

## 15.2 Exercises

1–2 Find  $\int_0^2 f(x, y) dx$  and  $\int_0^1 f(x, y) dy$ .

1.  $f(x, y) = 12x^2y^3$                       2.  $f(x, y) = y + xe^y$

3–14 Calculate the iterated integral.

3.  $\int_1^4 \int_0^2 (6x^2y - 2x) dy dx$       4.  $\int_0^1 \int_1^2 (4x^3 - 9x^2y^2) dy dx$

5.  $\int_0^2 \int_0^4 y^3 e^{2x} dy dx$               6.  $\int_{\pi/6}^{\pi/2} \int_{-1}^5 \cos y dx dy$

7.  $\int_{-3}^3 \int_0^{\pi/2} (y + y^2 \cos x) dx dy$     8.  $\int_1^3 \int_1^5 \frac{\ln y}{xy} dy dx$

9.  $\int_1^4 \int_1^2 \left( \frac{x}{y} + \frac{y}{x} \right) dy dx$       10.  $\int_0^1 \int_0^3 e^{x+3y} dx dy$

11.  $\int_0^1 \int_0^1 v(u + v^2)^4 du dv$       12.  $\int_0^1 \int_0^1 xy\sqrt{x^2 + y^2} dy dx$

13.  $\int_0^2 \int_0^{\pi} r \sin^2 \theta d\theta dr$           14.  $\int_0^1 \int_0^1 \sqrt{s+t} ds dt$

15–22 Calculate the double integral.

15.  $\iint_R \sin(x - y) dA$ ,  $R = \{(x, y) \mid 0 \leq x \leq \pi/2, 0 \leq y \leq \pi/2\}$

16.  $\iint_R (y + xy^{-2}) dA$ ,  $R = \{(x, y) \mid 0 \leq x \leq 2, 1 \leq y \leq 2\}$

17.  $\iint_R \frac{xy^2}{x^2 + 1} dA$ ,  $R = \{(x, y) \mid 0 \leq x \leq 1, -3 \leq y \leq 3\}$

18.  $\iint_R \frac{1 + x^2}{1 + y^2} dA$ ,  $R = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}$

19.  $\iint_R x \sin(x + y) dA$ ,  $R = [0, \pi/6] \times [0, \pi/3]$

20.  $\iint_R \frac{x}{1 + xy} dA$ ,  $R = [0, 1] \times [0, 1]$

21.  $\iint_R ye^{-xy} dA$ ,  $R = [0, 2] \times [0, 3]$

22.  $\iint_R \frac{1}{1 + x + y} dA$ ,  $R = [1, 3] \times [1, 2]$





23–24 Sketch the solid whose volume is given by the iterated integral.

23.  $\int_0^1 \int_0^1 (4 - x - 2y) dx dy$

24.  $\int_0^1 \int_0^1 (2 - x^2 - y^2) dy dx$

25. Find the volume of the solid that lies under the plane  $4x + 6y - 2z + 15 = 0$  and above the rectangle  $R = \{(x, y) \mid -1 \leq x \leq 2, -1 \leq y \leq 1\}$ .

26. Find the volume of the solid that lies under the hyperbolic paraboloid  $z = 3y^2 - x^2 + 2$  and above the rectangle  $R = [-1, 1] \times [1, 2]$ .

27. Find the volume of the solid lying under the elliptic paraboloid  $x^2/4 + y^2/9 + z = 1$  and above the rectangle  $R = [-1, 1] \times [-2, 2]$ .
28. Find the volume of the solid enclosed by the surface  $z = 1 + e^x \sin y$  and the planes  $x = \pm 1, y = 0, y = \pi$ , and  $z = 0$ .
29. Find the volume of the solid enclosed by the surface  $z = x \sec^2 y$  and the planes  $z = 0, x = 0, x = 2, y = 0$ , and  $y = \pi/4$ .
30. Find the volume of the solid in the first octant bounded by the cylinder  $z = 16 - x^2$  and the plane  $y = 5$ .
31. Find the volume of the solid enclosed by the paraboloid  $z = 2 + x^2 + (y - 2)^2$  and the planes  $z = 1, x = 1, x = -1, y = 0$ , and  $y = 4$ .
-  32. Graph the solid that lies between the surface  $z = 2xy/(x^2 + 1)$  and the plane  $z = x + 2y$  and is bounded by the planes  $x = 0, x = 2, y = 0$ , and  $y = 4$ . Then find its volume.
-  33. Use a computer algebra system to find the exact value of the integral  $\iint_R x^2 y^3 e^{xy} dA$ , where  $R = [0, 1] \times [0, 1]$ . Then use the CAS to draw the solid whose volume is given by the integral.
-  34. Graph the solid that lies between the surfaces  $z = e^{-x^2} \cos(x^2 + y^2)$  and  $z = 2 - x^2 - y^2$  for  $|x| \leq 1, |y| \leq 1$ . Use a computer algebra system to approximate the volume of this solid correct to four decimal places.
- 35–36 Find the average value of  $f$  over the given rectangle.
35.  $f(x, y) = x^2 y$ ,  $R$  has vertices  $(-1, 0), (-1, 5), (1, 5), (1, 0)$
36.  $f(x, y) = e^y \sqrt{x + e^y}$ ,  $R = [0, 4] \times [0, 1]$
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- 37–38 Use symmetry to evaluate the double integral.
37.  $\iint_R \frac{xy}{1 + x^4} dA$ ,  $R = \{(x, y) \mid -1 \leq x \leq 1, 0 \leq y \leq 1\}$
38.  $\iint_R (1 + x^2 \sin y + y^2 \sin x) dA$ ,  $R = [-\pi, \pi] \times [-\pi, \pi]$
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-  39. Use your CAS to compute the iterated integrals
- $$\int_0^1 \int_0^1 \frac{x - y}{(x + y)^3} dy dx \quad \text{and} \quad \int_0^1 \int_0^1 \frac{x - y}{(x + y)^3} dx dy$$
- Do the answers contradict Fubini's Theorem? Explain what is happening.
40. (a) In what way are the theorems of Fubini and Clairaut similar?  
 (b) If  $f(x, y)$  is continuous on  $[a, b] \times [c, d]$  and
- $$g(x, y) = \int_a^x \int_c^y f(s, t) dt ds$$
- for  $a < x < b, c < y < d$ , show that  $g_{xy} = g_{yx} = f(x, y)$ .