12.1 Exercises

1. Suppose you start at the origin, move along the x-axis a distance of 4 units in the positive direction, and then move downward a distance of 3 units. What are the coordinates of your position?

2. Sketch the points $(0, 5, 2)$, $(4, 0, -1)$, $(2, 4, 6)$, and $(1, -1, 2)$ on a single set of coordinate axes.

3. Which of the points $A(-4, 0, -1)$, $B(3, 1, -5)$, and $C(2, 4, 6)$ is closest to the yz-plane? Which point lies in the xz-plane?

4. What are the projections of the point $(2, 3, 5)$ on the xy-, yz-, and xz-planes? Draw a rectangular box with the origin and $(2, 3, 5)$ as opposite vertices and with its faces parallel to the coordinate planes. Label all vertices of the box. Find the length of the diagonal of the box.

5. Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $x + y = 2$.

6. (a) What does the equation $x = 4$ represent in $\mathbb{R}^2$? What does it represent in $\mathbb{R}^3$? Illustrate with sketches.
   (b) What does the equation $y = 3$ represent in $\mathbb{R}^3$? What does $z = 5$ represent? What does the pair of equations $y = 3$, $z = 5$ represent? In other words, describe the set of points (x, y, z) such that $y = 3$ and $z = 5$. Illustrate with a sketch.

7–8 Find the lengths of the sides of the triangle PQR. Is it a right triangle? Is it an isosceles triangle?

7. $P(3, -2, -3)$, $Q(7, 0, 1)$, $R(1, 2, 1)$

8. $P(2, -1, 0)$, $Q(4, 1, 1)$, $R(4, -5, 4)$

9. Determine whether the points lie on straight line.
   (a) $A(2, 4, 2)$, $B(3, 7, -2)$, $C(1, 3, 3)$
   (b) $D(0, -5, 5)$, $E(1, -2, 4)$, $F(3, 4, 2)$

10. Find the distance from $(4, -2, 6)$ to each of the following.
    (a) The xy-plane
    (b) The yz-plane
    (c) The xz-plane
    (d) The x-axis
    (e) The y-axis
    (f) The z-axis

11. Find an equation of the sphere with center $(-3, 2, 5)$ and radius 4. What is the intersection of this sphere with the yz-plane?

12. Find an equation of the sphere with center $(2, -6, 4)$ and radius 5. Describe its intersection with each of the coordinate planes.

13. Find an equation of the sphere that passes through the point $(4, 3, -1)$ and has center $(3, 8, 1)$.

14. Find an equation of the sphere that passes through the origin and whose center is $(1, 2, 3)$.

15–18 Show that the equation represents a sphere, and find its center and radius.

15. $x^2 + y^2 + z^2 - 2x - 4y + 8z = 15$

16. $x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$

17. $2x^2 + 2y^2 + 2z^2 = 8x - 24z + 1$

18. $3x^2 + 3y^2 + 3z^2 = 10 + 6y + 12z$

19. (a) Prove that the midpoint of the line segment from $P_1(x_1, y_1, z_1)$ to $P_2(x_2, y_2, z_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

(b) Find the lengths of the medians of the triangle with vertices $A(1, 2, 3)$, $B(-2, 0, 5)$, and $C(4, 1, 5)$.

20. Find an equation of a sphere if one of its diameters has endpoints $(2, 1, 4)$ and $(4, 3, 10)$.

21. Find equations of the spheres with center $(2, -3, 6)$ that touch (a) the xy-plane, (b) the yz-plane, (c) the xz-plane.

22. Find an equation of the largest sphere with center $(5, 4, 9)$ that is contained in the first octant.

23–34 Describe in words the region of $\mathbb{R}^3$ represented by the equations or inequalities.

23. $x = 5$

24. $y = -2$

25. $y < 8$

26. $x \geqslant -3$

27. $0 \leqslant z \leqslant 6$

28. $z^2 = 1$

29. $x^2 + y^2 = 4, \quad z = -1$

30. $y^2 + z^2 = 16$

31. $x^2 + y^2 + z^2 \leqslant 3$

32. $x = z$

33. $x^2 + z^2 \leqslant 9$

34. $x^2 + y^2 + z^2 > 2z$

35–38 Write inequalities to describe the region.

35. The region between the yz-plane and the vertical plane $x = 5$

36. The solid cylinder that lies on or below the plane $z = 8$ and on or above the disk in the xy-plane with center the origin and radius 2

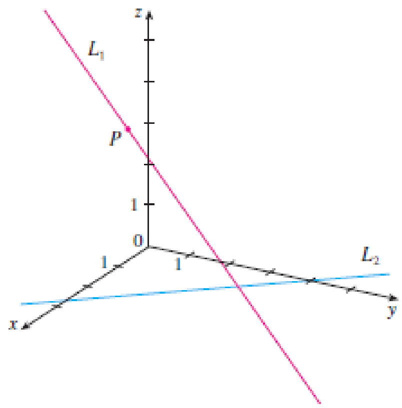
37. The region consisting of all points between (but not on) the spheres of radius r and R centered at the origin, where $r < R$

38. The solid upper hemisphere of the sphere of radius 2 centered at the origin

39. The figure shows a line L_1 in space and a second line L_2, which is the projection of L_1 on the xy-plane. (In other words, the points on L_2 are directly beneath, or above, the points on L_1.)

(a) Find the coordinates of the point P on the line L_1.

(b) Locate on the diagram the points A, B, and C, where the line L_1 intersects the xy-plane, the yz-plane, and the xz-plane, respectively.



40. Consider the points P such that the distance from P to $A(-1, 5, 3)$ is twice the distance from P to $B(6, 2, -2)$. Show that the set of all such points is a sphere, and find its center and radius.

41. Find an equation of the set of all points equidistant from the points $A(-1, 5, 3)$ and $B(6, 2, -2)$. Describe the set.

42. Find the volume of the solid that lies inside both of the spheres

$$x^2 + y^2 + z^2 + 4x - 2y + 4z + 5 = 0$$

and $$x^2 + y^2 + z^2 = 4$$

43. Find the distance between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 4x + 4y + 4z - 11$.

44. Describe and sketch a solid with the following properties. When illuminated by rays parallel to the z-axis, its shadow is a circular disk. If the rays are parallel to the y-axis, its shadow is a square. If the rays are parallel to the x-axis, its shadow is an isosceles triangle.