

15.7 Exercises

1. Evaluate the integral in Example 1, integrating first with respect to y , then z , and then x .

2. Evaluate the integral $\iiint_E (xy + z^2) dV$, where

$$E = \{(x, y, z) \mid 0 \leq x \leq 2, 0 \leq y \leq 1, 0 \leq z \leq 3\}$$

using three different orders of integration.

3–8 Evaluate the iterated integral.

3. $\int_0^2 \int_0^{2x} \int_0^{y-x} (2x - y) dx dy dz$ 4. $\int_0^1 \int_x^{2x} \int_0^y 2xyz dz dy dx$

5. $\int_1^2 \int_0^{2x} \int_0^{\ln x} x e^{-y} dy dx dz$ 6. $\int_0^1 \int_0^1 \int_0^{\sqrt{1-z^2}} \frac{z}{y+1} dx dz dy$

7. $\int_0^{\pi/2} \int_0^y \int_0^x \cos(x+y+z) dz dx dy$

8. $\int_0^{\sqrt{\pi}} \int_0^x \int_0^{xz} x^2 \sin y dy dz dx$

9–18 Evaluate the triple integral.

9. $\iiint_E y dV$, where

$$E = \{(x, y, z) \mid 0 \leq x \leq 3, 0 \leq y \leq x, x - y \leq z \leq x + y\}$$

10. $\iiint_E e^{z/y} dV$, where

$$E = \{(x, y, z) \mid 0 \leq y \leq 1, y \leq x \leq 1, 0 \leq z \leq xy\}$$

11. $\iiint_E \frac{z}{x^2 + z^2} dV$, where

$$E = \{(x, y, z) \mid 1 \leq y \leq 4, y \leq z \leq 4, 0 \leq x \leq z\}$$

12. $\iiint_E \sin y dV$, where E lies below the plane $z = x$ and above the triangular region with vertices $(0, 0, 0)$, $(\pi, 0, 0)$, and $(0, \pi, 0)$

13. $\iiint_E 6xy dV$, where E lies under the plane $z = 1 + x + y$ and above the region in the xy -plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$

14. $\iiint_E xy dV$, where E is bounded by the parabolic cylinders $y = x^2$ and $x = y^2$ and the planes $z = 0$ and $z = x + y$

15. $\iiint_T x^2 dV$, where T is the solid tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$

16. $\iiint_T xyz dV$, where T is the solid tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$, and $(1, 0, 1)$

17. $\iiint_E x dV$, where E is bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$

18. $\iiint_E z dV$, where E is bounded by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$, $y = 3x$, and $z = 0$ in the first octant

19–22 Use a triple integral to find the volume of the given solid.

19. The tetrahedron enclosed by the coordinate planes and the plane $2x + y + z = 4$

20. The solid enclosed by the paraboloids $y = x^2 + z^2$ and $y = 8 - x^2 - z^2$

21. The solid enclosed by the cylinder $y = x^2$ and the planes $z = 0$ and $y + z = 1$

22. The solid enclosed by the cylinder $x^2 + z^2 = 4$ and the planes $y = -1$ and $y + z = 4$

23. (a) Express the volume of the wedge in the first octant that is cut from the cylinder $y^2 + z^2 = 1$ by the planes $y = x$ and $x = 1$ as a triple integral.

CAS (b) Use either the Table of Integrals (on Reference Pages 6–10) or a computer algebra system to find the exact value of the triple integral in part (a).

24. (a) In the **Midpoint Rule for triple integrals** we use a triple Riemann sum to approximate a triple integral over a box B , where $f(x, y, z)$ is evaluated at the center $(\bar{x}_i, \bar{y}_j, \bar{z}_k)$ of the box B_{ijk} . Use the Midpoint Rule to estimate $\iiint_B \sqrt{x^2 + y^2 + z^2} dV$, where B is the cube defined by $0 \leq x \leq 4$, $0 \leq y \leq 4$, $0 \leq z \leq 4$. Divide B into eight cubes of equal size.

CAS (b) Use a computer algebra system to approximate the integral in part (a) correct to the nearest integer. Compare with the answer to part (a).

25–26 Use the Midpoint Rule for triple integrals (Exercise 24) to estimate the value of the integral. Divide B into eight sub-boxes of equal size.

25. $\iiint_B \cos(xyz) dV$, where

$$B = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1\}$$

26. $\iiint_B \sqrt{x} e^{xyz} dV$, where

$$B = \{(x, y, z) \mid 0 \leq x \leq 4, 0 \leq y \leq 1, 0 \leq z \leq 2\}$$

27–28 Sketch the solid whose volume is given by the iterated integral.

27. $\int_0^1 \int_0^{1-x} \int_0^{2-2x} dy dz dx$ 28. $\int_0^2 \int_0^{2-y} \int_0^{4-y^2} dx dz dy$

29–32 Express the integral $\iiint_E f(x, y, z) dV$ as an iterated integral in six different ways, where E is the solid bounded by the given surfaces.

29. $y = 4 - x^2 - 4z^2$, $y = 0$

15.8 Exercises

1–2 Plot the point whose cylindrical coordinates are given. Then find the rectangular coordinates of the point.

1. (a) $(4, \pi/3, -2)$ (b) $(2, -\pi/2, 1)$
 2. (a) $(\sqrt{2}, 3\pi/4, 2)$ (b) $(1, 1, 1)$

3–4 Change from rectangular to cylindrical coordinates.

3. (a) $(-1, 1, 1)$ (b) $(-2, 2\sqrt{3}, 3)$
 4. (a) $(2\sqrt{3}, 2, -1)$ (b) $(4, -3, 2)$

5–6 Describe in words the surface whose equation is given.

5. $\theta = \pi/4$ 6. $r = 5$

7–8 Identify the surface whose equation is given.

7. $z = 4 - r^2$ 8. $2r^2 + z^2 = 1$

9–10 Write the equations in cylindrical coordinates.

9. (a) $x^2 - x + y^2 + z^2 = 1$ (b) $z = x^2 - y^2$
 10. (a) $3x + 2y + z = 6$ (b) $-x^2 - y^2 + z^2 = 1$

11–12 Sketch the solid described by the given inequalities.

11. $0 \leq r \leq 2, -\pi/2 \leq \theta \leq \pi/2, 0 \leq z \leq 1$
 12. $0 \leq \theta \leq \pi/2, r \leq z \leq 2$

18. Evaluate $\iiint_E z \, dV$, where E is enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

19. Evaluate $\iiint_E (x + y + z) \, dV$, where E is the solid in the first octant that lies under the paraboloid $z = 4 - x^2 - y^2$.

20. Evaluate $\iiint_E x \, dV$, where E is enclosed by the planes $z = 0$ and $z = x + y + 5$ and by the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

21. Evaluate $\iiint_E x^2 \, dV$, where E is the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$, and below the cone $z^2 = 4x^2 + 4y^2$.

22. Find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.

23. Find the volume of the solid that is enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 2$.

24. Find the volume of the solid that lies between the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 2$.

25. (a) Find the volume of the region E bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.
 (b) Find the centroid of E (the center of mass in the case where the density is constant).

26. (a) Find the volume of the solid that the cylinder $r = a \cos \theta$ cuts out of the sphere of radius a centered at the origin.
 (b) Illustrate the solid of part (a) by graphing the sphere and the cylinder on the same screen.

