6.4 Exercises

1. Explain why the natural logarithmic function $y = \ln x$ is used much more frequently in calculus than the other logarithmic functions $y = \log_a x$.

2–26 Differentiate the function.

2. $f(x) = x \ln x - x$

3. $f(x) = \sin(\ln x)$

4. $f(x) = \ln(\sin^2 x)$

5. $f(x) = \ln \dfrac{1}{x}$

6. $y = \dfrac{1}{\ln x}$

7. $f(x) = \log_{10}(x^3 + 1)$

8. $f(x) = \log_5(xe^x)$

9. $f(x) = \sin x \ln(5x)$

10. $f(u) = \dfrac{u}{1 + \ln u}$

11. $G(y) = \ln \dfrac{(2y + 1)^5}{\sqrt{y^2 + 1}}$

12. $h(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$

13. $g(x) = \ln\left(x\sqrt{x^2 - 1}\right)$

14. $g(r) = r^2 \ln(2r + 1)$

15. $f(u) = \dfrac{\ln u}{1 + \ln(2u)}$

16. $y = \ln |1 + t - t^3|$

17. $f(x) = x^5 + 5^x$

18. $g(x) = x \sin(2^x)$

19. $y = \tan[\ln(ax + b)]$

20. $H(z) = \ln\sqrt{\dfrac{a^2 - z^2}{a^2 + z^2}}$

21. $y = \ln(e^{-x} + xe^{-x})$

22. $y = \ln|\cos(\ln x)|$

23. $y = 2x \log_{10}\sqrt{x}$

24. $y = \log_2(e^{-x}\cos \pi x)$

25. $f(t) = 10^{\sqrt{t}}$

26. $F(t) = 3^{\cos 2t}$

27–30 Find y' and y''.

27. $y = x^2 \ln(2x)$

28. $y = \dfrac{\ln x}{x^2}$

29. $y = \ln\left(x + \sqrt{1 + x^2}\right)$

30. $y = \ln(\sec x + \tan x)$

31–34 Differentiate f and find the domain of f.

31. $f(x) = \dfrac{x}{1 - \ln(x - 1)}$

32. $f(x) = \sqrt{2 + \ln x}$

33. $f(x) = \ln(x^2 - 2x)$

34. $f(x) = \ln \ln \ln x$

35. If $f(x) = \dfrac{\ln x}{1 + x^2}$, find $f'(1)$.

36. If $f(x) = \ln(1 + e^{2x})$, find $f'(0)$.

37–38 Find an equation of the tangent line to the curve at the given point.

37. $y = \ln(x^2 - 3x + 1)$, $(3, 0)$

38. $y = x^2 \ln x$, $(1, 0)$

39. If $f(x) = \sin x + \ln x$, find $f'(x)$. Check that your answer is reasonable by comparing the graphs of f and f'.

40. Find equations of the tangent lines to the curve $y = (\ln x)/x$ at the points $(1, 0)$ and $(e, 1/e)$. Illustrate by graphing the curve and its tangent lines.

41. Let $f(x) = cx + \ln(\cos x)$. For what value of c is $f'(\pi/4) = 6$?

42. Let $f(x) = \log_a(3x^2 - 2)$. For what value of a is $f'(1) = 3$?

43–54 Use logarithmic differentiation to find the derivative of the function.

43. $y = (x^2 + 2)^2(x^4 + 4)^4$

44. $y = \dfrac{e^{-x}\cos^2 x}{x^2 + x + 1}$

45. $y = \sqrt{\dfrac{x - 1}{x^4 + 1}}$

46. $y = \sqrt{x}\, e^{x^2 - x}(x + 1)^{2/3}$

47. $y = x^x$

48. $y = x^{\cos x}$

49. $y = x^{\sin x}$

50. $y = \sqrt{x}^{\,x}$

51. $y = (\cos x)^x$

52. $y = (\sin x)^{\ln x}$

53. $y = (\tan x)^{1/x}$

54. $y = (\ln x)^{\cos x}$

55. Find y' if $y = \ln(x^2 + y^2)$.

56. Find y' if $x^y = y^x$.

57. Find a formula for $f^{(n)}(x)$ if $f(x) = \ln(x - 1)$.

58. Find $\dfrac{d^9}{dx^9}(x^8 \ln x)$.

59–60 Use a graph to estimate the roots of the equation correct to one decimal place. Then use these estimates as the initial approximations in Newton's method to find the roots correct to six decimal places.

59. $(x - 4)^2 = \ln x$

60. $\ln(4 - x^2) = x$

61. Find the intervals of concavity and the inflection points of the function $f(x) = (\ln x)/\sqrt{x}$.

62. Find the absolute minimum value of the function $f(x) = x \ln x$.

63–66 Discuss the curve under the guidelines of Section 3.5.

63. $y = \ln(\sin x)$

64. $y = \ln(\tan^2 x)$

65. $y = \ln(1 + x^2)$

66. $y = \ln(x^2 - 3x + 2)$

CAS 67. If $f(x) = \ln(2x + x \sin x)$, use the graphs of f, f', and f'' to estimate the intervals of increase and the inflection points of f on the interval $(0, 15]$.

68. Investigate the family of curves $f(x) = \ln(x^2 + c)$. What happens to the inflection points and asymptotes as c changes? Graph several members of the family to illustrate what you discover.

69. The flash unit on a camera operates by storing charge on a capacitor and releasing it suddenly when the flash is set off. The following data describe the charge Q remaining on the capacitor (measured in microcoulombs, μC) at time t (measured in seconds).

t	0.00	0.02	0.04	0.06	0.08	0.10
Q	100.00	81.87	67.03	54.88	44.93	36.76

(a) Use a graphing calculator or computer to find an exponential model for the charge.

(b) The derivative $Q'(t)$ represents the electric current (measured in microamperes, μA) flowing from the capacitor to the flash bulb. Use part (a) to estimate the current when $t = 0.04$ s. Compare with the result of Example 2 in Section 1.4.

70. The table gives the US population from 1790 to 1860.

Year	Population	Year	Population
1790	3,929,000	1830	12,861,000
1800	5,308,000	1840	17,063,000
1810	7,240,000	1850	23,192,000
1820	9,639,000	1860	31,443,000

(a) Use a graphing calculator or computer to fit an exponential function to the data. Graph the data points and the exponential model. How good is the fit?
(b) Estimate the rates of population growth in 1800 and 1850 by averaging slopes of secant lines.
(c) Use the exponential model in part (a) to estimate the rates of growth in 1800 and 1850. Compare these estimates with the ones in part (b).
(d) Use the exponential model to predict the population in 1870. Compare with the actual population of 38,558,000. Can you explain the discrepancy?

71–82 Evaluate the integral.

71. $\int_2^4 \frac{3}{x}\,dx$

72. $\int_0^3 \frac{dx}{5x+1}$

73. $\int_1^2 \frac{dt}{8-3t}$

74. $\int_4^9 \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$

75. $\int_1^e \frac{x^2+x+1}{x}\,dx$

76. $\int \frac{\sin(\ln x)}{x}\,dx$

77. $\int \frac{(\ln x)^2}{x}\,dx$

78. $\int \frac{\cos x}{2+\sin x}\,dx$

79. $\int \frac{\sin 2x}{1+\cos^2 x}\,dx$

80. $\int \frac{e^x}{e^x+1}\,dx$

81. $\int_1^2 10^t\,dt$

82. $\int x2^{x^2}\,dx$

83. Show that $\int \cot x\,dx = \ln|\sin x| + C$ by (a) differentiating the right side of the equation and (b) using the method of Example 11.

84. Find, correct to three decimal places, the area of the region above the hyperbola $y = 2/(x-2)$, below the x-axis, and between the lines $x = -4$ and $x = -1$.

85. Find the volume of the solid obtained by rotating the region under the curve

$$y = \frac{1}{\sqrt{x+1}}$$

from 0 to 1 about the x-axis.

86. Find the volume of the solid obtained by rotating the region under the curve

$$y = \frac{1}{x^2+1}$$

from 0 to 3 about the y-axis.

87. The work done by a gas when it expands from volume V_1 to volume V_2 is $W = \int_{V_1}^{V_2} P\,dV$, where $P = P(V)$ is the pressure as a function of the volume V. (See Exercise 27 in Section 5.4.) Boyle's Law states that when a quantity of gas expands at constant temperature, $PV = C$, where C is a constant. If the initial volume is 600 cm^3 and the initial pressure is 150 kPa, find the work done by the gas when it expands at constant temperature to 1000 cm^3.

88. Find f if $f''(x) = x^{-2}$, $x > 0$, $f(1) = 0$, and $f(2) = 0$.

89. If g is the inverse function of $f(x) = 2x + \ln x$, find $g'(2)$.

90. If $f(x) = e^x + \ln x$ and $h(x) = f^{-1}(x)$, find $h'(e)$.

91. For what values of m do the line $y = mx$ and the curve $y = x/(x^2+1)$ enclose a region? Find the area of the region.

92. (a) Find the linear approximation to $f(x) = \ln x$ near 1.
(b) Illustrate part (a) by graphing f and its linearization.
(c) For what values of x is the linear approximation accurate to within 0.1?

93. Use the definition of derivative to prove that

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$$

94. Show that $\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x$ for any $x > 0$.