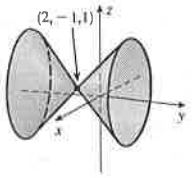


33. $x^2 + \dfrac{(y-2)^2}{4} + (z-3)^2 = 1$

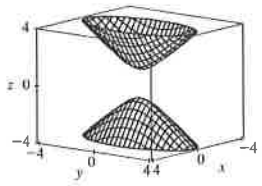
Ellipsoid with center (0, 2, 3)



35. $(y+1)^2 = (x-2)^2 + (z-1)^2$
Circular cone with vertex $(2, -1, 1)$ and axis parallel to the y-axis



37.



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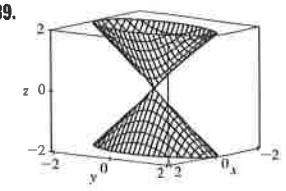

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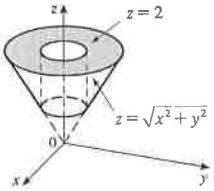
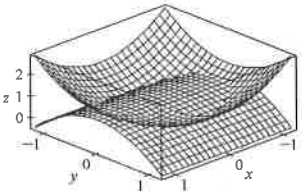


43. $y = x^2 + z^2$ **45.** $-4x = y^2 + z^2$, paraboloid

47. (a) $\dfrac{x^2}{(6378.137)^2} + \dfrac{y^2}{(6378.137)^2} + \dfrac{z^2}{(6356.523)^2} = 1$
(b) Circle (c) Ellipse

51.



CHAPTER 12 REVIEW ■ PAGE 858

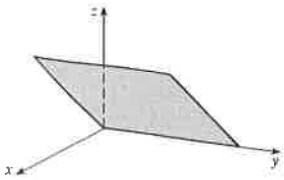
True-False Quiz

1. False **3.** False **5.** True **7.** True **9.** True
11. True **13.** True **15.** False **17.** False
19. False **21.** True

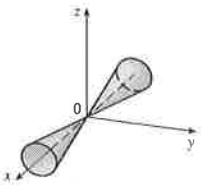
Exercises

1. (a) $(x+1)^2 + (y-2)^2 + (z-1)^2 = 69$
(b) $(y-2)^2 + (z-1)^2 = 68, x = 0$
(c) Center $(4, -1, -3)$, radius 5
3. $\mathbf{u} \cdot \mathbf{v} = 3\sqrt{2}$; $|\mathbf{u} \times \mathbf{v}| = 3\sqrt{2}$; out of the page
5. $-2, -4$ **7.** (a) 2 (b) -2 (c) -2 (d) 0
9. $\cos^{-1}\left(\frac{1}{3}\right) \approx 71°$ **11.** (a) $\langle 4, -3, 4 \rangle$ (b) $\sqrt{41}/2$
13. 166 N, 114 N
15. $x = 4 - 3t, y = -1 + 2t, z = 2 + 3t$
17. $x = -2 + 2t, y = 2 - t, z = 4 + 5t$
19. $-4x + 3y + z = -14$ **21.** $(1, 4, 4)$ **23.** Skew
25. $x + y + z = 4$ **27.** $22/\sqrt{26}$

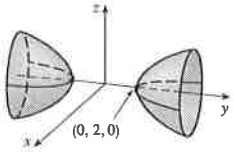
29. Plane



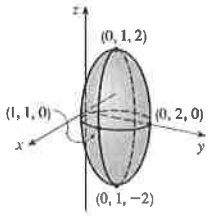
31. Cone



33. Hyperboloid of two sheets



35. Ellipsoid



37. $4x^2 + y^2 + z^2 = 16$

PROBLEMS PLUS ■ PAGE 861

1. $\left(\sqrt{3} - \frac{3}{2}\right)$ m

3. (a) $(x + 1)/(-2c) = (y - c)/(c^2 - 1) = (z - c)/(c^2 + 1)$

(b) $x^2 + y^2 = t^2 + 1, z = t$ (c) $4\pi/3$

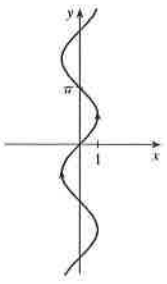
5. 20

CHAPTER 13

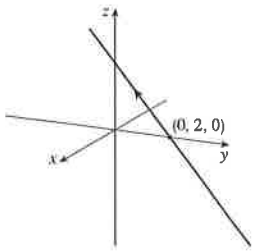
EXERCISES 13.1 ■ PAGE 869

1. $(-1, 2]$ **3.** $\mathbf{i} + \mathbf{j} + \mathbf{k}$ **5.** $\langle -1, \pi/2, 0 \rangle$

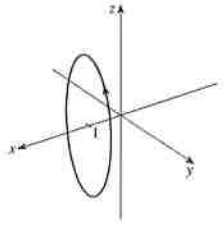
7.



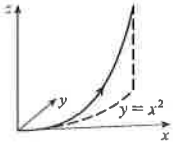
9.



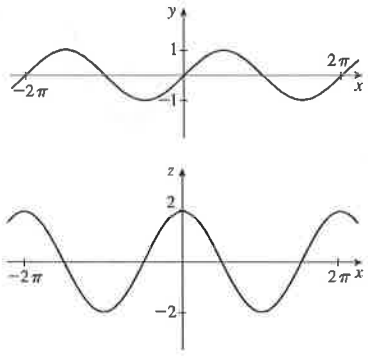
11.

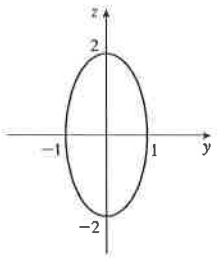


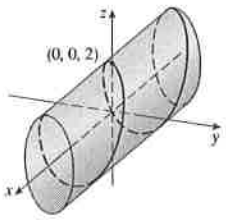
13.



15.





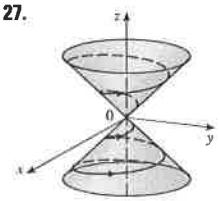


17. $\mathbf{r}(t) = \langle 2 + 4t, 2t, -2t \rangle, 0 \leq t \leq 1;$
$x = 2 + 4t, y = 2t, z = -2t, 0 \leq t \leq 1$

19. $\mathbf{r}(t) = \left\langle \frac{1}{2}t, -1 + \frac{4}{3}t, 1 - \frac{3}{4}t \right\rangle, 0 \leq t \leq 1;$
$x = \frac{1}{2}t, y = -1 + \frac{4}{3}t, z = 1 - \frac{3}{4}t, 0 \leq t \leq 1$

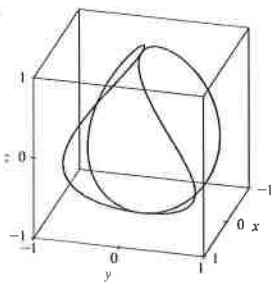
21. II **23.** V **25.** IV

27.

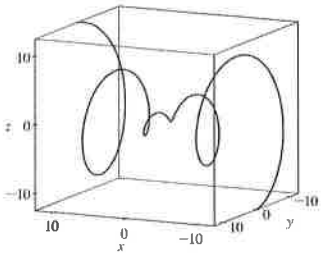


29. (0, 0, 0), (1, 0, 1)

31.



33.



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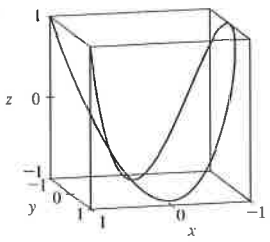
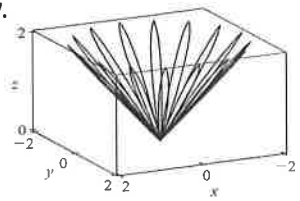


37.



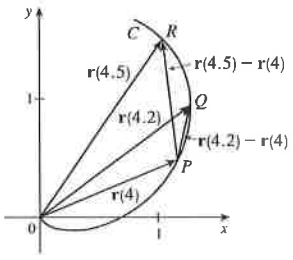
41. $\mathbf{r}(t) = t\,\mathbf{i} + \frac{1}{2}(t^2 - 1)\,\mathbf{j} + \frac{1}{2}(t^2 + 1)\,\mathbf{k}$

43. $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \cos 2t\,\mathbf{k},\ 0 \leq t \leq 2\pi$

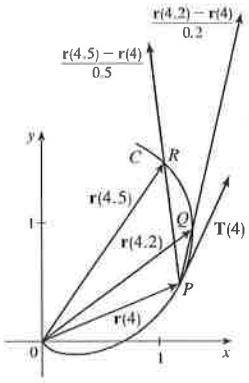
45. $x = 2\cos t,\ y = 2\sin t,\ z = 4\cos^2 t$ **47.** Yes

EXERCISES 13.2 ■ PAGE 876

1. (a)

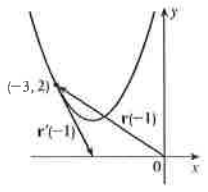


(b), (d)



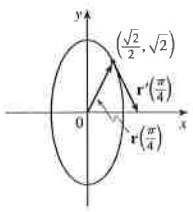
(c) $\mathbf{r}'(4) = \lim_{h \to 0} \dfrac{\mathbf{r}(4 + h) - \mathbf{r}(4)}{h}$; $\mathbf{T}(4) = \dfrac{\mathbf{r}'(4)}{|\mathbf{r}'(4)|}$

3. (a), (c)



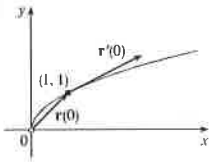
(b) $\mathbf{r}'(t) = \langle 1, 2t \rangle$

5. (a), (c)



(b) $\mathbf{r}'(t) = \cos t\,\mathbf{i} - 2\sin t\,\mathbf{j}$

7. (a), (c)



(b) $\mathbf{r}'(t) = 2e^{2t}\,\mathbf{i} + e^t\,\mathbf{j}$

9. $\mathbf{r}'(t) = \langle t\cos t + \sin t,\ 2t,\ \cos 2t - 2t\sin 2t \rangle$

11. $\mathbf{r}'(t) = \mathbf{i} + \left(1/\sqrt{t}\right)\mathbf{k}$

13. $\mathbf{r}'(t) = 2te^{t^2}\,\mathbf{i} + [3/(1 + 3t)]\,\mathbf{k}$ **15.** $\mathbf{r}'(t) = \mathbf{b} + 2t\mathbf{c}$

17. $\left\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \right\rangle$ **19.** $\frac{3}{5}\,\mathbf{j} + \frac{4}{5}\,\mathbf{k}$

21. $\langle 1, 2t, 3t^2 \rangle, \left\langle 1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14} \right\rangle, \langle 0, 2, 6t \rangle, \langle 6t^2, -6t, 2 \rangle$

23. $x = 3 + t,\ y = 2t,\ z = 2 + 4t$

25. $x = 1 - t,\ y = t,\ z = 1 - t$

27. $\mathbf{r}(t) = (3 - 4t)\,\mathbf{i} + (4 + 3t)\,\mathbf{j} + (2 - 6t)\,\mathbf{k}$

29. $x = t,\ y = 1 - t,\ z = 2t$

31. $x = -\pi - t,\ y = \pi + t,\ z = -\pi t$

33. 66° **35.** $2\,\mathbf{i} - 4\,\mathbf{j} + 32\,\mathbf{k}$ **37.** $\mathbf{i} + \mathbf{j} + \mathbf{k}$

39. $\tan t\,\mathbf{i} + \frac{1}{8}(t^2 + 1)^4\,\mathbf{j} + \left(\frac{1}{3}t^3 \ln t - \frac{1}{9}t^3\right)\mathbf{k} + \mathbf{C}$

41. $t^2\,\mathbf{i} + t^3\,\mathbf{j} + \left(\frac{2}{3}t^{3/2} - \frac{2}{3}\right)\mathbf{k}$

47. $2t\cos t + 2\sin t - 2\cos t\sin t$ **49.** 35

EXERCISES 13.3 ■ PAGE 884

1. $10\sqrt{10}$ **3.** $e - e^{-1}$ **5.** $\frac{1}{27}(13^{3/2} - 8)$ **7.** 18.6833

9. 1.2780 **11.** 42

13. $\mathbf{r}(t(s)) = \dfrac{2}{\sqrt{29}}s\,\mathbf{i} + \left(1 - \dfrac{3}{\sqrt{29}}s\right)\mathbf{j} + \left(5 + \dfrac{4}{\sqrt{29}}s\right)\mathbf{k}$

15. (3 sin 1, 4, 3 cos 1)

17. (a) $\left\langle 1/\sqrt{10}, \left(-3/\sqrt{10}\right)\sin t, \left(3/\sqrt{10}\right)\cos t \right\rangle$, $\langle 0, -\cos t, -\sin t \rangle$ (b) $\frac{3}{10}$

19. (a) $\dfrac{1}{e^{2t} + 1}\left\langle \sqrt{2}e^t, e^{2t}, -1 \right\rangle, \dfrac{1}{e^{2t} + 1}\left\langle 1 - e^{2t}, \sqrt{2}e^t, \sqrt{2}e^t \right\rangle$

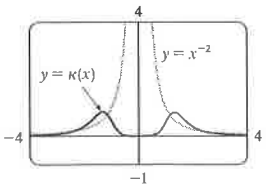
(b) $\sqrt{2}e^{2t}/(e^{2t} + 1)^2$

21. $6t^2/(9t^4 + 4t^2)^{3/2}$ **23.** $\frac{4}{25}$ **25.** $\frac{1}{7}\sqrt{\frac{19}{14}}$

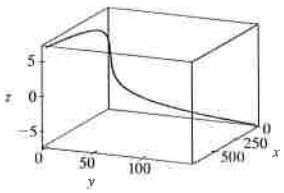
27. $12x^2/(1 + 16x^6)^{3/2}$ **29.** $e^x|x + 2|/[1 + (xe^x + e^x)^2]^{3/2}$

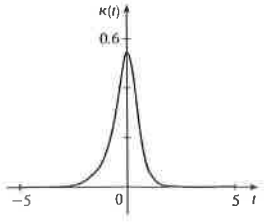
31. $\left(-\frac{1}{2}\ln 2,\ 1/\sqrt{2}\right)$, approaches 0 **33.** (a) P (b) 1.3, 0.7

35.



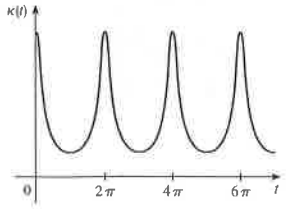
37.





39. a is $y = f(x)$, b is $y = \kappa(x)$

41. $\kappa(t) = \dfrac{6\sqrt{4\cos^2 t - 12\cos t + 13}}{(17 - 12\cos t)^{3/2}}$



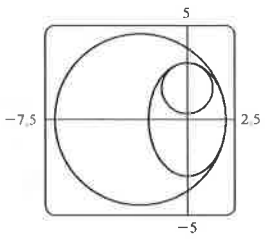
integer multiples of 2π

43. $6t^2/(4t^2 + 9t^4)^{3/2}$

45. $1/(\sqrt{2}e^t)$ **47.** $\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\rangle, \langle -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\rangle, \langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\rangle$

49. $y = 6x + \pi, x + 6y = 6\pi$

51. $(x + \frac{5}{2})^2 + y^2 = \frac{81}{4}, x^2 + (y - \frac{5}{3})^2 = \frac{16}{9}$



53. $(-1, -3, 1)$

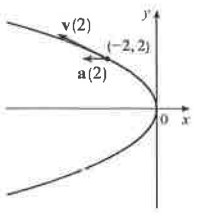
55. $2x + y + 4z = 7, 6x - 8y - z = -3$

63. $2/(t^4 + 4t^2 + 1)$ **65.** 2.07×10^{10} Å ≈ 2 m

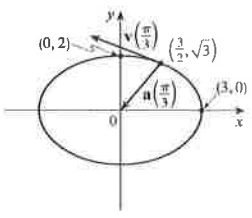
EXERCISES 13.4 ■ PAGE 894

1. (a) $1.8\mathbf{i} - 3.8\mathbf{j} - 0.7\mathbf{k}, 2.0\mathbf{i} - 2.4\mathbf{j} - 0.6\mathbf{k}, 2.8\mathbf{i} + 1.8\mathbf{j} - 0.3\mathbf{k}, 2.8\mathbf{i} + 0.8\mathbf{j} - 0.4\mathbf{k}$
(b) $2.4\mathbf{i} - 0.8\mathbf{j} - 0.5\mathbf{k}, 2.58$

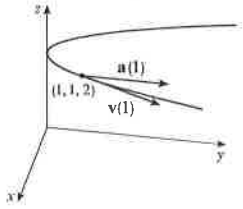
3. $\mathbf{v}(t) = \langle -t, 1\rangle$
$\mathbf{a}(t) = \langle -1, 0\rangle$
$|\mathbf{v}(t)| = \sqrt{t^2 + 1}$



5. $\mathbf{v}(t) = -3\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j}$
$\mathbf{a}(t) = -3\cos t\,\mathbf{i} - 2\sin t\,\mathbf{j}$
$|\mathbf{v}(t)| = \sqrt{5\sin^2 t + 4}$



7. $\mathbf{v}(t) = \mathbf{i} + 2t\,\mathbf{j}$
$\mathbf{a}(t) = 2\,\mathbf{j}$
$|\mathbf{v}(t)| = \sqrt{1 + 4t^2}$



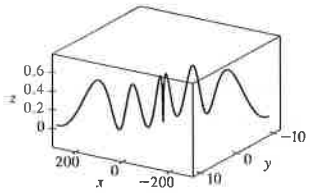
9. $\langle 2t + 1, 2t - 1, 3t^2\rangle, \langle 2, 2, 6t\rangle, \sqrt{9t^4 + 8t^2 + 2}$

11. $\sqrt{2}\,\mathbf{i} + e^t\,\mathbf{j} - e^{-t}\,\mathbf{k}, e^t\,\mathbf{j} + e^{-t}\,\mathbf{k}, e^t + e^{-t}$

13. $e^t[(\cos t - \sin t)\mathbf{i} + (\sin t + \cos t)\mathbf{j} + (t + 1)\mathbf{k}]$,
$e^t[-2\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j} + (t + 2)\mathbf{k}], e^t\sqrt{t^2 + 2t + 3}$

15. $\mathbf{v}(t) = t\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{k}, \mathbf{r}(t) = (\frac{1}{2}t^2 + 1)\,\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k}$

17. (a) $\mathbf{r}(t) = (\frac{1}{3}t^3 + t)\,\mathbf{i} + (t - \sin t + 1)\,\mathbf{j} + (\frac{1}{4} - \frac{1}{4}\cos 2t)\,\mathbf{k}$
(b)



19. $t = 4$ **21.** $\mathbf{r}(t) = t\,\mathbf{i} - t\,\mathbf{j} + \frac{5}{2}t^2\,\mathbf{k}, |\mathbf{v}(t)| = \sqrt{25t^2 + 2}$

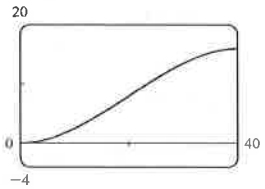
23. (a) ≈ 3535 m (b) ≈ 1531 m (c) 200 m/s

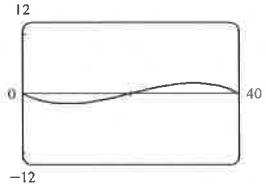
25. 30 m/s **27.** $\approx 10.2°, \approx 79.8°$

29. $13.0° < \theta < 36.0°, 55.4° < \theta < 85.5°$

31. $(250, -50, 0); 10\sqrt{93} \approx 96.4$ ft/s

33. (a) 16 m (b) $\approx 23.6°$ upstream





35. The path is contained in a circle that lies in a plane perpendicular to $\mathbf{c}$ with center on a line through the origin in the direction of $\mathbf{c}$.

37. $6t, 6$ **39.** $0, 1$ **41.** $e^t - e^{-t}, \sqrt{2}$

43. 4.5 cm/s^2, 9.0 cm/s^2 **45.** $t = 1$

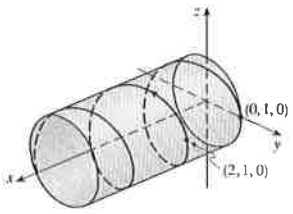
CHAPTER 13 REVIEW ■ PAGE 897

True-False Quiz

1. True **3.** False **5.** False **7.** False
9. True **11.** False **13.** True

Exercises

1. (a)



(b) $\mathbf{r}'(t) = \mathbf{i} - \pi\sin \pi t\,\mathbf{j} + \pi\cos \pi t\,\mathbf{k}$,
$\mathbf{r}''(t) = -\pi^2\cos \pi t\,\mathbf{j} - \pi^2\sin \pi t\,\mathbf{k}$

3. $\mathbf{r}(t) = 4\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j} + (5 - 4\cos t)\mathbf{k},\ 0 \leq t \leq 2\pi$

5. $\frac{1}{3}\mathbf{i} - (2/\pi^2)\mathbf{j} + (2/\pi)\mathbf{k}$ **7.** 86.631 **9.** $\pi/2$

11. (a) $\langle t^2, t, 1\rangle/\sqrt{t^4 + t^2 + 1}$
(b) $\langle t^3 + 2t, 1 - t^4, -2t^3 - t\rangle/\sqrt{t^8 + 5t^6 + 6t^4 + 5t^2 + 1}$
(c) $\sqrt{t^8 + 5t^6 + 6t^4 + 5t^2 + 1}/(t^4 + t^2 + 1)^2$

13. $12/17^{3/2}$ **15.** $x - 2y + 2\pi = 0$

17. $\mathbf{v}(t) = (1 + \ln t)\,\mathbf{i} + \mathbf{j} - e^{-t}\,\mathbf{k}$, $|\mathbf{v}(t)| = \sqrt{2 + 2\ln t + (\ln t)^2 + e^{-2t}}$, $\mathbf{a}(t) = (1/t)\,\mathbf{i} + e^{-t}\,\mathbf{k}$

19. (a) About 3.8 ft above the ground, 60.8 ft from the athlete (b) ≈21.4 ft (c) ≈64.2 ft from the athlete

21. (c) $-2e^{-t}\,\mathbf{v}_d + e^{-t}\,\mathbf{R}$

23. (a) $\mathbf{v} = \omega R(-\sin \omega t\,\mathbf{i} + \cos \omega t\,\mathbf{j})$ (c) $\mathbf{a} = -\omega^2\mathbf{r}$

PROBLEMS PLUS ■ PAGE 900

1. (a) 90°, $v_0^2/(2g)$

3. (a) ≈0.94 ft to the right of the table's edge, ≈15 ft/s (b) ≈7.6° (c) ≈2.13 ft to the right of the table's edge

5. 56°

7. $\mathbf{r}(u, v) = \mathbf{c} + u\,\mathbf{a} + v\,\mathbf{b}$ where $\mathbf{a} = \langle a_1, a_2, a_3\rangle$, $\mathbf{b} = \langle b_1, b_2, b_3\rangle$, $\mathbf{c} = \langle c_1, c_2, c_3\rangle$

CHAPTER 14

EXERCISES 14.1 ■ PAGE 912

1. (a) −27; a temperature of −15°C with wind blowing at 40 km/h feels equivalent to about −27°C without wind.
(b) When the temperature is −20°C, what wind speed gives a wind chill of −30°C? 20 km/h
(c) With a wind speed of 20 km/h, what temperature gives a wind chill of −49°C? −35°C
(d) A function of wind speed that gives wind-chill values when the temperature is −5°C
(e) A function of temperature that gives wind-chill values when the wind speed is 50 km/h

3. ≈94.2; the manufacturer's yearly production is valued at $94.2 million when 120,000 labor hours are spent and $20 million in capital is invested.

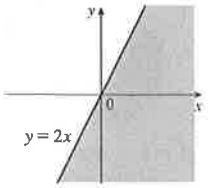
5. (a) ≈20.5; the surface area of a person 70 inches tall who weighs 160 pounds is approximately 20.5 square feet.

7. (a) 25; a 40-knot wind blowing in the open sea for 15 h will create waves about 25 ft high.
(b) $f(30, t)$ is a function of t giving the wave heights produced by 30-knot winds blowing for t hours.
(c) $f(v, 30)$ is a function of v giving the wave heights produced by winds of speed v blowing for 30 hours.

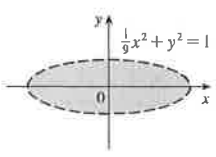
9. (a) 1 (b) $\mathbb{R}^2$ (c) $[-1, 1]$

11. (a) 3 (b) $\{(x, y, z) \mid x^2 + y^2 + z^2 < 4, x \geq 0, y \geq 0, z \geq 0\}$, interior of a sphere of radius 2, center the origin, in the first octant

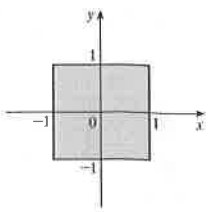
13. $\{(x, y) \mid y \leq 2x\}$



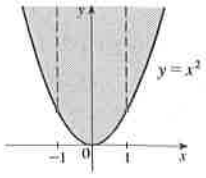
15. $\left\{(x, y) \mid \frac{1}{9}x^2 + y^2 < 1\right\}$, $(-\infty, \ln 9]$



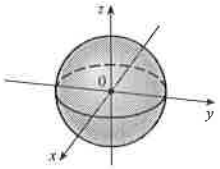
17. $\{(x, y) \mid -1 \leq x \leq 1, -1 \leq y \leq 1\}$



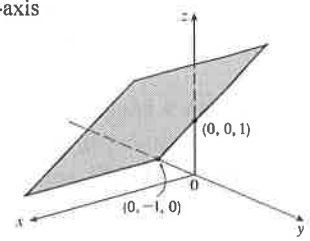
19. $\{(x, y) \mid y \geq x^2, x \neq \pm 1\}$



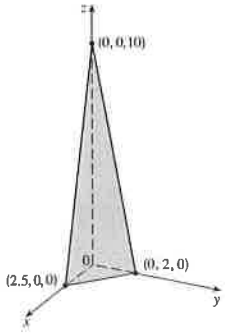
21. $\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$



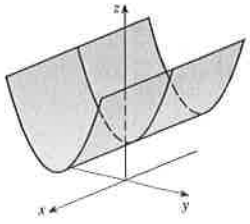
23. $z = 1 + y$, plane parallel to x-axis



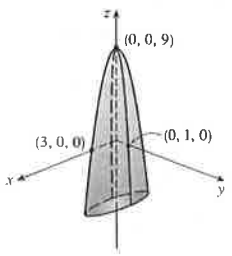
25. $4x + 5y + z = 10$, plane



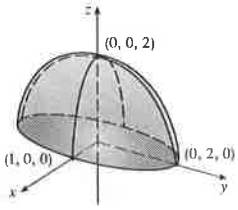
27. $z = y^2 + 1$, parabolic cylinder



29. $z = 9 - x^2 - 9y^2$, elliptic paraboloid

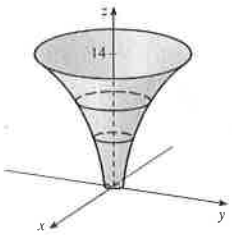


31. $z = \sqrt{4 - 4x^2 - y^2}$, top half of ellipsoid

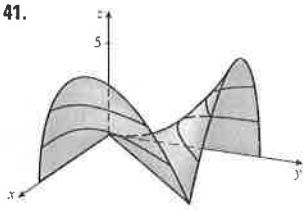


33. $\approx 56, \approx 35$ **35.** 11°C, 19.5°C **37.** Steep; nearly flat

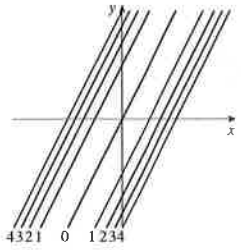
39.



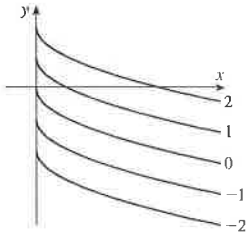
41.



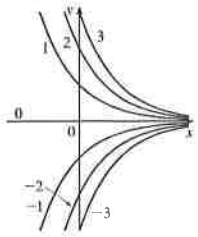
43. $(y - 2x)^2 = k$



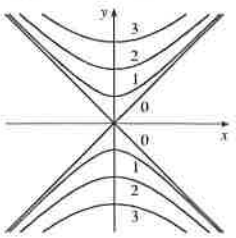
45. $y = -\sqrt{x} + k$



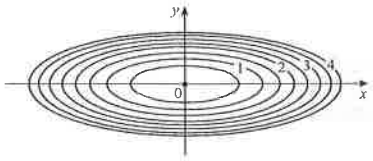
47. $y = ke^{-x}$

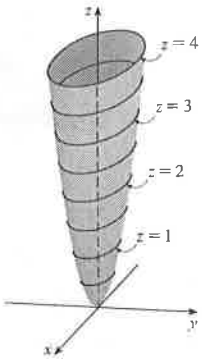


49. $y^2 - x^2 = k^2$

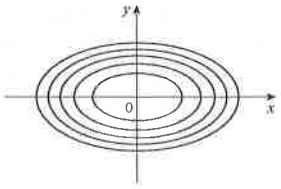


51. $x^2 + 9y^2 = k$





53.



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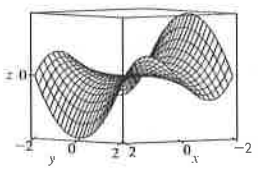

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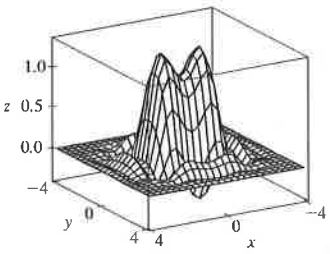


59. (a) C (b) II **61.** (a) F (b) I

63. (a) B (b) VI **65.** Family of parallel planes

67. Family of circular cylinders with axis the x-axis ($k > 0$)

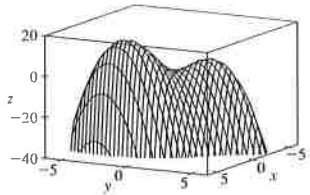
69. (a) Shift the graph of f upward 2 units

(b) Stretch the graph of f vertically by a factor of 2

(c) Reflect the graph of f about the xy-plane

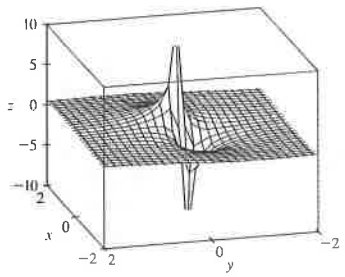
(d) Reflect the graph of f about the xy-plane and then shift it upward 2 units

71.



f appears to have a maximum value of about 15. There are two local maximum points but no local minimum point.

73.



The function values approach 0 as x, y become large; as (x, y) approaches the origin, f approaches $\pm\infty$ or 0, depending on the direction of approach.

75. If $c = 0$, the graph is a cylindrical surface. For $c > 0$, the level curves are ellipses. The graph curves upward as we leave the origin, and the steepness increases as c increases. For $c < 0$, the level curves are hyperbolas. The graph curves upward in the y-direction and downward, approaching the xy-plane, in the x-direction giving a saddle-shaped appearance near $(0, 0, 1)$.

77. $c = -2, 0, 2$ **79.** (b) $y = 0.75x + 0.01$

EXERCISES 14.2 ■ PAGE 923

1. Nothing; if f is continuous, $f(3, 1) = 6$ **3.** $-\frac{5}{2}$

5. 1 **7.** $\frac{2}{7}$ **9.** Does not exist **11.** Does not exist

13. 0 **15.** Does not exist **17.** 2

19. $\sqrt{3}$ **21.** Does not exist

23. The graph shows that the function approaches different numbers along different lines.

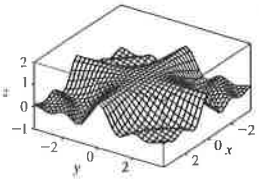
25. $h(x, y) = (2x + 3y - 6)^2 + \sqrt{2x + 3y - 6}$; $\{(x, y) \mid 2x + 3y \geqslant 6\}$

27. Along the line $y = x$ **29.** $\mathbb{R}^2$ **31.** $\{(x, y) \mid x^2 + y^2 \neq 1\}$

33. $\{(x, y) \mid x^2 + y^2 > 4\}$ **35.** $\{(x, y, z) \mid x^2 + y^2 + z^2 \leqslant 1\}$

37. $\{(x, y) \mid (x, y) \neq (0, 0)\}$ **39.** 0 **41.** -1

43.



f is continuous on $\mathbb{R}^2$

EXERCISES 14.3 ■ PAGE 935

1. (a) The rate of change of temperature as longitude varies, with latitude and time fixed; the rate of change as only latitude varies; the rate of change as only time varies.
(b) Positive, negative, positive

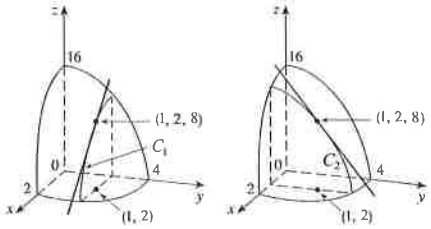
3. (a) $f_T(-15, 30) \approx 1.3$; for a temperature of -15°C and wind speed of 30 km/h, the wind-chill index rises by 1.3°C for each degree the temperature increases. $f_v(-15, 30) \approx -0.15$; for a temperature of -15°C and wind speed of 30 km/h, the wind-chill index decreases by 0.15°C for each km/h the wind speed increases.
(b) Positive, negative (c) 0

5. (a) Positive (b) Negative

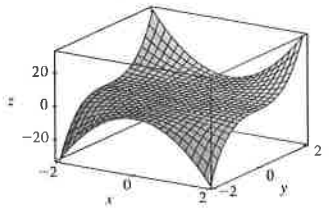
7. (a) Positive (b) Negative

9. $c = f$, $b = f_x$, $a = f_y$

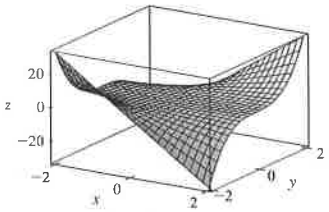
11. $f_x(1, 2) = -8$ = slope of C_1, $f_y(1, 2) = -4$ = slope of C_2



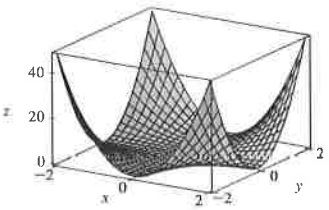
13.



$f(x, y) = x^2y^3$



$f_x(x, y) = 2xy^3$



$f_y(x, y) = 3x^2y^2$

15. $f_x(x, y) = -3y$, $f_y(x, y) = 5y^4 - 3x$

17. $f_x(x, t) = -\pi e^{-t} \sin \pi x$, $f_t(x, t) = -e^{-t} \cos \pi x$

19. $\partial z/\partial x = 20(2x + 3y)^9$, $\partial z/\partial y = 30(2x + 3y)^9$

21. $f_x(x, y) = 1/y$, $f_y(x, y) = -x/y^2$

23. $f_x(x, y) = \dfrac{(ad - bc)y}{(cx + dy)^2}$, $f_y(x, y) = \dfrac{(bc - ad)x}{(cx + dy)^2}$

25. $g_u(u, v) = 10uv(u^2v - v^3)^4$, $g_v(u, v) = 5(u^2 - 3v^2)(u^2v - v^3)^4$

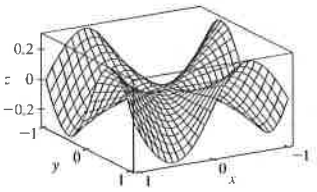
27. $R_p(p, q) = \dfrac{q^2}{1 + p^2q^4}$, $R_q(p, q) = \dfrac{2pq}{1 + p^2q^4}$

29. $F_x(x, y) = \cos(e^x)$, $F_y(x, y) = -\cos(e^y)$

31. $f_x = z - 10xy^3z^4$, $f_y = -15x^2y^2z^4$, $f_z = x - 20x^2y^3z^3$

33. $\partial w/\partial x = 1/(x + 2y + 3z)$, $\partial w/\partial y = 2/(x + 2y + 3z)$, $\partial w/\partial z = 3/(x + 2y + 3z)$

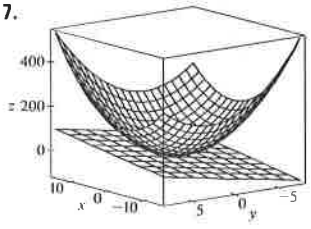
35. $\partial u/\partial x = y\sin^{-1}(yz)$, $\partial u/\partial y = x\sin^{-1}(yz) + xyz/\sqrt{1-y^2z^2}$, $\partial u/\partial z = xy^2/\sqrt{1-y^2z^2}$
37. $h_x = 2xy\cos(z/t)$, $h_y = x^2\cos(z/t)$, $h_z = (-x^2y/t)\sin(z/t)$, $h_t = (x^2yz/t^2)\sin(z/t)$
39. $\partial u/\partial x_i = x_i/\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$
41. $\frac{1}{5}$ **43.** $\frac{1}{4}$ **45.** $f_x(x,y) = y^2 - 3x^2y$, $f_y(x,y) = 2xy - x^3$
47. $\dfrac{\partial z}{\partial x} = -\dfrac{x}{3z}, \dfrac{\partial z}{\partial y} = -\dfrac{2y}{3z}$
49. $\dfrac{\partial z}{\partial x} = \dfrac{yz}{e^z - xy}, \dfrac{\partial z}{\partial y} = \dfrac{xz}{e^z - xy}$
51. (a) $f'(x), g'(y)$ (b) $f'(x+y), f'(x+y)$
53. $f_{xx} = 6xy^5 + 24x^2y$, $f_{xy} = 15x^2y^4 + 8x^3 = f_{yx}$, $f_{yy} = 20x^3y^3$
55. $w_{uu} = v^2/(u^2+v^2)^{3/2}$, $w_{uv} = -uv/(u^2+v^2)^{3/2} = w_{vu}$, $w_{vv} = u^2/(u^2+v^2)^{3/2}$
57. $z_{xx} = -2x/(1+x^2)^2$, $z_{xy} = 0 = z_{yx}$, $z_{yy} = -2y/(1+y^2)^2$
63. $24xy^2 - 6y, 24x^2y - 6x$ **65.** $(2x^2y^2z^5 + 6xyz^3 + 2z)e^{xyz^2}$
67. $\theta e^{r\theta}(2\sin\theta + \theta\cos\theta + r\theta\sin\theta)$ **69.** $4/(y+2z)^3, 0$
71. $6yz^2$ **73.** $\approx 12.2, \approx 16.8, \approx 23.25$ **83.** R^2/R_1^2
87. $\dfrac{\partial T}{\partial P} = \dfrac{V-nb}{nR}, \dfrac{\partial P}{\partial V} = \dfrac{2n^2a}{V^3} - \dfrac{nRT}{(V-nb)^2}$
93. No **95.** $x = 1+t, y = 2, z = 2-2t$ **99.** -2
101. (a)



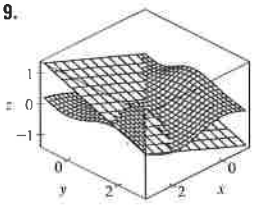
(b) $f_x(x,y) = \dfrac{x^4y + 4x^2y^3 - y^5}{(x^2+y^2)^2}$, $f_y(x,y) = \dfrac{x^5 - 4x^3y^2 - xy^4}{(x^2+y^2)^2}$
(c) 0, 0 (e) No, since f_{xy} and f_{yx} are not continuous.

EXERCISES 14.4 ■ PAGE 946

1. $z = -7x - 6y + 5$ **3.** $x + y - 2z = 0$
5. $x + y + z = 0$
7.



9.



11. $6x + 4y - 23$ **13.** $\frac{1}{9}x - \frac{2}{9}y + \frac{2}{3}$ **15.** $1 - \pi y$
19. 6.3 **21.** $\frac{3}{7}x + \frac{2}{7}y + \frac{6}{7}z$; 6.9914
23. $4T + H - 329$; 129°F
25. $dz = -2e^{-2x}\cos 2\pi t\,dx - 2\pi e^{-2x}\sin 2\pi t\,dt$
27. $dm = 5p^4q^3\,dp + 3p^5q^2\,dq$
29. $dR = \beta^2\cos\gamma\,d\alpha + 2\alpha\beta\cos\gamma\,d\beta - \alpha\beta^2\sin\gamma\,d\gamma$
31. $\Delta z = 0.9225$, $dz = 0.9$ **33.** 5.4 cm² **35.** 16 cm³
37. $\approx -0.0165mg$; decrease
39. $\frac{1}{17} \approx 0.059\ \Omega$ **41.** 2.3% **43.** $\varepsilon_1 = \Delta x$, $\varepsilon_2 = \Delta y$

EXERCISES 14.5 ■ PAGE 954

1. $(2x+y)\cos t + (2y+x)e^t$
3. $[(x/t) - y\sin t]/\sqrt{1+x^2+y^2}$
5. $e^{y/z}[2t - (x/z) - (2xy/z^2)]$
7. $\partial z/\partial s = 2xy^3\cos t + 3x^2y^2\sin t$, $\partial z/\partial t = -2sxy^3\sin t + 3sx^2y^2\cos t$
9. $\partial z/\partial s = t^2\cos\theta\cos\phi - 2st\sin\theta\sin\phi$, $\partial z/\partial t = 2st\cos\theta\cos\phi - s^2\sin\theta\sin\phi$
11. $\dfrac{\partial z}{\partial s} = e^r\left(t\cos\theta - \dfrac{s}{\sqrt{s^2+t^2}}\sin\theta\right)$, $\dfrac{\partial z}{\partial t} = e^r\left(s\cos\theta - \dfrac{t}{\sqrt{s^2+t^2}}\sin\theta\right)$
13. 62 **15.** 7, 2
17. $\dfrac{\partial u}{\partial r} = \dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial r} + \dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial r}$, $\dfrac{\partial u}{\partial s} = \dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial s}$, $\dfrac{\partial u}{\partial t} = \dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial t}$
19. $\dfrac{\partial w}{\partial x} = \dfrac{\partial w}{\partial r}\dfrac{\partial r}{\partial x} + \dfrac{\partial w}{\partial s}\dfrac{\partial s}{\partial x} + \dfrac{\partial w}{\partial t}\dfrac{\partial t}{\partial x}$, $\dfrac{\partial w}{\partial y} = \dfrac{\partial w}{\partial r}\dfrac{\partial r}{\partial y} + \dfrac{\partial w}{\partial s}\dfrac{\partial s}{\partial y} + \dfrac{\partial w}{\partial t}\dfrac{\partial t}{\partial y}$
21. 1582, 3164, −700 **23.** $2\pi, -2\pi$
25. $\frac{5}{144}, -\frac{5}{96}, \frac{5}{144}$ **27.** $\dfrac{2x + y\sin x}{\cos x - 2y}$
29. $\dfrac{1 + x^4y^2 + y^2 + x^4y^4 - 2xy}{x^2 - 2xy - 2x^5y^3}$
31. $-\dfrac{x}{3z}, -\dfrac{2y}{3z}$ **33.** $\dfrac{yz}{e^z - xy}, \dfrac{xz}{e^z - xy}$
35. 2°C/s **37.** ≈ -0.33 m/s per minute
39. (a) 6 m³/s (b) 10 m²/s (c) 0 m/s
41. ≈ -0.27 L/s **43.** $-1/(12\sqrt{3})$ rad/s
45. (a) $\partial z/\partial r = (\partial z/\partial x)\cos\theta + (\partial z/\partial y)\sin\theta$, $\partial z/\partial\theta = -(\partial z/\partial x)r\sin\theta + (\partial z/\partial y)r\cos\theta$
51. $4rs\,\partial^2z/\partial x^2 + (4r^2 + 4s^2)\partial^2z/\partial x\,\partial y + 4rs\,\partial^2z/\partial y^2 + 2\,\partial z/\partial y$

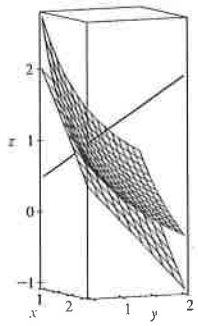
EXERCISES 14.6 ■ PAGE 967

1. ≈ -0.08 mb/km **3.** ≈ 0.778 **5.** $2 + \sqrt{3}/2$
7. (a) $\nabla f(x,y) = \langle 2\cos(2x+3y), 3\cos(2x+3y)\rangle$ (b) $\langle 2, 3\rangle$ (c) $\sqrt{3} - \frac{3}{2}$
9. (a) $\langle 2xyz - yz^3, x^2z - xz^3, x^2y - 3xyz^2\rangle$ (b) $\langle -3, 2, 2\rangle$ (c) $\frac{2}{5}$
11. $\dfrac{4 - 3\sqrt{3}}{10}$ **13.** $-8/\sqrt{10}$ **15.** $4/\sqrt{30}$
17. $\frac{23}{42}$ **19.** 2/5 **21.** $\sqrt{65}$, $\langle 1, 8\rangle$
23. 1, $\langle 0, 1\rangle$ **25.** 1, $\langle 3, 6, -2\rangle$
27. (b) $\langle -12, 92\rangle$
29. All points on the line $y = x + 1$
31. (a) $-40/(3\sqrt{3})$
33. (a) $32/\sqrt{3}$ (b) $\langle 38, 6, 12\rangle$ (c) $2\sqrt{406}$
35. $\frac{327}{13}$ **39.** $\frac{774}{25}$
41. (a) $x + y + z = 11$ (b) $x - 3 = y - 3 = z - 5$

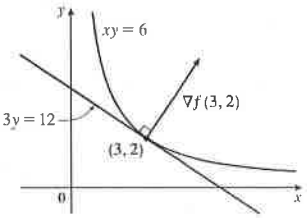
43. (a) $2x + 3y + 12z = 24$ (b) $\dfrac{x-3}{2} = \dfrac{y-2}{3} = \dfrac{z-1}{12}$

45. (a) $x + y + z = 1$ (b) $x = y = z - 1$

47.



49. $\langle 2, 3\rangle$, $2x + 3y = 12$



55. No **59.** $\left(-\frac{5}{4}, -\frac{5}{4}, \frac{25}{8}\right)$

63. $x = -1 - 10t$, $y = 1 - 16t$, $z = 2 - 12t$

67. If $\mathbf{u} = \langle a, b\rangle$ and $\mathbf{v} = \langle c, d\rangle$, then $af_x + bf_y$ and $cf_x + df_y$ are known, so we solve linear equations for f_x and f_y.

EXERCISES 14.7 ■ PAGE 977

1. (a) f has a local minimum at (1, 1).
(b) f has a saddle point at (1, 1).

3. Local minimum at (1, 1), saddle point at (0, 0)

5. Minimum $f\left(\frac{1}{3}, -\frac{2}{3}\right) = -\frac{1}{3}$

7. Saddle points at (1, 1), (−1, −1)

9. Maximum $f(0, 0) = 2$, minimum $f(0, 4) = -30$, saddle points at (2, 2), (−2, 2)

11. Minimum $f(2, 1) = -8$, saddle point at (0, 0)

13. None **15.** Minimum $f(0, 0) = 0$, saddle points at $(\pm 1, 0)$

17. Minima $f(0, 1) = f(\pi, -1) = f(2\pi, 1) = -1$, saddle points at $(\pi/2, 0)$, $(3\pi/2, 0)$

21. Minima $f(1, \pm 1) = 3$, $f(-1, \pm 1) = 3$

23. Maximum $f(\pi/3, \pi/3) = 3\sqrt{3}/2$, minimum $f(5\pi/3, 5\pi/3) = -3\sqrt{3}/2$, saddle point at (π, π)

25. Minima $f(0, -0.794) \approx -1.191$, $f(\pm 1.592, 1.267) \approx -1.310$, saddle points $(\pm 0.720, 0.259)$, lowest points $(\pm 1.592, 1.267, -1.310)$

27. Maximum $f(0.170, -1.215) \approx 3.197$, minima $f(-1.301, 0.549) \approx -3.145$, $f(1.131, 0.549) \approx -0.701$, saddle points $(-1.301, -1.215)$, $(0.170, 0.549)$, $(1.131, -1.215)$, no highest or lowest point

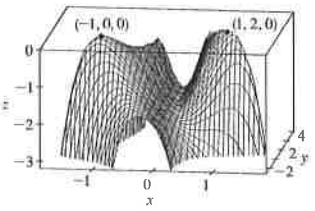
29. Maximum $f(0, \pm 2) = 4$, minimum $f(1, 0) = -1$

31. Maximum $f(\pm 1, 1) = 7$, minimum $f(0, 0) = 4$

33. Maximum $f(3, 0) = 83$, minimum $f(1, 1) = 0$

35. Maximum $f(1, 0) = 2$, minimum $f(-1, 0) = -2$

37.



39. $2/\sqrt{3}$ **41.** $(2, 1, \sqrt{5})$, $(2, 1, -\sqrt{5})$ **43.** $\frac{100}{3}, \frac{100}{3}, \frac{100}{3}$

45. $8r^3/(3\sqrt{3})$ **47.** $\frac{4}{3}$ **49.** Cube, edge length $c/12$

51. Square base of side 40 cm, height 20 cm **53.** $L^3/(3\sqrt{3})$

EXERCISES 14.8 ■ PAGE 987

1. ≈59, 30

3. No maximum, minimum $f(1, 1) = f(-1, -1) = 2$

5. Maximum $f(0, \pm 1) = 1$, minimum $f(\pm 2, 0) = -4$

7. Maximum $f(2, 2, 1) = 9$, minimum $f(-2, -2, -1) = -9$

9. Maximum $2/\sqrt{3}$, minimum $-2/\sqrt{3}$

11. Maximum $\sqrt{3}$, minimum 1

13. Maximum $f\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) = 2$, minimum $f\left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right) = -2$

15. Maximum $f(1, \sqrt{2}, -\sqrt{2}) = 1 + 2\sqrt{2}$, minimum $f(1, -\sqrt{2}, \sqrt{2}) = 1 - 2\sqrt{2}$

17. Maximum $\frac{3}{2}$, minimum $\frac{1}{2}$

19. Maximum $f(3/\sqrt{2}, -3/\sqrt{2}) = 9 + 12\sqrt{2}$, minimum $f(-2, 2) = -8$

21. Maximum $f\left(\pm 1/\sqrt{2}, \mp 1/(2\sqrt{2})\right) = e^{1/4}$, minimum $f\left(\pm 1/\sqrt{2}, \pm 1/(2\sqrt{2})\right) = e^{-1/4}$

29–41. See Exercises 39–53 in Section 14.7.

43. Nearest $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$, farthest $(-1, -1, 2)$

45. Maximum ≈9.7938, minimum ≈−5.3506

47. (a) c/n (b) When $x_1 = x_2 = \cdots = x_n$

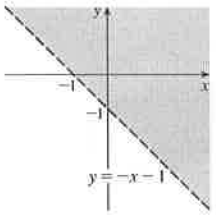
CHAPTER 14 REVIEW ■ PAGE 991

True-False Quiz

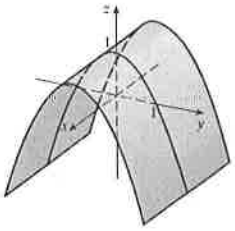
1. True **3.** False **5.** False **7.** True **9.** False **11.** True

Exercises

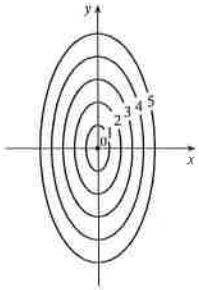
1. $\{(x, y) \mid y > -x - 1\}$



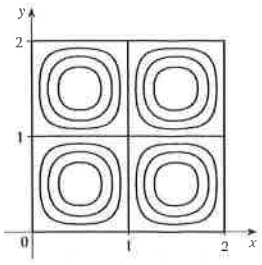
3.



5.



7.



9. $\frac{2}{3}$

11. (a) $\approx 3.5°C/m, -3.0°C/m$ (b) $\approx 0.35°C/m$ by Equation 14.6.9 (Definition 14.6.2 gives $\approx 1.1°C/m$.) (c) -0.25

13. $f_x = 32xy(5y^3 + 2x^2y)^7, f_y = (16x^2 + 120y^2)(5y^3 + 2x^2y)^7$

15. $F_\alpha = \dfrac{2\alpha^3}{\alpha^2 + \beta^2} + 2\alpha \ln(\alpha^2 + \beta^2), F_\beta = \dfrac{2\alpha^2\beta}{\alpha^2 + \beta^2}$

17. $S_u = \arctan\left(v\sqrt{w}\right), S_v = \dfrac{u\sqrt{w}}{1 + v^2w}, S_w = \dfrac{uv}{2\sqrt{w}(1 + v^2w)}$

19. $f_{xx} = 24x, f_{xy} = -2y = f_{yx}, f_{yy} = -2x$

21. $f_{xx} = k(k-1)x^{k-2}y^lz^m, f_{xy} = klx^{k-1}y^{l-1}z^m = f_{yx}$, $f_{xz} = kmx^{k-1}y^lz^{m-1} = f_{zx}, f_{yy} = l(l-1)x^ky^{l-2}z^m$, $f_{yz} = lmx^ky^{l-1}z^{m-1} = f_{zy}, f_{zz} = m(m-1)x^ky^lz^{m-2}$

25. (a) $z = 8x + 4y + 1$ (b) $\dfrac{x-1}{8} = \dfrac{y+2}{4} = \dfrac{z-1}{-1}$

27. (a) $2x - 2y - 3z = 3$ (b) $\dfrac{x-2}{4} = \dfrac{y+1}{-4} = \dfrac{z-1}{-6}$

29. (a) $x + 2y + 5z = 0$ (b) $x = 2 + t, y = -1 + 2t, z = 5t$

31. $\left(2, \frac{1}{2}, -1\right), \left(-2, -\frac{1}{2}, 1\right)$

33. $60x + \frac{24}{5}y + \frac{32}{5}z - 120; 38.656$

35. $2xy^3(1 + 6p) + 3x^2y^2(pe^p + e^p) + 4z^3(p\cos p + \sin p)$

37. $-47, 108$

43. $\langle 2xe^{yz^2}, x^2z^2e^{yz^2}, 2x^2yze^{yz^2}\rangle$ **45.** $-\frac{4}{5}$

47. $\sqrt{145}/2, \langle 4, \frac{9}{2}\rangle$ **49.** $\approx \frac{5}{8}$ knot/mi

51. Minimum $f(-4, 1) = -11$

53. Maximum $f(1, 1) = 1$; saddle points (0, 0), (0, 3), (3, 0)

55. Maximum $f(1, 2) = 4$, minimum $f(2, 4) = -64$

57. Maximum $f(-1, 0) = 2$, minima $f(1, \pm 1) = -3$, saddle points $(-1, \pm 1)$, (1, 0)

59. Maximum $f\left(\pm\sqrt{2/3}, 1/\sqrt{3}\right) = 2/\left(3\sqrt{3}\right)$, minimum $f\left(\pm\sqrt{2/3}, -1/\sqrt{3}\right) = -2/\left(3\sqrt{3}\right)$

61. Maximum 1, minimum -1

63. $\left(\pm 3^{-1/4}, 3^{-1/4}\sqrt{2}, \pm 3^{1/4}\right), \left(\pm 3^{-1/4}, -3^{-1/4}\sqrt{2}, \pm 3^{1/4}\right)$

65. $P\left(2 - \sqrt{3}\right), P\left(3 - \sqrt{3}\right)/6, P\left(2\sqrt{3} - 3\right)/3$

PROBLEMS PLUS ■ PAGE 995

1. $L^2W^2, \frac{1}{4}L^2W^2$ **3.** (a) $x = w/3$, base $= w/3$ (b) Yes

7. $\sqrt{3/2}, 3/\sqrt{2}$

CHAPTER 15

EXERCISES 15.1 ■ PAGE 1005

1. (a) 288 (b) 144 **3.** (a) 0.990 (b) 1.151

5. (a) 4 (b) -8 **7.** $U < V < L$

9. (a) ≈ 248 (b) ≈ 15.5 **11.** 60 **13.** 3

15. 1.141606, 1.143191, 1.143535, 1.143617, 1.143637, 1.143642

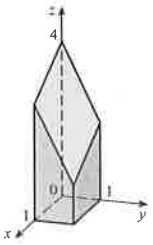
EXERCISES 15.2 ■ PAGE 1011

1. $500y^3, 3x^2$ **3.** 222 **5.** $32(e^4 - 1)$ **7.** 18

9. $\frac{21}{2}\ln 2$ **11.** $\frac{31}{30}$ **13.** π **15.** 0

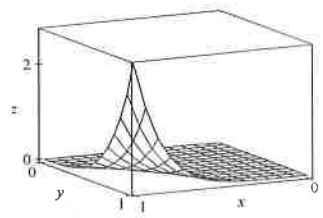
17. $9 \ln 2$ **19.** $\frac{1}{2}\left(\sqrt{3} - 1\right) - \frac{1}{12}\pi$ **21.** $\frac{1}{2}e^{-6} + \frac{5}{2}$

23.



25. 51 **27.** $\frac{166}{27}$ **29.** 2 **31.** $\frac{64}{3}$

33. $21e - 57$



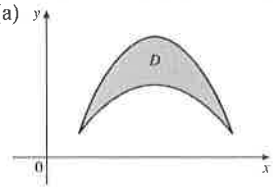
35. $\frac{5}{6}$ **37.** 0

39. Fubini's Theorem does not apply. The integrand has an infinite discontinuity at the origin.

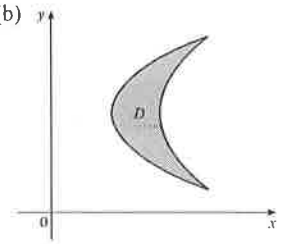
EXERCISES 15.3 ■ PAGE 1019

1. 32 **3.** $\frac{3}{10}$ **5.** $\frac{1}{3}\sin 1$ **7.** $\frac{4}{3}$ **9.** π

11. (a)



(b)



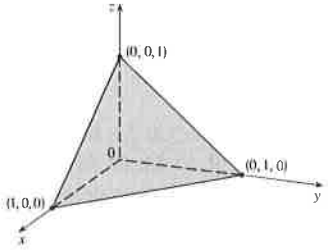
13. Type I: $D = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq x\}$, type II: $D = \{(x, y) \mid 0 \leq y \leq 1, y \leq x \leq 1\}; \frac{1}{3}$

15. $\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} y\,dy\,dx + \int_1^4 \int_{x-2}^{\sqrt{x}} y\,dy\,dx = \int_{-1}^2 \int_{y^2}^{y+2} y\,dx\,dy = \frac{9}{4}$

17. $\frac{1}{2}(1 - \cos 1)$ **19.** $\frac{11}{3}$ **21.** 0 **23.** $\frac{17}{60}$ **25.** $\frac{31}{8}$

27. 6 **29.** $\frac{128}{15}$ **31.** $\frac{1}{3}$ **33.** 0, 1.213; 0.713 **35.** $\frac{64}{3}$

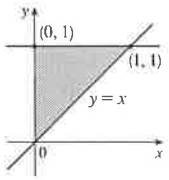
37.



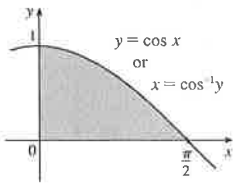
39. 13,984,735,616/14,549,535

41. $\pi/2$

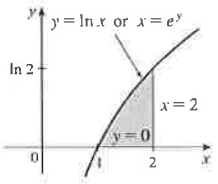
43. $\int_0^1 \int_x^1 f(x, y)\, dy\, dx$



45. $\int_0^1 \int_0^{\cos^{-1} y} f(x, y)\, dx\, dy$



47. $\int_0^{\ln 2} \int_{e^y}^2 f(x, y)\, dx\, dy$

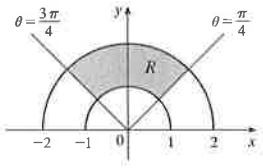


49. $\frac{1}{6}(e^9 - 1)$ **51.** $\frac{1}{3} \ln 9$ **53.** $\frac{1}{3}(2\sqrt{2} - 1)$ **55.** 1
57. $(\pi/16)e^{-1/16} \le \iint_Q e^{-(x^2+y^2)^2}\, dA \le \pi/16$ **59.** $\frac{3}{4}$ **63.** 9π
65. $a^2 b + \frac{3}{2}ab^2$ **67.** $\pi a^2 b$

EXERCISES 15.4 ■ PAGE 1026

1. $\int_0^{3\pi/2} \int_0^4 f(r\cos\theta, r\sin\theta) r\, dr\, d\theta$ **3.** $\int_{-1}^1 \int_0^{(x+1)/2} f(x, y)\, dy\, dx$

5. $3\pi/4$



7. $\frac{1250}{3}$ **9.** $(\pi/4)(\cos 1 - \cos 9)$
11. $(\pi/2)(1 - e^{-4})$ **13.** $\frac{3}{64}\pi^2$ **15.** $\pi/12$
17. $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$ **19.** $\frac{16}{3}\pi$ **21.** $\frac{4}{3}\pi$ **23.** $\frac{4}{3}\pi a^3$
25. $(2\pi/3)\left[1 - (1/\sqrt{2})\right]$ **27.** $(8\pi/3)(64 - 24\sqrt{3})$
29. $\frac{1}{2}\pi(1 - \cos 9)$ **31.** $2\sqrt{2}/3$ **33.** 4.5951
35. 1800π ft^3 **37.** $2/(a + b)$ **39.** $\frac{15}{16}$
41. (a) $\sqrt{\pi}/4$ (b) $\sqrt{\pi}/2$

EXERCISES 15.5 ■ PAGE 1036

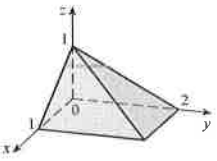
1. 285 C **3.** $42k, \left(2, \frac{85}{28}\right)$ **5.** $6, \left(\frac{3}{4}, \frac{3}{2}\right)$ **7.** $\frac{8}{15}k, \left(0, \frac{4}{7}\right)$
9. $L/4, (L/2, 16/(9\pi))$ **11.** $\left(\frac{3}{8}, 3\pi/16\right)$ **13.** $(0, 45/(14\pi))$
15. $(2a/5, 2a/5)$ if vertex is (0, 0) and sides are along positive axes
17. $\frac{64}{315}k, \frac{8}{105}k, \frac{88}{315}k$
19. $7ka^6/180, 7ka^6/180, 7ka^6/90$ if vertex is (0, 0) and sides are along positive axes
21. $\rho bh^3/3, \rho b^3 h/3; b/\sqrt{3}, h/\sqrt{3}$
23. $\rho a^4\pi/16, \rho a^4\pi/16; a/2, a/2$
25. $m = 3\pi/64, (\bar{x}, \bar{y}) = \left(\frac{16384\sqrt{2}}{10395\pi}, 0\right)$,
$I_x = \frac{5\pi}{384} - \frac{4}{105}, I_y = \frac{5\pi}{384} + \frac{4}{105}, I_0 = \frac{5\pi}{192}$
27. (a) $\frac{1}{2}$ (b) 0.375 (c) $\frac{5}{48} \approx 0.1042$
29. (b) (i) $e^{-0.2} \approx 0.8187$
(ii) $1 + e^{-1.8} - e^{-0.8} - e^{-1} \approx 0.3481$ (c) 2, 5
31. (a) ≈ 0.500 (b) ≈ 0.632
33. (a) $\iint_D k\left[1 - \frac{1}{20}\sqrt{(x - x_0)^2 + (y - y_0)^2}\right] dA$, where D is the disk with radius 10 mi centered at the center of the city
(b) $200\pi k/3 \approx 209k$, $200(\pi/2 - \frac{8}{9})k \approx 136k$, on the edge

EXERCISES 15.6 ■ PAGE 1040

1. $15\sqrt{26}$ **3.** $3\sqrt{14}$ **5.** $12\sin^{-1}\left(\frac{2}{3}\right)$
7. $(\pi/6)(17\sqrt{17} - 5\sqrt{5})$ **9.** $(2\pi/3)(2\sqrt{2} - 1)$
11. $a^2(\pi - 2)$ **13.** 13.9783 **15.** (a) ≈ 1.83 (b) ≈ 1.8616
17. $\frac{45}{8}\sqrt{14} + \frac{15}{16}\ln\left[(11\sqrt{5} + 3\sqrt{70})/(3\sqrt{5} + \sqrt{70})\right]$
19. 3.3213 **23.** $(\pi/6)(101\sqrt{101} - 1)$

EXERCISES 15.7 ■ PAGE 1049

1. $\frac{27}{4}$ **3.** $\frac{16}{15}$ **5.** $\frac{5}{3}$ **7.** $-\frac{1}{3}$ **9.** $\frac{27}{2}$ **11.** $9\pi/8$
13. $\frac{65}{28}$ **15.** $\frac{1}{60}$ **17.** $16\pi/3$ **19.** $\frac{16}{3}$ **21.** $\frac{8}{15}$
23. (a) $\int_0^1 \int_0^x \int_0^{\sqrt{1-y^2}} dz\, dy\, dx$ (b) $\frac{1}{4}\pi - \frac{1}{3}$
25. 0.985
27.



29. $\int_{-2}^2 \int_0^{4-x^2} \int_{-\sqrt{4-x^2-y}/2}^{\sqrt{4-x^2-y}/2} f(x, y, z)\, dz\, dy\, dx$
$= \int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} \int_{-\sqrt{4-x^2-y}/2}^{\sqrt{4-x^2-y}/2} f(x, y, z)\, dz\, dx\, dy$
$= \int_{-1}^1 \int_0^{4-4z^2} \int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x, y, z)\, dx\, dy\, dz$
$= \int_0^4 \int_{-\sqrt{4-y}/2}^{\sqrt{4-y}/2} \int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x, y, z)\, dx\, dz\, dy$
$= \int_{-2}^2 \int_{-\sqrt{4-x^2}/2}^{\sqrt{4-x^2}/2} \int_0^{4-x^2-4z^2} f(x, y, z)\, dy\, dz\, dx$
$= \int_{-1}^1 \int_{-\sqrt{4-4z^2}}^{\sqrt{4-4z^2}} \int_0^{4-x^2-4z^2} f(x, y, z)\, dy\, dx\, dz$

31. $\int_{-2}^2 \int_{x^2}^4 \int_0^{2-y/2} f(x, y, z)\, dz\, dy\, dx$
$= \int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \int_0^{2-y/2} f(x, y, z)\, dz\, dx\, dy$
$= \int_0^2 \int_0^{4-2z} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z)\, dx\, dy\, dz$
$= \int_0^4 \int_0^{2-y/2} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z)\, dx\, dz\, dy$
$= \int_{-2}^2 \int_0^{2-x^2/2} \int_{x^2}^{4-2z} f(x, y, z)\, dy\, dz\, dx$
$= \int_0^2 \int_{-\sqrt{4-2z}}^{\sqrt{4-2z}} \int_{x^2}^{4-2z} f(x, y, z)\, dy\, dx\, dz$

33. $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z)\, dz\, dy\, dx = \int_0^1 \int_0^{y^2} \int_0^{1-y} f(x, y, z)\, dz\, dx\, dy$
$= \int_0^1 \int_0^{1-z} \int_0^{y^2} f(x, y, z)\, dx\, dy\, dz = \int_0^1 \int_0^{1-y} \int_0^{y^2} f(x, y, z)\, dx\, dz\, dy$
$= \int_0^1 \int_0^{1-\sqrt{x}} \int_{\sqrt{x}}^{1-z} f(x, y, z)\, dy\, dz\, dx = \int_0^1 \int_0^{(1-z)^2} \int_{\sqrt{x}}^{1-z} f(x, y, z)\, dy\, dx\, dz$

35. $\int_0^1 \int_y^1 \int_0^y f(x, y, z)\, dz\, dx\, dy = \int_0^1 \int_0^x \int_0^y f(x, y, z)\, dz\, dy\, dx$
$= \int_0^1 \int_z^1 \int_y^1 f(x, y, z)\, dx\, dy\, dz = \int_0^1 \int_0^y \int_y^1 f(x, y, z)\, dx\, dz\, dy$
$= \int_0^1 \int_0^x \int_z^x f(x, y, z)\, dy\, dz\, dx = \int_0^1 \int_z^1 \int_z^x f(x, y, z)\, dy\, dx\, dz$

37. 64π **39.** $\frac{79}{30}, \left(\frac{358}{553}, \frac{33}{79}, \frac{571}{553}\right)$

41. a^5, $(7a/12, 7a/12, 7a/12)$

43. $I_x = I_y = I_z = \frac{2}{3}kL^5$ **45.** $\frac{1}{2}\pi kha^4$

47. (a) $m = \int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} \sqrt{x^2 + y^2}\, dz\, dy\, dx$
(b) $(\bar{x}, \bar{y}, \bar{z})$, where
$\bar{x} = (1/m) \int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} x\sqrt{x^2 + y^2}\, dz\, dy\, dx$
$\bar{y} = (1/m) \int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} y\sqrt{x^2 + y^2}\, dz\, dy\, dx$
$\bar{z} = (1/m) \int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} z\sqrt{x^2 + y^2}\, dz\, dy\, dx$
(c) $\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} (x^2 + y^2)^{3/2}\, dz\, dy\, dx$

49. (a) $\frac{3}{32}\pi + \frac{11}{24}$

(b) $\left(\dfrac{28}{9\pi + 44}, \dfrac{30\pi + 128}{45\pi + 220}, \dfrac{45\pi + 208}{135\pi + 660}\right)$

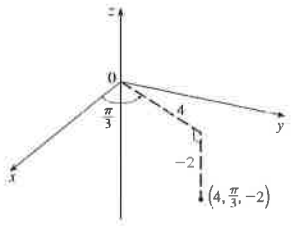
(c) $\frac{1}{240}(68 + 15\pi)$

51. (a) $\frac{1}{8}$ (b) $\frac{1}{64}$ (c) $\frac{1}{3760}$ **53.** $L^3/8$

55. (a) The region bounded by the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$
(b) $4\sqrt{6}\pi/45$

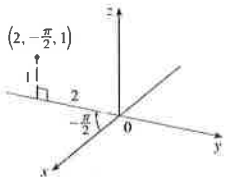
EXERCISES 15.8 ■ PAGE 1055

1. (a)



$\left(2, 2\sqrt{3}, -2\right)$

(b)



$(0, -2, 1)$

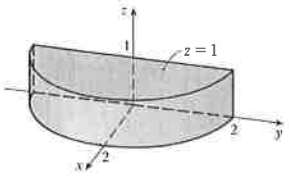
3. (a) $\left(\sqrt{2}, 3\pi/4, 1\right)$ (b) $(4, 2\pi/3, 3)$

5. Vertical half-plane through the z-axis

7. Circular paraboloid

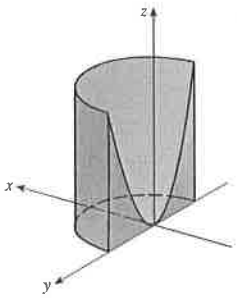
9. (a) $z^2 = 1 + r\cos\theta - r^2$ (b) $z = r^2\cos 2\theta$

11.



13. Cylindrical coordinates: $6 \le r \le 7$, $0 \le \theta \le 2\pi$, $0 \le z \le 20$

15.



4π

17. 384π **19.** $\frac{8}{3}\pi + \frac{128}{15}$ **21.** $2\pi/5$ **23.** $\frac{4}{3}\pi\left(\sqrt{2} - 1\right)$

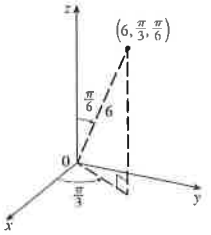
25. (a) 162π (b) $(0, 0, 15)$

27. $\pi Ka^2/8$, $(0, 0, 2a/3)$ **29.** 0

31. (a) $\iiint_C h(P)g(P)\, dV$, where C is the cone
(b) $\approx 3.1 \times 10^{19}$ ft-lb

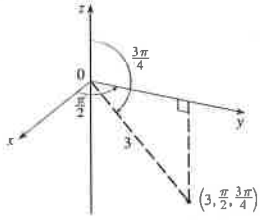
EXERCISES 15.9 ■ PAGE 1061

1. (a)



$\left(\dfrac{3}{2}, \dfrac{3\sqrt{3}}{2}, 3\sqrt{3}\right)$

(b)



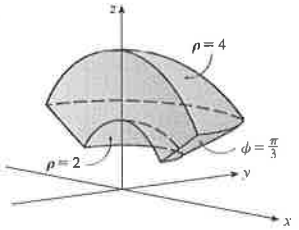
$\left(0, \dfrac{3\sqrt{2}}{2}, -\dfrac{3\sqrt{2}}{2}\right)$

3. (a) $(2, 3\pi/2, \pi/2)$ (b) $(2, 3\pi/4, 3\pi/4)$

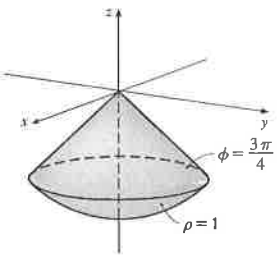
5. Half-cone **7.** Sphere, radius $\frac{1}{2}$, center $\left(0, \frac{1}{2}, 0\right)$

9. (a) $\cos^2\phi = \sin^2\phi$ (b) $\rho^2(\sin^2\phi\cos^2\theta + \cos^2\phi) = 9$

11.

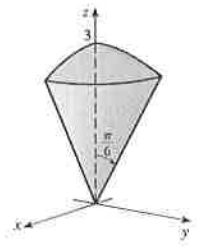


13.

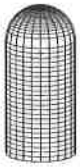


15. $0 \le \phi \le \pi/4$, $0 \le \rho \le \cos\phi$

17. $(9\pi/4)(2-\sqrt{3})$



19. $\int_0^{\pi/2}\int_0^3\int_0^2 f(r\cos\theta, r\sin\theta, z)\, r\, dz\, dr\, d\theta$
21. $312{,}500\pi/7$ **23.** $1688\pi/15$ **25.** $\pi/8$
27. $(\sqrt{3}-1)\pi a^3/3$ **29.** (a) 10π (b) $(0, 0, 2.1)$
31. (a) $(0, 0, \frac{7}{12})$ (b) $11K\pi/960$
33. (a) $(0, 0, \frac{3}{8}a)$ (b) $4K\pi a^5/15$
35. $\frac{1}{3}\pi(2-\sqrt{2}), (0, 0, 3/[8(2-\sqrt{2})])$
37. $5\pi/6$ **39.** $(4\sqrt{2}-5)/15$ **41.** $4096\pi/21$
43.



45. $136\pi/99$

EXERCISES 15.10 ■ PAGE 1071

1. 16 **3.** $\sin^2\theta - \cos^2\theta$ **5.** 0
7. The parallelogram with vertices (0, 0), (6, 3), (12, 1), (6, −2)
9. The region bounded by the line $y = 1$, the y-axis, and $y = \sqrt{x}$
11. $x = \frac{1}{3}(v-u)$, $y = \frac{1}{3}(u+2v)$ is one possible transformation, where $S = \{(u, v) \mid -1 \le u \le 1, 1 \le v \le 3\}$
13. $x = u\cos v$, $y = u\sin v$ is one possible transformation, where $S = \{(u, v) \mid 1 \le u \le \sqrt{2}, 0 \le v \le \pi/2\}$
15. -3 **17.** 6π **19.** $2\ln 3$
21. (a) $\frac{4}{3}\pi abc$ (b) $1.083 \times 10^{12}\ \text{km}^3$ (c) $\frac{4}{15}\pi(a^2+b^2)abck$
23. $\frac{8}{5}\ln 8$ **25.** $\frac{3}{2}\sin 1$ **27.** $e - e^{-1}$

CHAPTER 15 REVIEW ■ PAGE 1073

True-False Quiz

1. True **3.** True **5.** True **7.** True **9.** False

Exercises

1. ≈64.0 **3.** $4e^2 - 4e + 3$ **5.** $\frac{1}{2}\sin 1$ **7.** $\frac{2}{3}$
9. $\int_0^{\pi}\int_2^4 f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$
11. The region inside the loop of the four-leaved rose $r = \sin 2\theta$ in the first quadrant
13. $\frac{1}{2}\sin 1$ **15.** $\frac{1}{2}e^6 - \frac{7}{2}$ **17.** $\frac{1}{4}\ln 2$ **19.** 8
21. $81\pi/5$ **23.** $\frac{81}{2}$ **25.** $\pi/96$ **27.** $\frac{64}{15}$
29. 176 **31.** $\frac{2}{3}$ **33.** $2ma^3/9$
35. (a) $\frac{1}{4}$ (b) $(\frac{1}{3}, \frac{8}{15})$
(c) $I_x = \frac{1}{12}$, $I_y = \frac{1}{24}$; $\bar{y} = 1/\sqrt{3}$, $\bar{x} = 1/\sqrt{6}$
37. (a) $(0, 0, h/4)$ (b) $\pi a^4 h/10$
39. $\ln(\sqrt{2}+\sqrt{3}) + \sqrt{2}/3$ **41.** $\frac{486}{5}$ **43.** 0.0512
45. (a) $\frac{1}{15}$ (b) $\frac{1}{3}$ (c) $\frac{1}{45}$
47. $\int_0^1\int_0^{1-z}\int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z)\, dx\, dy\, dz$ **49.** $-\ln 2$ **51.** 0

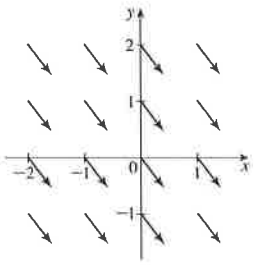
PROBLEMS PLUS ■ PAGE 1077

1. 30 **3.** $\frac{1}{2}\sin 1$ **7.** (b) 0.90
13. $abc\pi\left(\frac{2}{3} - \frac{8}{9\sqrt{3}}\right)$

CHAPTER 16

EXERCISES 16.1 ■ PAGE 1085

1.



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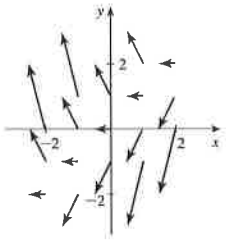

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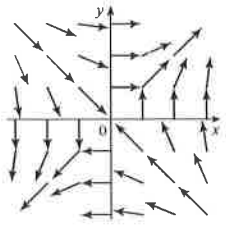
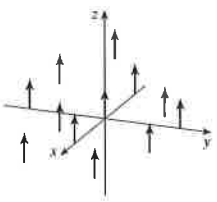
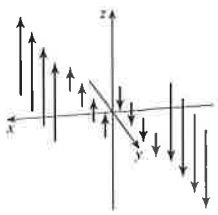


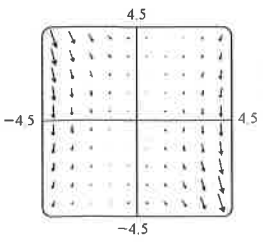
7.



9.



11. IV **13.** I **15.** IV **17.** III

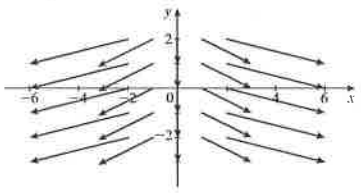
19. 

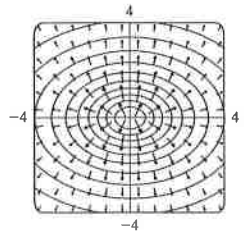
The line $y = 2x$

21. $\nabla f(x, y) = (xy + 1)e^{xy}\,\mathbf{i} + x^2e^{xy}\,\mathbf{j}$

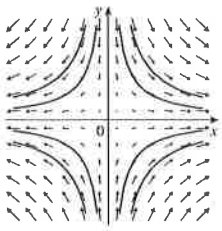
23. $\nabla f(x, y, z) = \dfrac{x}{\sqrt{x^2 + y^2 + z^2}}\,\mathbf{i} + \dfrac{y}{\sqrt{x^2 + y^2 + z^2}}\,\mathbf{j} + \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}\,\mathbf{k}$

25. $\nabla f(x, y) = 2x\,\mathbf{i} - \mathbf{j}$



27. 

29. III **31.** II **33.** (2.04, 1.03)

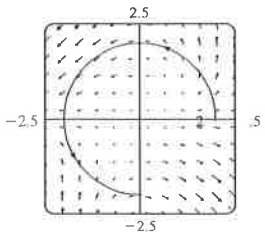
35. (a) 

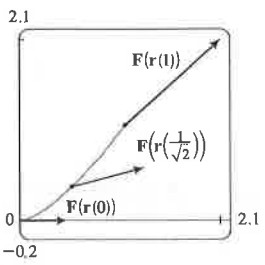
(b) $y = 1/x, x > 0$

$y = C/x$

EXERCISES 16.2 ■ PAGE 1096

1. $\frac{1}{54}(145^{3/2} - 1)$ **3.** 1638.4 **5.** $\frac{243}{8}$ **7.** $\frac{5}{2}$
9. $\sqrt{5}\pi$ **11.** $\frac{1}{12}\sqrt{14}(e^6 - 1)$ **13.** $\frac{2}{5}(e - 1)$ **15.** $\frac{35}{3}$
17. (a) Positive (b) Negative **19.** 45
21. $\frac{6}{5} - \cos 1 - \sin 1$ **23.** 1.9633 **25.** 15.0074
27. $3\pi + \frac{2}{3}$



29. (a) $\frac{11}{8} - 1/e$ (b) 

31. $\frac{172,704}{5,632,705}\sqrt{2}(1 - e^{-14\pi})$ **33.** $2\pi k$, $(4/\pi, 0)$
35. (a) $\bar{x} = (1/m)\int_C x\rho(x, y, z)\,ds$,
$\bar{y} = (1/m)\int_C y\rho(x, y, z)\,ds$,
$\bar{z} = (1/m)\int_C z\rho(x, y, z)\,ds$, where $m = \int_C \rho(x, y, z)\,ds$
(b) $(0, 0, 3\pi)$
37. $I_x = k\left(\frac{1}{2}\pi - \frac{4}{3}\right)$, $I_y = k\left(\frac{1}{2}\pi - \frac{2}{3}\right)$ **39.** $2\pi^2$ **41.** $\frac{7}{3}$
43. (a) $2ma\,\mathbf{i} + 6mbt\,\mathbf{j}$, $0 \leq t \leq 1$ (b) $2ma^2 + \frac{9}{2}mb^2$
45. $\approx 1.67 \times 10^4$ ft-lb **47.** (b) Yes **51.** ≈ 22 J

EXERCISES 16.3 ■ PAGE 1106

1. 40 **3.** $f(x, y) = x^2 - 3xy + 2y^2 - 8y + K$
5. Not conservative **7.** $f(x, y) = ye^x + x\sin y + K$
9. $f(x, y) = x\ln y + x^2y^3 + K$
11. (b) 16 **13.** (a) $f(x, y) = \frac{1}{2}x^2y^2$ (b) 2
15. (a) $f(x, y, z) = xyz + z^2$ (b) 77
17. (a) $f(x, y, z) = ye^{xz}$ (b) 4 **19.** $4/e$
21. It doesn't matter which curve is chosen.
23. 30 **25.** No **27.** Conservative
31. (a) Yes (b) Yes (c) Yes
33. (a) No (b) Yes (c) Yes

EXERCISES 16.4 ■ PAGE 1113

1. 8π **3.** $\frac{2}{3}$ **5.** 12 **7.** $\frac{1}{3}$ **9.** -24π **11.** $-\frac{16}{3}$
13. 4π **15.** $-8e + 48e^{-1}$ **17.** $-\frac{1}{12}$ **19.** 3π **21.** (c) $\frac{9}{2}$
23. $(4a/3\pi, 4a/3\pi)$ if the region is the portion of the disk $x^2 + y^2 = a^2$ in the first quadrant
27. 0

EXERCISES 16.5 ■ PAGE 1121

1. (a) **0** (b) 3
3. (a) $ze^x\,\mathbf{i} + (xye^z - yze^x)\,\mathbf{j} - xe^z\,\mathbf{k}$ (b) $y(e^z + e^x)$
5. (a) **0** (b) $2/\sqrt{x^2 + y^2 + z^2}$
7. (a) $\langle -e^y\cos z, -e^z\cos x, -e^x\cos y\rangle$
(b) $e^x\sin y + e^y\sin z + e^z\sin x$
9. (a) Negative (b) curl $\mathbf{F} = \mathbf{0}$
11. (a) Zero (b) curl $\mathbf{F}$ points in the negative z-direction
13. $f(x, y, z) = xy^2z^3 + K$ **15.** Not conservative
17. $f(x, y, z) = xe^{yz} + K$ **19.** No

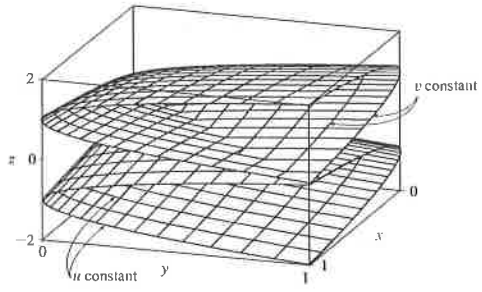
EXERCISES 16.6 ■ PAGE 1132

1. P: no; Q: yes

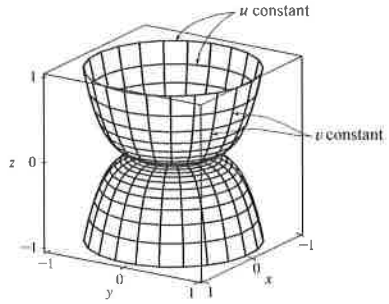
3. Plane through (0, 3, 1) containing vectors $\langle 1, 0, 4\rangle$, $\langle 1, -1, 5\rangle$

5. Hyperbolic paraboloid

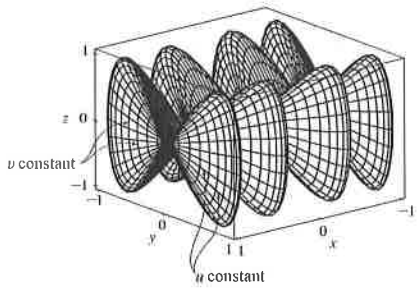
7.



9.



11.



13. IV **15.** II **17.** III

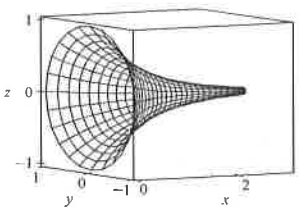
19. $x = u, y = v - u, z = -v$

21. $y = y, z = z, x = \sqrt{1 + y^2 + \frac{1}{4}z^2}$

23. $x = 2\sin\phi\cos\theta, y = 2\sin\phi\sin\theta,$
$z = 2\cos\phi, 0 \le \phi \le \pi/4, 0 \le \theta \le 2\pi$
$\left[\text{or } x = x, y = y, z = \sqrt{4 - x^2 - y^2}, x^2 + y^2 \le 2\right]$

25. $x = x, y = 4\cos\theta, z = 4\sin\theta, 0 \le x \le 5, 0 \le \theta \le 2\pi$

29. $x = x, y = e^{-x}\cos\theta,$
$z = e^{-x}\sin\theta, 0 \le x \le 3,$
$0 \le \theta \le 2\pi$



31. (a) Direction reverses (b) Number of coils doubles

33. $3x - y + 3z = 3$ **35.** $\frac{\sqrt{3}}{2}x - \frac{1}{2}y + z = \frac{\pi}{3}$

37. $-x + 2z = 1$ **39.** $3\sqrt{14}$ **41.** $\sqrt{14}\,\pi$

43. $\frac{4}{15}(3^{5/2} - 2^{7/2} + 1)$ **45.** $(2\pi/3)(2\sqrt{2} - 1)$

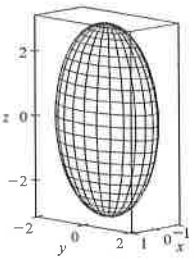
47. $\frac{1}{2}\sqrt{21} + \frac{17}{4}\left[\ln\left(2 + \sqrt{21}\right) - \ln\sqrt{17}\right]$ **49.** 4

51. $A(S) \le \sqrt{3}\,\pi R^2$ **53.** 13.9783

55. (a) 24.2055 (b) 24.2476

57. $\frac{45}{8}\sqrt{14} + \frac{15}{16}\ln\left[\left(11\sqrt{5} + 3\sqrt{70}\right)/\left(3\sqrt{5} + \sqrt{70}\right)\right]$

59. (b)



(c) $\int_0^{2\pi}\int_0^{\pi}\sqrt{36\sin^4u\cos^2v + 9\sin^4u\sin^2v + 4\cos^2u\sin^2u}\,du\,dv$

61. 4π. **63.** $2a^2(\pi - 2)$

EXERCISES 16.7 ■ PAGE 1144

1. 49.09 **3.** 900π **5.** $11\sqrt{14}$ **7.** $\frac{2}{3}\left(2\sqrt{2} - 1\right)$

9. $171\sqrt{14}$ **11.** $\sqrt{21}/3$ **13.** $364\sqrt{2}\,\pi/3$

15. $(\pi/60)\left(391\sqrt{17} + 1\right)$ **17.** 16π **19.** 12 **21.** 4

23. $\frac{713}{180}$ **25.** $-\frac{4}{3}\pi$ **27.** 0 **29.** 48 **31.** $2\pi + \frac{8}{3}$

33. 4.5822 **35.** 3.4895

37. $\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_D [P(\partial h/\partial x) - Q + R(\partial h/\partial z)]\,dA,$
where D = projection of S on xz-plane

39. $(0, 0, a/2)$

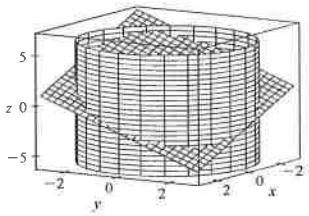
41. (a) $I_z = \iint_S (x^2 + y^2)\rho(x, y, z)\,dS$ (b) $4329\sqrt{2}\,\pi/5$

43. 0 kg/s **45.** $\frac{8}{3}\pi a^3\varepsilon_0$ **47.** 1248π

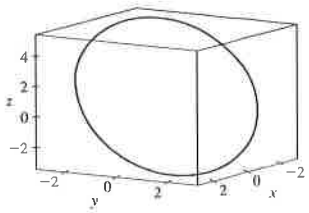
EXERCISES 16.8 ■ PAGE 1151

3. 0 **5.** 0 **7.** -1 **9.** 80π

11. (a) $81\pi/2$ (b)



(c) $x = 3\cos t, y = 3\sin t,$
$z = 1 - 3(\cos t + \sin t),$
$0 \le t \le 2\pi$



17. 3