6.1 Exercises

1. (a) What is a one-to-one function?
   (b) How can you tell from the graph of a function whether it is one-to-one?

2. (a) Suppose f is a one-to-one function with domain A and range B. How is the inverse function f^{-1} defined? What is the domain of f^{-1}? What is the range of f^{-1}?
   (b) If you are given a formula for f, how do you find a formula for f^{-1}?
   (c) If you are given the graph of f, how do you find the graph of f^{-1}?

3–16 A function is given by a table of values, a graph, a formula, or a verbal description. Determine whether it is one-to-one.

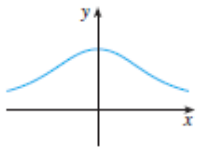
3.

x	1	2	3	4	5	6
$f(x)$	1.5	2.0	3.6	5.3	2.8	2.0

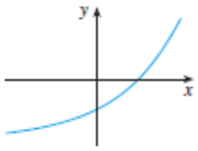
4.

x	1	2	3	4	5	6
$f(x)$	1.0	1.9	2.8	3.5	3.1	2.9

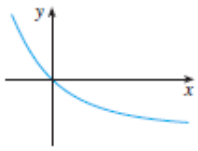
5.



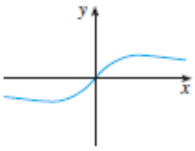
6.



7.



8.



9. $f(x) = x^2 - 2x$

10. $f(x) = 10 - 3x$

11. $g(x) = 1/x$

12. $g(x) = |x|$

13. $h(x) = 1 + \cos x$

14. $h(x) = 1 + \cos x, \quad 0 \leq x \leq \pi$

15. $f(t)$ is the height of a football t seconds after kickoff.

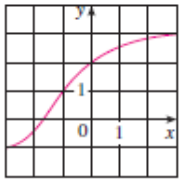
16. $f(t)$ is your height at age t.

17. Assume that f is a one-to-one function.
   (a) If $f(6) = 17$, what is $f^{-1}(17)$?
   (b) If $f^{-1}(3) = 2$, what is $f(2)$?

18. If $f(x) = x^5 + x^3 + x$, find $f^{-1}(3)$ and $f(f^{-1}(2))$.

19. If $h(x) = x + \sqrt{x}$, find $h^{-1}(6)$.

20. The graph of f is given.
   (a) Why is f one-to-one?
   (b) What are the domain and range of f^{-1}?
   (c) What is the value of $f^{-1}(2)$?
   (d) Estimate the value of $f^{-1}(0)$.



21. The formula $C = \frac{5}{9}(F - 32)$, where $F \geq -459.67$, expresses the Celsius temperature C as a function of the Fahrenheit temperature F. Find a formula for the inverse function and interpret it. What is the domain of the inverse function?

22. In the theory of relativity, the mass of a particle with speed v is

$$m = f(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is the rest mass of the particle and c is the speed of light in a vacuum. Find the inverse function of f and explain its meaning.

23–28 Find a formula for the inverse of the function.

23. $f(x) = 3 - 2x$

24. $f(x) = \dfrac{4x - 1}{2x + 3}$

25. $f(x) = 1 + \sqrt{2 + 3x}$

26. $y = x^2 - x, \quad x \geq \frac{1}{2}$

27. $y = \dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}$

28. $f(x) = 2x^2 - 8x, \quad x \geq 2$

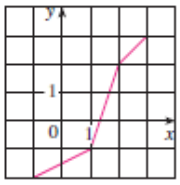
29–30 Find an explicit formula for f^{-1} and use it to graph f^{-1}, f, and the line $y = x$ on the same screen. To check your work, see whether the graphs of f and f^{-1} are reflections about the line.

29. $f(x) = x^4 + 1, \quad x \geq 0$

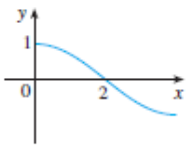
30. $f(x) = \sqrt{x^2 + 2x}, \quad x > 0$

31–32 Use the given graph of f to sketch the graph of f^{-1}.

31.



32.



33. Let $f(x) = \sqrt{1 - x^2}$, $0 \leq x \leq 1$.
(a) Find f^{-1}. How is it related to f?
(b) Identify the graph of f and explain your answer to part (a).

34. Let $g(x) = \sqrt[3]{1 - x^3}$.
(a) Find g^{-1}. How is it related to g?
(b) Graph g. How do you explain your answer to part (a)?

35–38
(a) Show that f is one-to-one.
(b) Use Theorem 7 to find $(f^{-1})'(a)$.
(c) Calculate $f^{-1}(x)$ and state the domain and range of f^{-1}.
(d) Calculate $(f^{-1})'(a)$ from the formula in part (c) and check that it agrees with the result of part (b).
(e) Sketch the graphs of f and f^{-1} on the same axes.

35. $f(x) = x^3, \quad a = 8$

36. $f(x) = \sqrt{x - 2}, \quad a = 2$

37. $f(x) = 9 - x^2, \quad 0 \leq x \leq 3, \quad a = 8$

38. $f(x) = 1/(x - 1), \quad x > 1, \quad a = 2$

39–42 Find $(f^{-1})'(a)$.

39. $f(x) = 2x^3 + 3x^2 + 7x + 4, \quad a = 4$

40. $f(x) = x^3 + 3 \sin x + 2 \cos x, \quad a = 2$

41. $f(x) = 3 + x^2 + \tan(\pi x/2), \quad -1 < x < 1, \quad a = 3$

42. $f(x) = \sqrt{x^3 + x^2 + x + 1}, \quad a = 2$

43. Suppose f^{-1} is the inverse function of a differentiable function f and $f(4) = 5$, $f'(4) = \frac{2}{3}$. Find $(f^{-1})'(5)$.

44. If g is an increasing function such that $g(2) = 8$ and $g'(2) = 5$, calculate $(g^{-1})'(8)$.

45. If $f(x) = \int_3^x \sqrt{1 + t^3}\, dt$, find $(f^{-1})'(0)$.

46. Suppose f^{-1} is the inverse function of a differentiable function f and let $G(x) = 1/f^{-1}(x)$. If $f(3) = 2$ and $f'(3) = \frac{1}{9}$, find $G'(2)$.

47. Graph the function $f(x) = \sqrt{x^3 + x^2 + x + 1}$ and explain why it is one-to-one. Then use a computer algebra system to find an explicit expression for $f^{-1}(x)$. (Your CAS will produce three possible expressions. Explain why two of them are irrelevant in this context.)

48. Show that $h(x) = \sin x$, $x \in \mathbb{R}$, is not one-to-one, but its restriction $f(x) = \sin x$, $-\pi/2 \leq x \leq \pi/2$, is one-to-one. Compute the derivative of $f^{-1} = \sin^{-1}$ by the method of Note 2.

49. (a) If we shift a curve to the left, what happens to its reflection about the line $y = x$? In view of this geometric principle, find an expression for the inverse of $g(x) = f(x + c)$, where f is a one-to-one function.
(b) Find an expression for the inverse of $h(x) = f(cx)$, where $c \neq 0$.

50. (a) If f is a one-to-one, twice differentiable function with inverse function g, show that

$$g''(x) = -\frac{f''(g(x))}{[f'(g(x))]^3}$$

(b) Deduce that if f is increasing and concave upward, then its inverse function is concave downward.