REMARKS

(*i*) In Problems 27–36 in Exercises 4.4 you are asked to solve initial-value problems, and in Problems 37–40 you are asked to solve boundary-value problems. As illustrated in Example 8, be sure to apply the initial conditions or the boundary conditions to the general solution $y = y_c + y_p$. Students often make the mistake of applying these conditions only to the complementary function y_c because it is that part of the solution that contains the constants $c_1, c_2, \ldots, c_n$.

(*ii*) From the "Form Rule for Case I" on page 144 of this section you see why the method of undetermined coefficients is not well suited to nonhomogeneous linear DEs when the input function $g(x)$ is something other than one of the four basic types highlighted in color on page 140. For example, if $P(x)$ is a polynomial, then continued differentiation of $P(x)e^{\alpha x}\sin\beta x$ will generate an independent set containing only a *finit* number of functions—all of the same type, namely, a polynomial times $e^{\alpha x}\sin\beta x$ or a polynomial times $e^{\alpha x}\cos\beta x$. On the other hand, repeated differentiation of input functions such as $g(x) = \ln x$ or $g(x) = \tan^{-1}x$ generates an independent set containing an *infinit* number of functions:

$$\textit{derivatives of } \ln x: \quad \frac{1}{x}, \frac{-1}{x^2}, \frac{2}{x^3}, \ldots,$$

$$\textit{derivatives of } \tan^{-1}x: \quad \frac{1}{1+x^2}, \frac{-2x}{(1+x^2)^2}, \frac{-2+6x^2}{(1+x^2)^3}, \ldots$$

EXERCISES 4.4

Answers to selected odd-numbered problems begin on page ANS-5.

In Problems 1–26 solve the given differential equation by undetermined coefficients

1. $y'' + 3y' + 2y = 6$
2. $4y'' + 9y = 15$
3. $y'' - 10y' + 25y = 30x + 3$
4. $y'' + y' - 6y = 2x$
5. $\frac{1}{4}y'' + y' + y = x^2 - 2x$
6. $y'' - 8y' + 20y = 100x^2 - 26xe^x$
7. $y'' + 3y = -48x^2e^{3x}$
8. $4y'' - 4y' - 3y = \cos 2x$
9. $y'' - y' = -3$
10. $y'' + 2y' = 2x + 5 - e^{-2x}$
11. $y'' - y' + \frac{1}{4}y = 3 + e^{x/2}$
12. $y'' - 16y = 2e^{4x}$
13. $y'' + 4y = 3\sin 2x$
14. $y'' - 4y = (x^2 - 3)\sin 2x$
15. $y'' + y = 2x\sin x$
16. $y'' - 5y' = 2x^3 - 4x^2 - x + 6$
17. $y'' - 2y' + 5y = e^x\cos 2x$
18. $y'' - 2y' + 2y = e^{2x}(\cos x - 3\sin x)$
19. $y'' + 2y' + y = \sin x + 3\cos 2x$
20. $y'' + 2y' - 24y = 16 - (x + 2)e^{4x}$
21. $y''' - 6y'' = 3 - \cos x$
22. $y''' - 2y'' - 4y' + 8y = 6xe^{2x}$
23. $y''' - 3y'' + 3y' - y = x - 4e^x$
24. $y''' - y'' - 4y' + 4y = 5 - e^x + e^{2x}$
25. $y^{(4)} + 2y'' + y = (x - 1)^2$
26. $y^{(4)} - y'' = 4x + 2xe^{-x}$

In Problems 27–36 solve the given initial-value problem.

27. $y'' + 4y = -2, \quad y(\pi/8) = \frac{1}{2}, y'(\pi/8) = 2$
28. $2y'' + 3y' - 2y = 14x^2 - 4x - 11, \quad y(0) = 0, y'(0) = 0$
29. $5y'' + y' = -6x, \quad y(0) = 0, y'(0) = -10$
30. $y'' + 4y' + 4y = (3 + x)e^{-2x}, \quad y(0) = 2, y'(0) = 5$
31. $y'' + 4y' + 5y = 35e^{-4x}, \quad y(0) = -3, y'(0) = 1$

32. $y'' - y = \cosh x, \quad y(0) = 2, y'(0) = 12$

33. $\dfrac{d^2x}{dt^2} + \omega^2 x = F_0 \sin \omega t, \quad x(0) = 0, x'(0) = 0$

34. $\dfrac{d^2x}{dt^2} + \omega^2 x = F_0 \cos \quad t, \quad x(0) = 0, x'(0) = 0$

35. $y''' - 2y'' + y' = 2 - 24e^x + 40e^{5x}, \quad y(0) = \frac{1}{2},$
$y'(0) = \frac{5}{2}, y''(0) = -\frac{9}{2}$

36. $y''' + 8y = 2x - 5 + 8e^{-2x}, \quad y(0) = -5, y'(0) = 3,$
$y''(0) = -4$

In Problems 37–40 solve the given boundary-value problem.

37. $y'' + y = x^2 + 1, \quad y(0) = 5, y(1) = 0$

38. $y'' - 2y' + 2y = 2x - 2, \quad y(0) = 0, y(\pi) = \pi$

39. $y'' + 3y = 6x, \quad y(0) = 0, y(1) + y'(1) = 0$

40. $y'' + 3y = 6x, \quad y(0) + y'(0) = 0, y(1) = 0$

In Problems 41 and 42 solve the given initial-value problem in which the input function $g(x)$ is discontinuous. [*Hint*: Solve each problem on two intervals, and then find a solution so that y and y' are continuous at $x = \pi/2$ (Problem 41) and at $x = \pi$ (Problem 42).]

41. $y'' + 4y = g(x), \quad y(0) = 1, y'(0) = 2,$ where

$$g(x) = \begin{cases} \sin x, & 0 \le x \le \pi/2 \\ 0, & x > \pi/2 \end{cases}$$

42. $y'' - 2y' + 10y = g(x), \quad y(0) = 0, y'(0) = 0,$ where

$$g(x) = \begin{cases} 20, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$$

Discussion Problems

43. Consider the differential equation $ay'' + by' + cy = e^{kx}$, where a, b, c, and k are constants. The auxiliary equation of the associated homogeneous equation is $am^2 + bm + c = 0$.
   (a) If k is not a root of the auxiliary equation, show that we can find a particular solution of the form $y_p = Ae^{kx}$, where $A = 1/(ak^2 + bk + c)$.
   (b) If k is a root of the auxiliary equation of multiplicity one, show that we can find a particular solution of the form $y_p = Axe^{kx}$, where $A = 1/(2ak + b)$. Explain how we know that $k \neq -b/(2a)$.
   (c) If k is a root of the auxiliary equation of multiplicity two, show that we can find a particular solution of the form $y = Ax^2e^{kx}$, where $A = 1/(2a)$.

44. Discuss how the method of this section can be used to find a particular solution of $y'' + y = \sin x \cos 2x$. Carry out your idea.

45. Without solving, match a solution curve of $y'' + y = f(x)$ shown in the figure with one of the following functions:
   (*i*) $f(x) = 1$, (*ii*) $f(x) = e^{-x}$,
   (*iii*) $f(x) = e^x$, (*iv*) $f(x) = \sin 2x$,
   (*v*) $f(x) = e^x \sin x$, (*vi*) $f(x) = \sin x$.
   Briefly discuss your reasoning

(a)

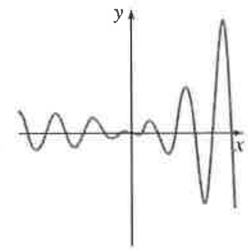

FIGURE 4.4.1 Solution curve

(b)

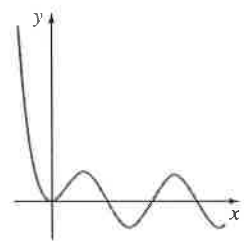

FIGURE 4.4.2 Solution curve

(c)

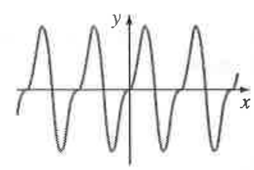

FIGURE 4.4.3 Solution curve

(d)

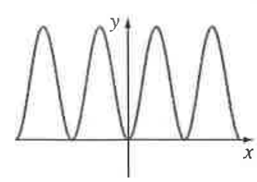

FIGURE 4.4.4 Solution curve

Computer Lab Assignments

In Problems 46 and 47 find a particular solution of the given differential equation. Use a CAS as an aid in carrying out differentiations, simplifications, and algebra

46. $y'' - 4y' + 8y = (2x^2 - 3x)e^{2x} \cos 2x + (10x^2 - x - 1)e^{2x} \sin 2x$

47. $y^{(4)} + 2y'' + y = 2 \cos x - 3x \sin x$