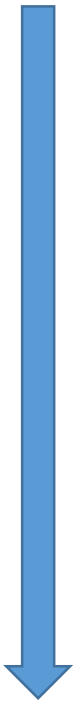
7.2 Exercises

1–49 Evaluate the integral.

1. $\int \sin^2 x \cos^3 x \, dx$
2. $\int \sin^3\theta \cos^4\theta \, d\theta$
3. $\int_0^{\pi/2} \sin^7\theta \cos^5\theta \, d\theta$
4. $\int_0^{\pi/2} \sin^5 x \, dx$
5. $\int \sin^2(\pi x) \cos^5(\pi x) \, dx$
6. $\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}} \, dx$
7. $\int_0^{\pi/2} \cos^2\theta \, d\theta$
8. $\int_0^{2\pi} \sin^2\left(\frac{1}{3}\theta\right) d\theta$
9. $\int_0^{\pi} \cos^4(2t) \, dt$
10. $\int_0^{\pi} \sin^2 t \cos^4 t \, dt$
11. $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$
12. $\int_0^{\pi/2} (2 - \sin\theta)^2 \, d\theta$
13. $\int t \sin^2 t \, dt$
14. $\int \cos\theta \cos^5(\sin\theta) \, d\theta$
15. $\int \frac{\cos^5\alpha}{\sqrt{\sin\alpha}} \, d\alpha$
16. $\int x \sin^3 x \, dx$



17. $\int \cos^2 x \tan^3 x\, dx$

18. $\int \cot^5\theta \sin^4\theta\, d\theta$

19. $\int \dfrac{\cos x + \sin 2x}{\sin x}\, dx$

20. $\int \cos^2 x \sin 2x\, dx$

21. $\int \tan x \sec^3 x\, dx$

22. $\int \tan^2\theta \sec^4\theta\, d\theta$

23. $\int \tan^2 x\, dx$

24. $\int (\tan^2 x + \tan^4 x)\, dx$

25. $\int \tan^4 x \sec^6 x\, dx$

26. $\int_0^{\pi/4} \sec^4\theta \tan^4\theta\, d\theta$

27. $\int_0^{\pi/3} \tan^5 x \sec^4 x\, dx$

28. $\int \tan^5 x \sec^3 x\, dx$

29. $\int \tan^3 x \sec x\, dx$

30. $\int_0^{\pi/4} \tan^4 t\, dt$

31. $\int \tan^5 x\, dx$

32. $\int \tan^2 x \sec x\, dx$

33. $\int x \sec x \tan x\, dx$

34. $\int \dfrac{\sin\phi}{\cos^3\phi}\, d\phi$

35. $\int_{\pi/6}^{\pi/2} \cot^2 x\, dx$

36. $\int_{\pi/4}^{\pi/2} \cot^3 x\, dx$

37. $\int_{\pi/4}^{\pi/2} \cot^5\phi \csc^3\phi\, d\phi$

38. $\int \csc^4 x \cot^6 x\, dx$

39. $\int \csc x\, dx$

40. $\int_{\pi/6}^{\pi/3} \csc^3 x\, dx$

41. $\int \sin 8x \cos 5x\, dx$

42. $\int \cos \pi x \cos 4\pi x\, dx$

43. $\int \sin 5\theta \sin\theta\, d\theta$

44. $\int \dfrac{\cos x + \sin x}{\sin 2x}\, dx$

45. $\int_0^{\pi/6} \sqrt{1 + \cos 2x}\, dx$

46. $\int_0^{\pi/4} \sqrt{1 - \cos 4\theta}\, d\theta$

47. $\int \dfrac{1 - \tan^2 x}{\sec^2 x}\, dx$

48. $\int \dfrac{dx}{\cos x - 1}$

49. $\int x \tan^2 x\, dx$

---

50. If $\int_0^{\pi/4} \tan^6 x \sec x\, dx = I$, express the value of $\int_0^{\pi/4} \tan^8 x \sec x\, dx$ in terms of I.

51–54 Evaluate the indefinite integral. Illustrate, and check that your answer is reasonable, by graphing both the integrand and its antiderivative (taking $C = 0$).

51. $\int x \sin^2(x^2)\, dx$

52. $\int \sin^5 x \cos^3 x\, dx$

53. $\int \sin 3x \sin 6x\, dx$

54. $\int \sec^4 \dfrac{x}{2}\, dx$

---

55. Find the average value of the function $f(x) = \sin^2 x \cos^3 x$ on the interval $[-\pi, \pi]$.

56. Evaluate $\int \sin x \cos x\, dx$ by four methods:
(a) the substitution $u = \cos x$
(b) the substitution $u = \sin x$
(c) the identity $\sin 2x = 2 \sin x \cos x$
(d) integration by parts
Explain the different appearances of the answers.

57–58 Find the area of the region bounded by the given curves.

57. $y = \sin^2 x$, $y = \cos^2 x$, $-\pi/4 \le x \le \pi/4$

58. $y = \sin^3 x$, $y = \cos^3 x$, $\pi/4 \le x \le 5\pi/4$

59–60 Use a graph of the integrand to guess the value of the integral. Then use the methods of this section to prove that your guess is correct.

59. $\int_0^{2\pi} \cos^3 x\, dx$

60. $\int_0^2 \sin 2\pi x \cos 5\pi x\, dx$

61–64 Find the volume obtained by rotating the region bounded by the given curves about the specified axis.

61. $y = \sin x$, $y = 0$, $\pi/2 \le x \le \pi$; about the x-axis

62. $y = \sin^2 x$, $y = 0$, $0 \le x \le \pi$; about the x-axis

63. $y = \sin x$, $y = \cos x$, $0 \le x \le \pi/4$; about $y = 1$

64. $y = \sec x$, $y = \cos x$, $0 \le x \le \pi/3$; about $y = -1$

---

65. A particle moves on a straight line with velocity function $v(t) = \sin \omega t \cos^2 \omega t$. Find its position function $s = f(t)$ if $f(0) = 0$.

66. Household electricity is supplied in the form of alternating current that varies from 155 V to −155 V with a frequency of 60 cycles per second (Hz). The voltage is thus given by the equation

$$E(t) = 155 \sin(120\pi t)$$

where t is the time in seconds. Voltmeters read the RMS (root-mean-square) voltage, which is the square root of the average value of $[E(t)]^2$ over one cycle.
(a) Calculate the RMS voltage of household current.
(b) Many electric stoves require an RMS voltage of 220 V. Find the corresponding amplitude A needed for the voltage $E(t) = A \sin(120\pi t)$.

67–69 Prove the formula, where m and n are positive integers.

67. $\int_{-\pi}^{\pi} \sin mx \cos nx\,dx = 0$

68. $\int_{-\pi}^{\pi} \sin mx \sin nx\,dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$

69. $\int_{-\pi}^{\pi} \cos mx \cos nx\,dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$

70. A *finite Fourier series* is given by the sum

$$f(x) = \sum_{n=1}^{N} a_n \sin nx$$
$$= a_1 \sin x + a_2 \sin 2x + \cdots + a_N \sin Nx$$

Show that the mth coefficient a_m is given by the formula

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin mx\,dx$$