16.5 Exercises

1–8 Find (a) the curl and (b) the divergence of the vector field.

1. $\mathbf{F}(x, y, z) = (x + yz)\,\mathbf{i} + (y + xz)\,\mathbf{j} + (z + xy)\,\mathbf{k}$

2. $\mathbf{F}(x, y, z) = xy^2z^3\,\mathbf{i} + x^3yz^2\,\mathbf{j} + x^2y^3z\,\mathbf{k}$

3. $\mathbf{F}(x, y, z) = xye^z\,\mathbf{i} + yze^x\,\mathbf{k}$

4. $\mathbf{F}(x, y, z) = \sin yz\,\mathbf{i} + \sin zx\,\mathbf{j} + \sin xy\,\mathbf{k}$

5. $\mathbf{F}(x, y, z) = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}}(x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k})$

6. $\mathbf{F}(x, y, z) = e^{xy}\sin z\,\mathbf{j} + y\tan^{-1}(x/z)\,\mathbf{k}$

7. $\mathbf{F}(x, y, z) = \langle e^x \sin y,\ e^y \sin z,\ e^z \sin x\rangle$

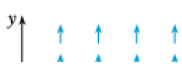
8. $\mathbf{F}(x, y, z) = \left\langle \dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x} \right\rangle$

9–11 The vector field **F** is shown in the xy-plane and looks the same in all other horizontal planes. (In other words, **F** is independent of z and its z-component is 0.)

(a) Is div **F** positive, negative, or zero? Explain.

(b) Determine whether curl $\mathbf{F} = \mathbf{0}$. If not, in which direction does curl **F** point?

9.



10.

12. Let f be a scalar field and **F** a vector field. State whether each expression is meaningful. If not, explain why. If so, state whether it is a scalar field or a vector field.

(a) curl f
(b) grad f
(c) div **F**
(d) curl(grad f)
(e) grad **F**
(f) grad(div **F**)
(g) div(grad f)
(h) grad(div f)
(i) curl(curl **F**)
(j) div(div **F**)
(k) (grad f) × (div **F**)
(l) div(curl(grad f))

13–18 Determine whether or not the vector field is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

13. $\mathbf{F}(x, y, z) = y^2z^3\,\mathbf{i} + 2xyz^3\,\mathbf{j} + 3xy^2z^2\,\mathbf{k}$

14. $\mathbf{F}(x, y, z) = xyz^2\,\mathbf{i} + x^2yz^2\,\mathbf{j} + x^2y^2z\,\mathbf{k}$

15. $\mathbf{F}(x, y, z) = 3xy^2z^2\,\mathbf{i} + 2x^2yz^3\,\mathbf{j} + 3x^2y^2z^2\,\mathbf{k}$

16. $\mathbf{F}(x, y, z) = \mathbf{i} + \sin z\,\mathbf{j} + y\cos z\,\mathbf{k}$

17. $\mathbf{F}(x, y, z) = e^{yz}\,\mathbf{i} + xze^{yz}\,\mathbf{j} + xye^{yz}\,\mathbf{k}$

18. $\mathbf{F}(x, y, z) = e^x \sin yz\,\mathbf{i} + ze^x \cos yz\,\mathbf{j} + ye^x \cos yz\,\mathbf{k}$

38. Maxwell's equations relating the electric field **E** and magnetic field **H** as they vary with time in a region containing no charge and no current can be stated as follows:

$$\text{div}\,\mathbf{E} = 0 \qquad \text{div}\,\mathbf{H} = 0$$

$$\text{curl}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} \qquad \text{curl}\,\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

where c is the speed of light. Use these equations to prove the following:

(a) $\nabla \times (\nabla \times \mathbf{E}) = -\dfrac{1}{c^2}\dfrac{\partial^2 \mathbf{E}}{\partial t^2}$

(b) $\nabla \times (\nabla \times \mathbf{H}) = -\dfrac{1}{c^2}\dfrac{\partial^2 \mathbf{H}}{\partial t^2}$

(c) $\nabla^2 \mathbf{E} = \dfrac{1}{c^2}\dfrac{\partial^2 \mathbf{E}}{\partial t^2}$ [*Hint:* Use Exercise 29.]

(d) $\nabla^2 \mathbf{H} = \dfrac{1}{c^2}\dfrac{\partial^2 \mathbf{H}}{\partial t^2}$

29. $\text{curl}(\text{curl}\,\mathbf{F}) = \text{grad}(\text{div}\,\mathbf{F}) - \nabla^2\mathbf{F}$